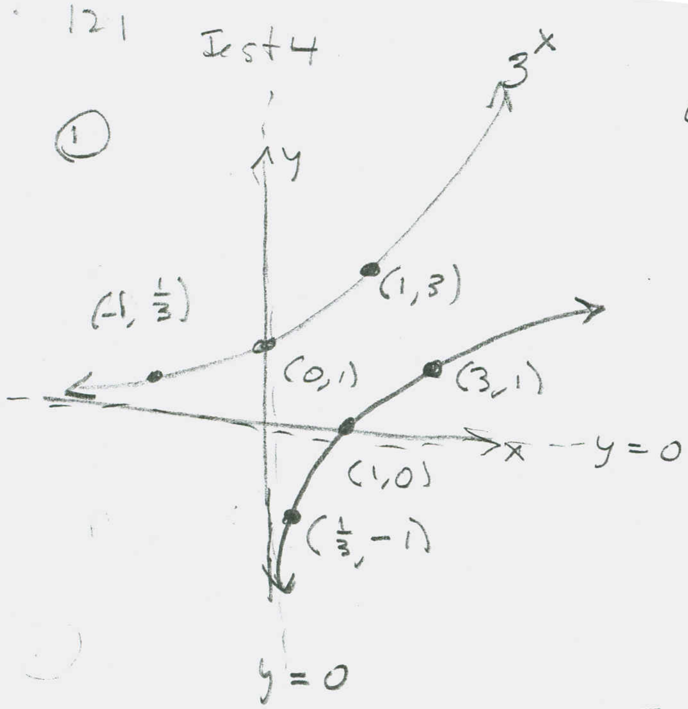
121 Test 4

ONLINE 121

①



Throw out #9.
Teacher made a typing error.

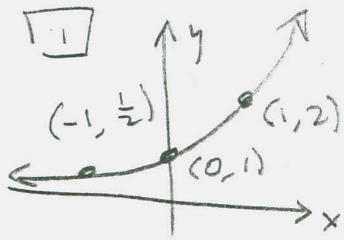
I awarded bonus for nailing the x-intercept.

② $3 \cdot 2^{x+1} - 5 = g(x)$

$f(x) = 2^x \longrightarrow 2^{x+1} \longrightarrow 3 \cdot 2^{x+1} \longrightarrow 3 \cdot 2^{x+1} - 5$

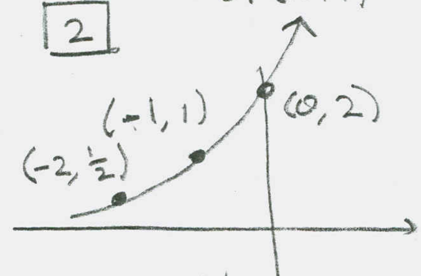
left 1, $f(x+1)$ — times 3, $3f(x+1)$ — down 5, $3f(x+1) - 5$

$\boxed{1}$



$f(x) = 2^x$

$\boxed{2}$



left 1
x-values minus 1.

$f(x+1) = 2^{x+1}$

Bonus:

