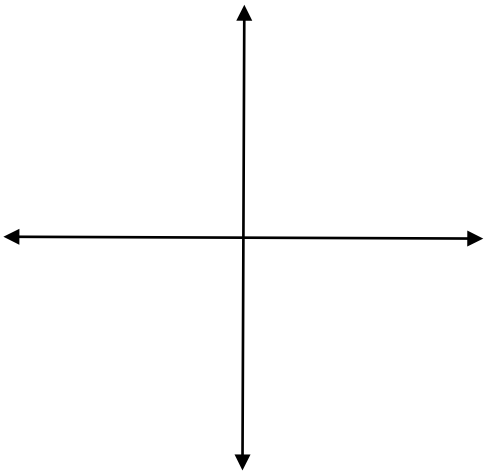
MAT 121-G81 Test 4 – Fall, 2010 Name_____

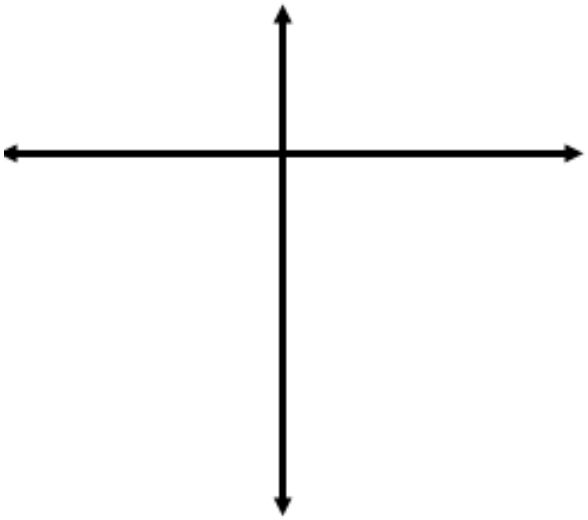
100 Points Covers Chapter 4, plus 7.1, 7.3

Graph:

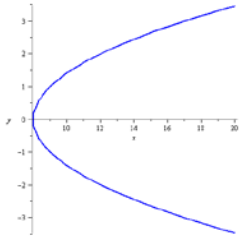
1. (10 pts) Graph $f(x) = 3^x$ and $g(x) = f^{-1}(x) = \log_3 x$ on the same set of coordinate axes. Show all intercepts, asymptotes, etc.

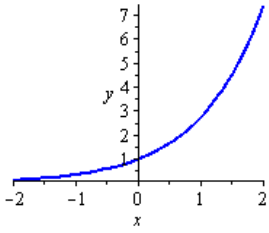


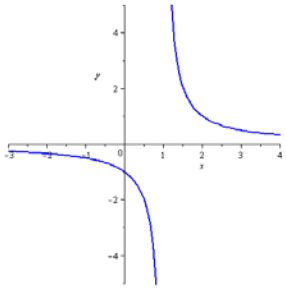
2. (5 pts) Graph $f(x) = 3 \cdot 2^{x+1} - 5$

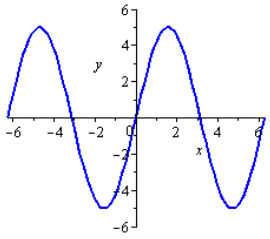


3. (5 pts) Determine which of the following are one-to-one. Indicate by writing "Yes" or "No" below the graphs. Tell me which one isn't a function.









4. For $f(x) = x^2 + 3$ and $g(x) = \sqrt{x-3}$, determine the following composite functions, *simplify them*, and state their domains:

a. (5 pts) $(f \circ g)(x)$

b. (5 pts) $(g \circ f)(x)$

5. (5 pts) For $f(x) = \log_2 x$ and $g(x) = 3x + 5$, find the domain of $(f \circ g)(x)$.

6. (5 pts) What is the domain of $\ln\left(\dfrac{x-2}{(x-3)(x+1)^3}\right)$? (This is like a Chapter 3 question!)

7. (5 pts) Let $f(x) = 3^{x+1} - 5$. Find $f^{-1}(x)$.

8. (5 pts) Find functions *f* and *g* so that *f* o *g* = *H*, given that $H(x) = \sqrt{x^2 - 1}$.

9. (5 pts) Evaluate $\log_2(96) - \log_3(3)$ *without a calculator* !!

10. A fast-growing city is growing exponentially (obeying the law of uninhibited growth) with a growth rate of 6%. The population was 30,000 in 2005.

a. (5 pts) Write an exponential function to model the situation. Tell what each variable represents.

b. (5 pts) In what year will the population reach 90,000?

11. (5 pts) Solve *without a calculator*: $\pi^{x+1} = e^x$. All I want is a symbolic answer and symbolic manipulations you perform to *get* there. For full credit, your answer should have a symbolic π in it.

12. (5 pts) Write the following as the logarithm of a single expression. Assume that variables represent positive numbers. $3\log_5(x+7) - 2\log_5(x-7) + \log_5 9$

13. (10 pts) Solve: $\log_5(x-4) + \log_5(x+2) = \log_5 7$

14. (5 pts) The half-life of carbon-14 is (approximately) 5800 years. (I think it's 5600 years in the textbook, but let's roll with 5800.) Using this half-life, we obtain an exponential decay function

$$A(t) = A_0 e^{-kt} = A_0 e^{-\frac{\ln 2}{5800}t} \approx A_0 e^{-0.00011950813t}.$$

How old is a sample from a neolithic fire pit if it is found that 12% of naturally-occurring carbon-14 is present in the sample? For ease of solving this problem, you may want to just use a symbolic k until the last step. Round your final answer to the nearest year.

15. Find the geometric sums:

a. (5 pts) $\frac{3}{4}+\frac{1}{2}+\frac{1}{3}+.....+\frac{32}{729}$ (Be careful finding your a, r, and n in $a \cdot r^{n-1}$)

b. (5 pts) $\sum_{k=1}^{\infty} \frac{2}{3} \cdot \left(\frac{3}{5}\right)^{k-1}$