Practice Final Solns

1a

$$x^2-2x = 35$$
$$x^2-2x+1^2 = 35+1^2$$
$$(x-1)^2 = 36$$
$$x-1 = \pm\sqrt{36} = \pm 6$$
$$x = 1 \pm 6 \rightarrow \boxed{x=7 \text{ OR } x=-5}$$

1b

$$x^2-2x-35=0$$
$$(x-7)(x+5)=0$$
$$\boxed{x=-5 \text{ OR } x=7}$$

1c

$$x^2-2x-35=0$$
$$a=1,\ b=-2,\ c=-35$$
$$b^2-4ac = (-2)^2-4(1)(-35)$$
$$= 4+140 = 144$$
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{2\pm\sqrt{144}}{2(1)} = \frac{2\pm 12}{2}$$
$$= \frac{2(1\pm 6)}{2} = 1\pm 6 \rightarrow \boxed{x=7 \text{ OR } x=-5}$$

3

$$(x-1)^2(x+2)^3(2x-3) \geq 0$$
$$x = 1, -2, \tfrac{3}{2} = -2, 1, \tfrac{3}{2}$$

$x=-2$ $m=3$ change

$x=1$ $m=2$ don't change

$x=\frac{3}{2}$ $m=1$ change

$$f(2) = (1)^2(4)^3(2(2)-3) = +$$

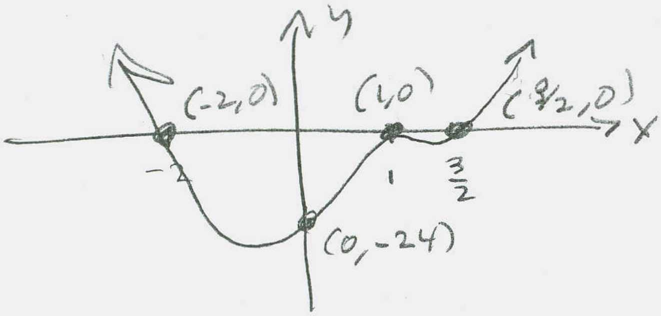
From sign pattern & ≥ 0 required:

$$(-\infty,-2] \cup [\tfrac{3}{2},\infty)$$
$$= \{x \mid -\infty < x \leq 2 \text{ OR } \tfrac{3}{2} \leq x < \infty\}$$

4 From sign pattern & $f(0) = (-1)^2(2)^3(-3)$

$= (8)(-3) = -24 \rightsquigarrow (0,-24)$

we obtain



Graph of f

$f(x) = (x-1)^2(x+2)^3(2x-3)$

5 Domain of $f(x)$ is $\mathbb{R} = (-\infty,\infty)$ It's a polynomial

6 $f(x) = \sqrt{(x-1)^2(x+2)^3(2x-3)}$. For $\mathcal{D}(f)$, we need $(x-1)^2(x+2)^3(2x-3) \geq 0$. This is $\boxed{\#3 \text{ ANS}}$

7 $f(x) = \sqrt{\dfrac{(x-1)^2(2x-3)}{(x+2)^3}}$ For $\mathcal{D}(f)$,

we need $\dfrac{(x-1)^2(2x-3)}{(x+2)^3} \geq 0$. SAME sign pattern as #s 3, 6, but $x \neq -2$.

