121 ONLINE FINAL, FALL 2010

1a $f(x) = \dfrac{x^2+5x+17}{x^2-5x-14} \Longrightarrow$

$\mathcal{D} = \{x \mid x^2-5x-14 \neq 0\}$

$= \{x \mid (x-7)(x+2) \neq 0\}$

$\boxed{= \{x \mid x \neq 7 \;\&\; x \neq -2\} = (-\infty,-2) \cup (-2,7) \cup (7,\infty)}$

1b $x^2 - 5x - 14 \geq 0$

$\Longrightarrow \mathcal{D}\left(\sqrt{x^2-5x-14}\right)$

$= \boxed{(-\infty,-2] \cup [7,\infty)}$

1c Want $\mathcal{D}(\ln(x^2-5x-14)) \Longrightarrow$

want $x^2-5x-14 > 0 \Longrightarrow$

$\mathcal{D}(\ln(x^2-5x-14)) = \boxed{(-\infty,-2) \cup (7,\infty)}$

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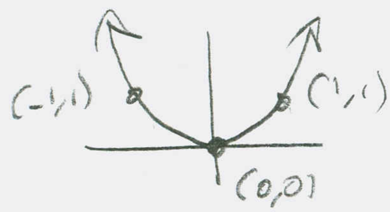
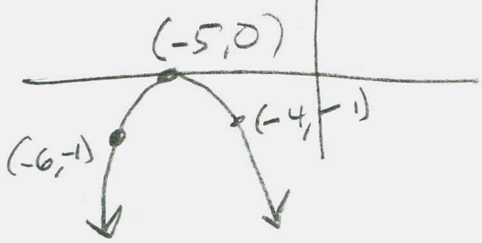
(2) $g(x) = -(x+5)^2 + 3$



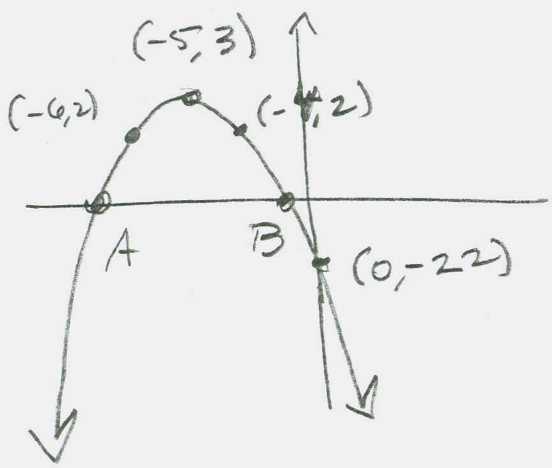
$f(x) = x^2$

$-f(x+5) = -(x+5)^2$



$*(0,-25)$

$$-(x+5)^2 + 3 = g(x) = -f(x+5) + 3$$



$$-(x+5)^2 + 3 = 0$$

$$(x+5)^2 = 3$$

$$x+5 = \pm\sqrt{3}$$

$$x = -5 \pm \sqrt{3}$$

$$\boxed{\begin{array}{l} A = (-5-\sqrt{3}, 0) \\ B = (-5+\sqrt{3}, 0) \end{array}}$$

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3a $x^2-6x+8=(x-4)(x-2)=0 \longrightarrow$

$\boxed{x \in \{2,4\}}$

3b $a=1,\ b=-6,\ c=8 \longrightarrow$

$b^2-4ac=(-6)^2-4(1)(8)$

$= 36-32$

$= 4$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{6\pm\sqrt{4}}{2}=\dfrac{6\pm2}{2}=3\pm1$

$\Longrightarrow \boxed{x \in \{2,4\}}$

3c $x^2-6x = -8$

$x^2-6x+3^2=-8+9$

$(x-3)^2=1$

$x-3=\pm\sqrt{1}$

$x=3\pm1 \longrightarrow \boxed{x \in \{2,4\}}$

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(4) $f(x) = (x+2)^2(x-3)(x-1)^3$

(a)