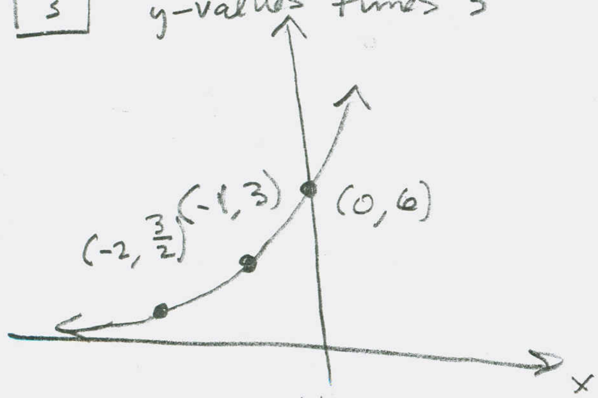
$3 \cdot 2^{x+1} - 5 = 0$

$2^{x+1} = \frac{5}{3}$

$x + 1 = \log_2(\frac{5}{3})$

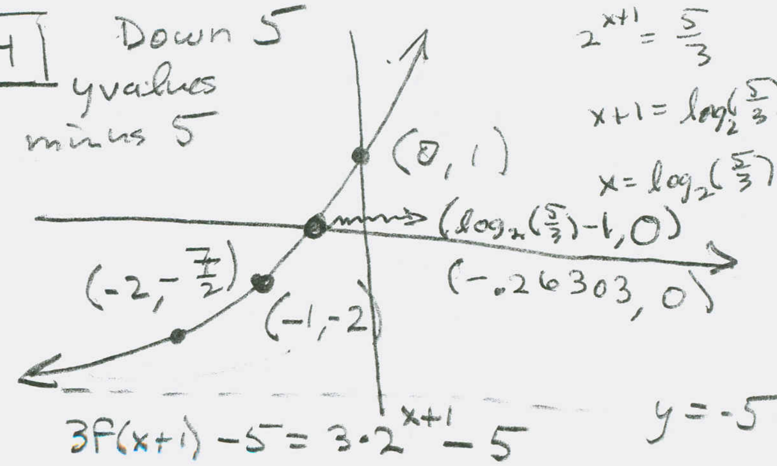
$x = \log_2(\frac{5}{3}) - 1$

$\boxed{3}$ Vertical times 3
y-values times 3



$3f(x+1) = 3 \cdot 2^{x+1}$

$\boxed{4}$ Down 5
y values minus 5



$3f(x+1) - 5 = 3 \cdot 2^{x+1} - 5$

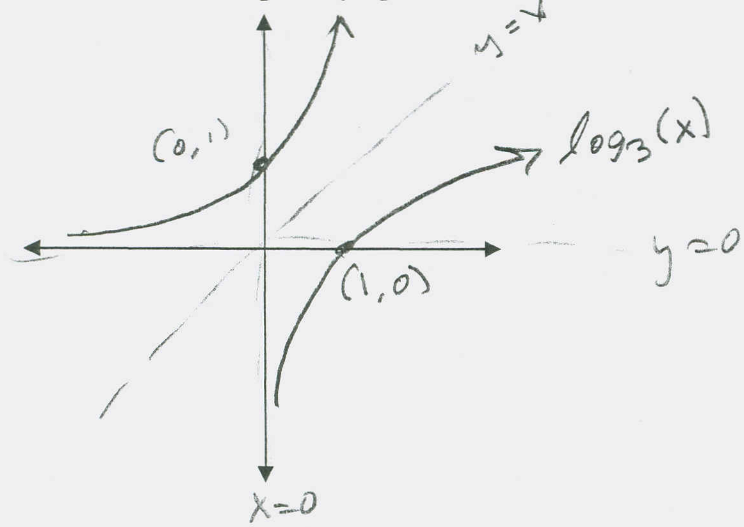
$\frac{3}{2} - 5 = \frac{3}{2} - \frac{10}{2} = -\frac{7}{2}$

MAT 121-G81 Test 4 – Fall, 2010 Name KEY

100 Points Covers Chapter 4, plus 7.1, 7.3

Graph:

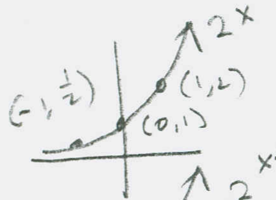
1. (10 pts) Graph $f(x) = 3^x$ and $g(x) = f^{-1}(x) = \log_3 x$ on the same set of coordinate axes. Show all intercepts, asymptotes, etc.



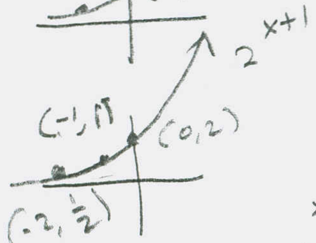
Don't like this work, Steve. Give separate 1st page.

2. (5 pts) Graph $f(x) = 3 \cdot 2^{x+1} - 5$

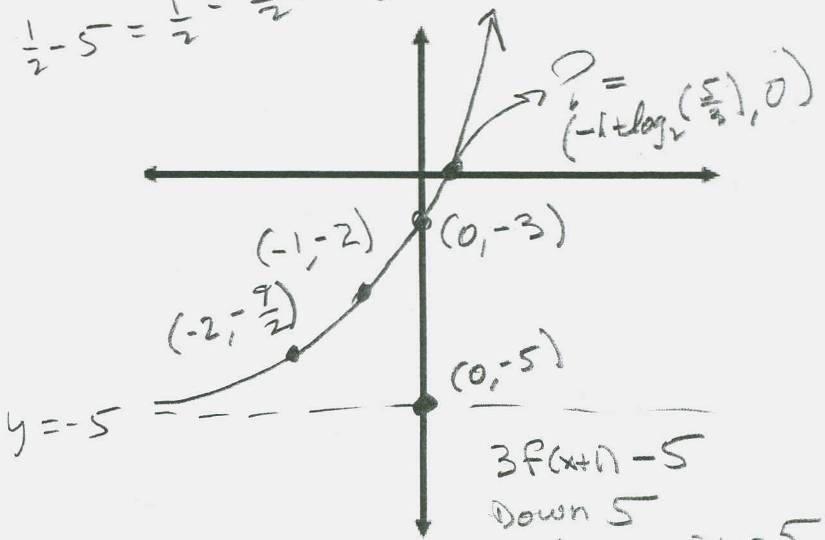
$\frac{1}{2} - 5 = \frac{1}{2} - \frac{10}{2} = -\frac{9}{2}$



f(x)

