MAT 121 Final Test
200 Points Name (5 pts) _____

1. Find the domain of the following:

   a. (10 pts) $f(x) = \dfrac{x^2 + 5x + 17}{x^2 - 5x - 14}$

   b. (10 pts) $f(x) = \sqrt{x^2 - 5x - 14}$

   c. (10 pts) $f(x) = \ln\left(x^2 - 5x - 14\right)$

2. (10 pts) Graph $g(x) = -(x + 5)^2 + 3$ using the techniques of shifting and reflecting. Start with the graph of the basic function and show all stages. In the final graph, indicate (label as ordered pairs) the *x*- and *y*-intercepts.

3. Let $f(x) = x^2 - 6x + 8$.

   a. (10 pts) Find the zeros of f by factoring.

   b. (10 pts) Find the zeros of f by quadratic formula.

   c. (10 pts) Find the zeros of f by completing the square.

4. Let $f(x) = (x + 2)^2(x - 3)(x - 1)^3$.

a. (5 pts) List each real zero and its multiplicity. Determine whether the graph of $f(x)$ touches or crosses the *x*-axis at each *x*-intercept.

b. (5 pts) Determine the power function that $f(x)$ resembles for large $|x|$. This is the End Behavior part of the question.

c. (5 pts) Use the information you reported to obtain a rough graph of $f(x)$. Show all intercepts, including the *y*-intercept.

5. (10 pts) Form a polynomial with real coefficients that has the given zeros and has degree 6. Please do not expand the polynomial.

Zeros: -4, multiplicity 1; 2, multiplicity 3; $3 + 2i$, multiplicity 1.

6. (10 pts) Let $f(x) = x^4 - 11x^3 + 42x^2 - 14x - 68$. Use synthetic division to determine $f(1)$.

7. (10 pts) Evaluate $\log_2(96) - \log_2(3)$ *without a calculator* !! (This is one that I mistyped on Test 4. But now it should work the way it ought to have worked.)

8. (10 pts) The half-life of carbon-14 is (approximately) 5500 years. (I think it's 5600 years in the textbook, but let's roll with 5500.) Use this half-life to obtain an exponential decay function

$$A(t) = A_0 e^{-kt}.$$

Find k symbolically (in terms of natural logarithm). This answer will be exact. Then estimate k to the 6th decimal place.

Exact: $k =$ Approximate: $k \approx$

Population model (using approximate k): $A(t) \approx$

9. Find the geometric sums:

a. (5 pts) $\sum_{k=1}^{50} 3(1.2)^{k-1}$

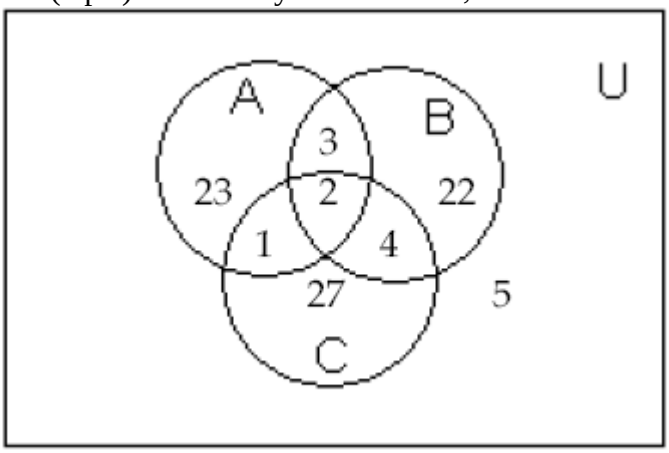
b. (5 pts) $\sum_{k=1}^{\infty} \frac{3}{4}\left(\frac{2}{5}\right)^{k-1}$

10. (10 pts) Solve the system $\begin{aligned} x + y &= 7 \\ 3x - 2y &= 8 \end{aligned}$ by substitution.

11. (10 pts) Solve the system $\begin{aligned} x + y &= 7 \\ 3x - 2y &= 8 \end{aligned}$ by elimination. (Matrices are fine.)

12. (5 pts) If $n(A) = 10, n(B) = 15,$ and $n(A \cup B) = 20$, what is $n(A \cap B)$?

13. (5 pts) How many are in B or C, but not in A?



Answer: _____

14. (10 pts) Graph the system of inequalities:

$$\begin{aligned} x + y &= 7 \\ 5x - 2y &\le 20 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

15. Evaluate the following:

a. (5 pts) $P(5,3)$

b. (5 pts) $C(5,3) = \binom{5}{3}$

16. (10 pts) Expand $(2x - 3y)^5$