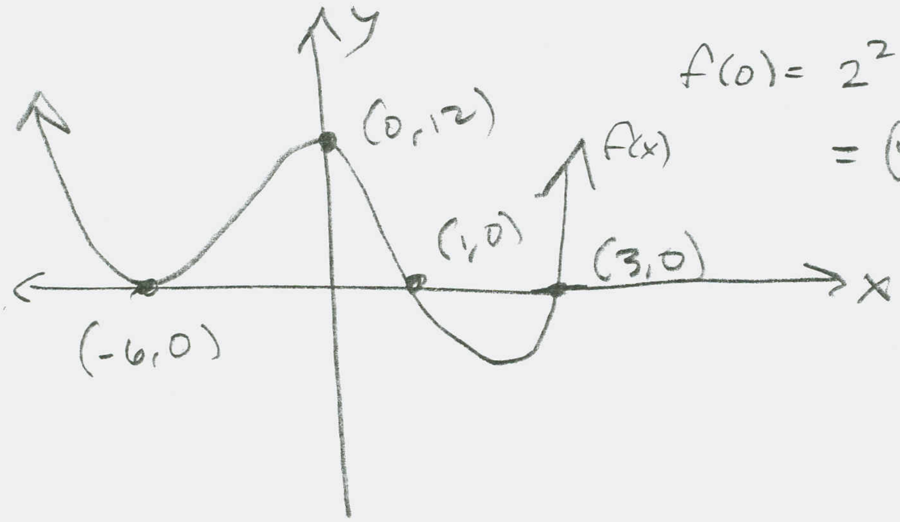
$x = -2$ $m = 2$ touch

$x = +1$ $m = 3$ cross

$x = +3$ $m = 1$ cross

(b) $f(x) \xrightarrow[]{x \to \infty} (x)^2(x)(x)^3 = \boxed{x^6}$

(c)



$f(0) = 2^2(-3)(-1)^3$

$= (4)(3) = 12$

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$$(x+4)(x-2)^3(x-(3+2i))(x-(3-2i))$$

$x=4,\ m=1;\ x=2,\ m=3;\ x=3+2i,\ m=1$

6 $f(x) = x^4 - 11x^3 + 42x^2 - 14x - 68$

Find $f(1)$:

$$\begin{array}{r|rrrrr} 1 & 1 & -11 & 42 & -14 & -68 \\ & & 1 & -10 & 32 & 18 \\ \hline & 1 & -10 & 32 & 18 & \boxed{-50 = f(1)} \end{array}$$

7 $\log_2(96) - \log_2(3)$

$= \log_2(32 \cdot 3) - \log_2(3)$

$= \log_2(32) + \log_2(3) - \log_2(3) = \boxed{5}$

8 $A_0 e^{-5500k} = \frac{1}{2} A_0$

$e^{-5500k} = \frac{1}{2}$

$-5500k = \ln(\frac{1}{2})$

$k = \dfrac{\ln(\frac{1}{2})}{-5500} = \boxed{\dfrac{\ln 2}{5500} = k \approx .0001260267601}$

$$\boxed{A(t) = A_0 e^{-.0001260267601 t}}$$

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9a $\sum_{k=1}^{50} 3(1.2)^{k-1}$ $a=3,\ r=1.2,\ n=50$

$$\frac{a(1-r^n)}{1-r} = \frac{3(1-1.2^{50})}{1-1.2}$$

$$\approx \boxed{136491.5723}$$

9b $\sum_{k=1}^{\infty} \frac{3}{4}\left(\frac{2}{5}\right)^{k-1}$ $a=\frac{3}{4},\ r=\frac{2}{5}.\ \infty$

$$\frac{a}{1-r} = \frac{\frac{3}{4}}{1-\frac{2}{5}} = \frac{\frac{3}{4}}{\frac{3}{5}} = \left(\frac{3}{4}\right)\left(\frac{5}{3}\right) = \boxed{\frac{5}{4}}$$

10 $x+y=7$
$3x-2y=8$ w/ substitution

$$x=7-y \Longrightarrow 3(7-y)-2y = 21-3y-2y = 21-5y = 8$$

$$\Longrightarrow -5y=-13 \longrightarrow \boxed{y=\frac{13}{5}} \Longrightarrow x = 7-\frac{13}{5}$$

$$= \frac{35-13}{5} = \boxed{\frac{22}{5}=x}$$

11 Elimination:

$$\left[\begin{array}{cc|c} 1 & 1 & 7 \\ 3 & -2 & 8 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 7 \\ 0 & -5 & -13 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 7 \\ 0 & 1 & \frac{13}{5} \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & \frac{22}{5} \\ 0 & 1 & \frac{13}{5} \end{array}\right] \Longrightarrow \boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{22}{5} \\ \frac{13}{5} \end{bmatrix}}$$

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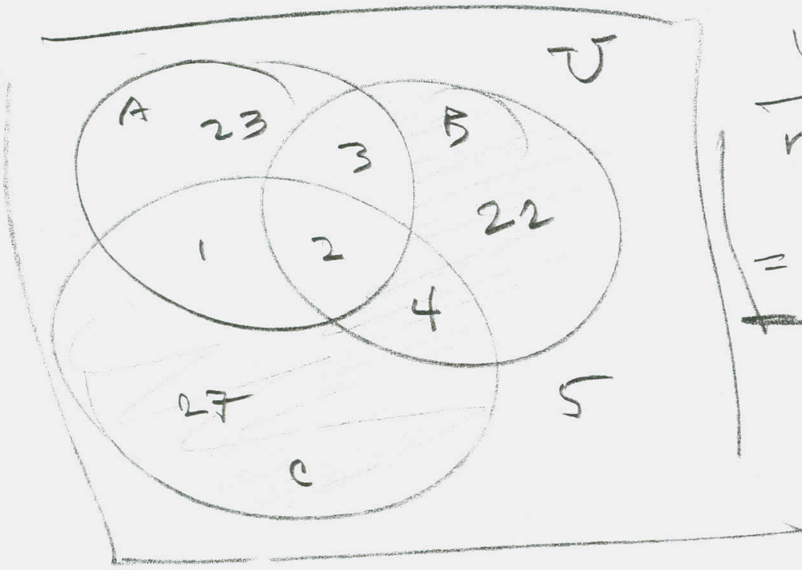
(12) $n(A)=10,\ n(B)=15,\ n(A\cup B)=20$