$\mathcal{D} = (-\infty,-2) \cup [\frac{3}{2},\infty)$

$= \{x \mid -\infty < x < -2 \text{ OR } \frac{3}{2} \leq x < \infty\}$

8 From sign pattern in #3 & the $(x+2)^3$ in the denominator, we have:

zeros: $x = 1, \frac{3}{2}$

V.A.: $x = -2$

121 PF cont'd

#8 cont'd

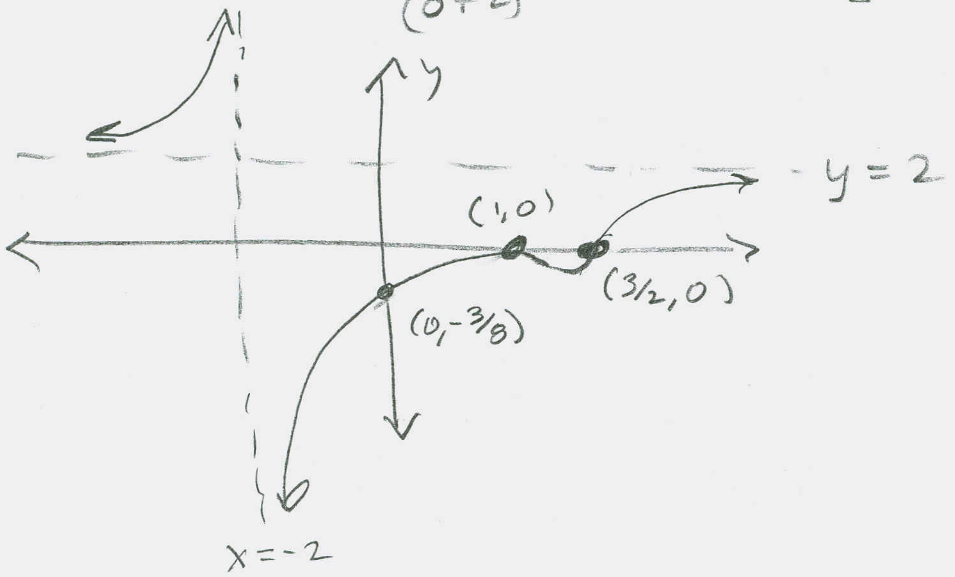
Now End Behavior: $\dfrac{\deg = 3}{\deg = 3} \Longrightarrow$ Horizontal Asymptote.

To get H.A. Take $x \to$ BIG:

$$f(x) = \frac{(x-1)^2(2x-3)}{(x+2)^3} \longrightarrow \frac{(x)^2(2x)}{(x)^3} = \frac{2x^3}{x^3} = 2 = y$$

$\boxed{y = 2 \text{ is H.A.}}$

$$f(0) = \frac{(0-1)^2(2(0)-3)}{(0+2)^3} = \frac{(-1)^2(-3)}{2^3} = \frac{-3}{8} \rightsquigarrow (0, -\tfrac{3}{8})$$



#9 $f(x) = \dfrac{(2x+10)(x-1)^2(2x-3)}{(x+2)^3(x+5)} = \dfrac{2\cancel{(x+5)}(x-1)^2(2x-3)}{\cancel{(x+5)}(x+2)^3}$

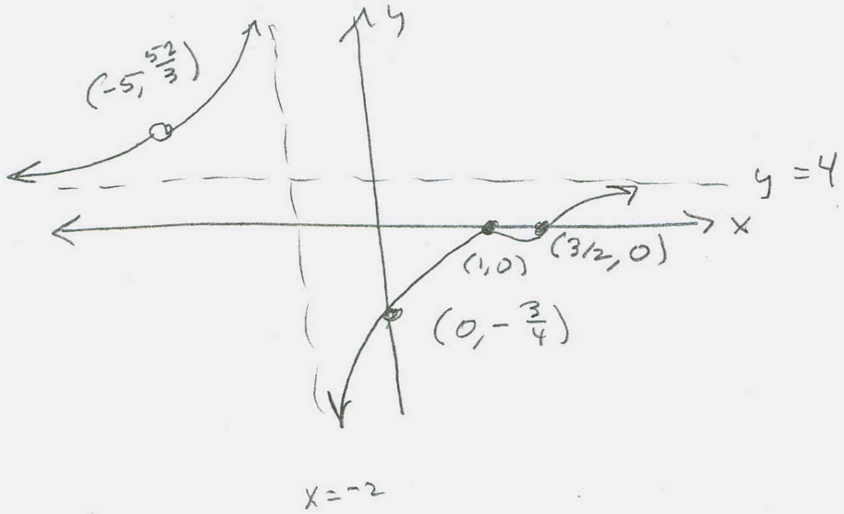
$= 2\dfrac{(x-1)^2(2x-3)}{(x+2)^3} =$ 2 times previous graph with hole @ $x = -5$

$$\text{"}f(-5)\text{"} = 2\frac{(-5-1)^2(2(-5)-3)}{(-5+2)^3} = \frac{(-6)^2(-13)}{(-3)^3} = \frac{(36)(13)}{27}$$

$$= \frac{(12)(13)}{9} = \frac{(4)(13)}{3} = \frac{52}{3} \rightsquigarrow \text{HOLE}: (-5, \tfrac{52}{3})$$

PF cnt'd

#9 cnt'd 2 Previous function, hole @ $(-5, \frac{52}{3})$



(10) $f(x) = \log_3\left((x-1)^2(x+2)^3(2x-3)\right)$

Same sign pattern, but need $(x-1)^2(x+2)^3(2x-3) > 0$.

So throw out endpoints in #3.

$(-\infty, -2) \cup (\frac{3}{2}, \infty)$

$= \{x \mid -\infty < x < -2 \text{ OR } 3/2 < x < \infty\} = \mathcal{D}$

(11) $\mathcal{D}$ for $\log_3\left(\frac{(x-1)^2(2x-3)}{(x+2)^3}\right)$ is same as

#10. We need $\frac{(x-2)^2(2x-3)}{(x+2)^3} > 0$.

(12) $f(x) = \frac{x-2}{x+1}$, $g(x) = \sqrt{x+2}$

(a) $f(g(x)) = \frac{\sqrt{x+2} - 2}{\sqrt{x+2} + 1}$ $\mathcal{D} = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$\mathcal{D}(g) = \{x \mid x \geq -2\}$, from

$x + 2 \geq 0$. Now $g(x) \in \mathcal{D}(f)$ part!

121 fF cnt'd

#12 cnt'd

$g(x) \in \mathcal{D}(f)$ means we need

$\sqrt{x+2} + 1 \neq 0 \implies$

$\sqrt{x+2} \neq -1$. Since $\sqrt{x+2} \geq 0$ by defn.,
this is always true, so

$$\boxed{\mathcal{D}(f \circ g) = \{x \mid x \geq -2\} = [-2, \infty)}$$

12b $(g \circ f)(x) = \boxed{g(f(x)) = \sqrt{\frac{x-2}{x+1} + 2}}$

$\mathcal{D}(g \circ f) = \{x \mid x \in \mathcal{D}(f) \text{ and } f(x) \in \mathcal{D}(g)\}$

$\mathcal{D}(f) = \{x \mid x \neq -1\} = (-\infty, -1) \cup (-1, \infty)$, from

NEED $x + 1 \neq 0$.

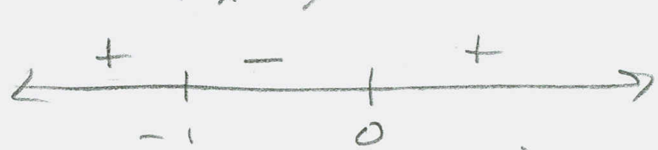
Now, the $f(x) \in \mathcal{D}(g)$ part:

Need $\frac{x-2}{x+1} + 2 \geq 0 \implies$

$$\frac{x-2+2(x+1)}{x+1} \geq 0$$

$$\frac{x-2+2x+2}{x+1} \geq 0$$

$\frac{3x}{x+1} \geq 0$ key vals:
$x=0, x=-1$



$(-\infty, -1]$ OR $[0, \infty)$

This gives: $\{x \mid x \leq -1 \text{ OR } x \geq 0 \text{ AND } x \neq -1\}$

$$= \boxed{\{x \mid x < -1 \text{ OR } x \geq 0\} = (-\infty, -1) \cup [0, \infty)}$$

121 PF cnt'd

(13) $\sqrt{x^2} = |x|$

(24) $\frac{1}{2}$-life is 39 (years)

$A(t) = A_0 e^{-kt}$. Given $A_0 e^{-39k} = \frac{1}{2} A_0$

$\Longrightarrow e^{-39k} = \frac{1}{2}$

$-39k = \ln(\frac{1}{2})$

$k = \frac{\ln(\frac{1}{2})}{-39} = \frac{-\ln(\frac{1}{2})}{39} = \frac{\ln 2}{39} = k$

$$\boxed{A(t) = A_0 e^{-\frac{\ln 2}{39} t}}$$

The rest of the questions show up on other tests I've already shared with you. But ask if you can't find 'em.