$$n(A\cup B)=n(A)+n(B)-n(A\cap B) \implies$$

$$20 = 10 + 15 - n(A\cap B) \implies$$

$$\boxed{n(A\cap B)=25-20=5}$$

(13)



Want

$$\boxed{n((B\cup C)\cap A') = 22+4+27=53}$$

(14)

$x+y=7$

$5x-2y\le 20$

$x\ge 0$

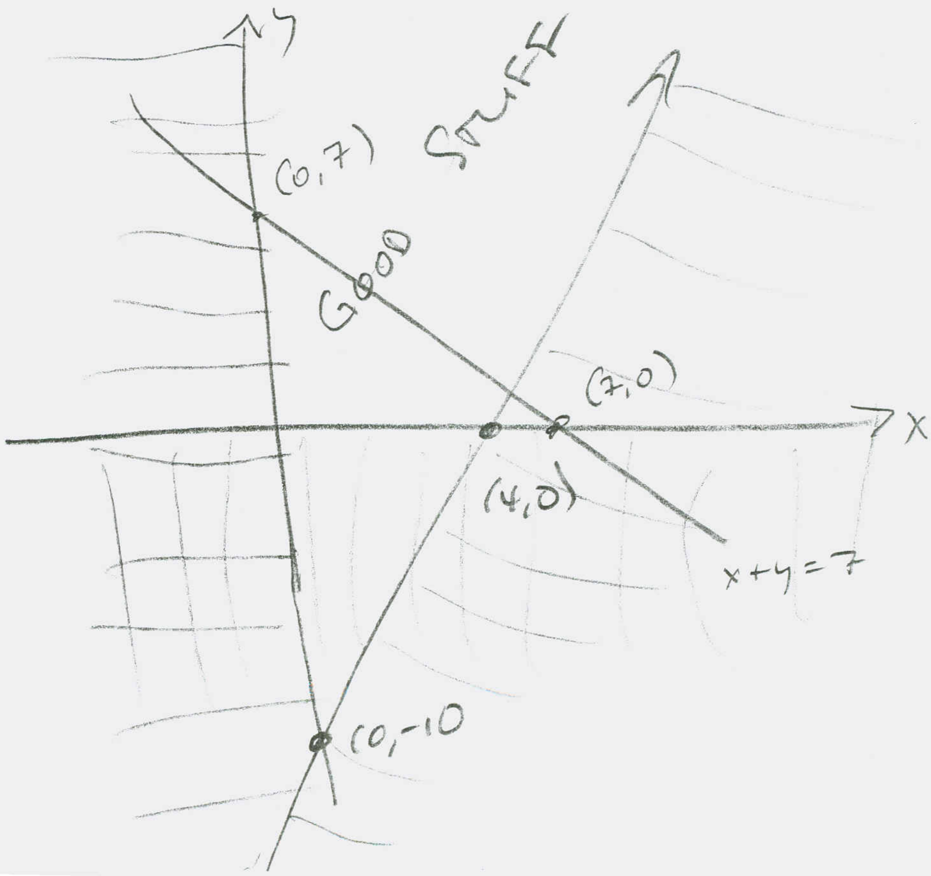
$y\ge 0$

$5x-2y=20$

x	y
0	-10
4	0

$0\le 20$

(0,0) Good



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15 (a) $P(5,3) = \dfrac{5!}{(5-3)!} = \dfrac{5!}{2!} = 5 \cdot 4 \cdot 3 = 60$

(b) $C(5,3) = \dfrac{5!}{2!\,3!} = \dfrac{5 \cdot 4}{2} = 10$

10 $(2x-3y)^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$=$

$(2x)^5(-3y)^0 + 5(2x)^4(-3y) + 10(2x)^3(-3y)^2 + 10(2x)^2(-3y)^3$

$+ 5(2x)(-3y)^4 + (-3y)^5$

$$\boxed{= 32x^5 - 240x^4y + 720x^3y^2 - 1080x^2y^3 + 810xy^4 - 243y^5}$$