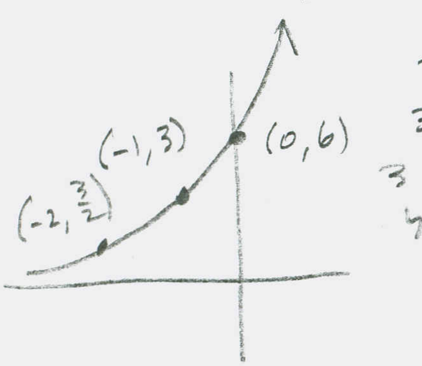


f(x+1)
left 1
x-values minus 1



3f(x+1) − 5
Down 5
y-values minus 5

$3 \cdot 2^{x+1}$



3f(x+1)
3times vertical
3 times y-values

?: $3 \cdot 2^{x+1} - 5 = 0$

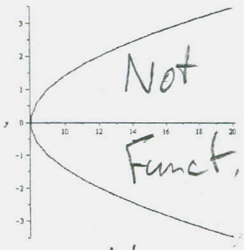
$3 \cdot 2^{x+1} = 5$

$2^{x+1} = \frac{5}{3}$

$x + 1 = \log_2\left(\frac{5}{3}\right)$

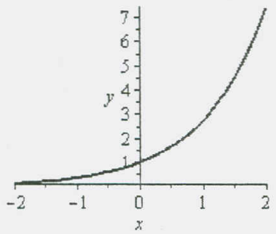
$x = -1 + \log_2\left(\frac{5}{3}\right)$

3. (5 pts) Determine which of the following are one-to-one. Indicate by writing "Yes" or "No" below the graphs. Tell me which one isn't a function.

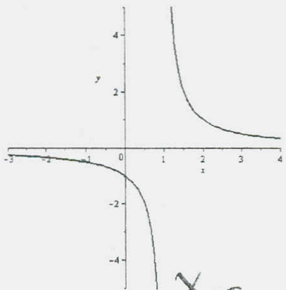


Not Funct,

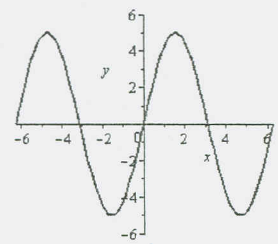
No



Yes



Yes



No

4. For $f(x) = x^2 + 3$ and $g(x) = \sqrt{x-3}$, determine the following composite functions, *simplify them*, and state their domains:

a. (5 pts) $(f \circ g)(x)$ $= f(g(x)) = (\sqrt{x-3})^2 + 3 = x - 3 + 3 = x$!

$\mathcal{D} = \{x \mid x \geq 3\}$ from $x \in \mathcal{D}(g)$ and
$g(x) \in \mathcal{D}(f)$

b. (5 pts) $(g \circ f)(x)$ $= g(f(x)) = \sqrt{x^2+3-3} = \sqrt{x^2} = |x|$

It's domain is all real numbers

5. (5 pts) For $f(x) = \log_2 x$ and $g(x) = 3x + 5$, find the domain of $(f \circ g)(x)$.

$f(g(x)) = \log_2(3x+5)$. Need $3x+5 > 0$

$3x > -5$ $\qquad \mathcal{D} = \{x \mid x > -\frac{5}{3}\}$

$x > -\frac{5}{3}$ $\qquad$ OR $(-\frac{5}{3}, \infty)$

6. (5 pts) What is the domain of $\ln\left(\dfrac{x-2}{(x-3)(x+1)^3}\right)$? (This is like a Chapter 3 question!)

Need

$$\frac{x-2}{(x-3)(x+1)^3} > 0$$

$x=2$ $m=1$
$x=3$ $m=1$
$x=-1$ $m=3$

$\left.\right\}$ change sign each ~~at~~ spot

$x = 4$;

$$\frac{4-2}{(4-3)(4+1)^3} = \frac{2}{(1)(5)^3} \text{ is } +$$

So, $D = (-1,2)\cup(3,\infty)$

7. (5 pts) Let $f(x) = 3^{x+1} - 5$. Find $f^{-1}(x)$.

$x = 3^{y+1} - 5$

$3^{y+1} - 5 = x$

$3^{y+1} = x + 5$

$y + 1 = \log_3(x+5)$

$y = \boxed{\log_3(x+5) - 1 = f^{-1}(x)}$

8. (5 pts) Find functions *f* and *g* so that $f \circ g = H$, given that $H(x) = \sqrt{x^2 - 1}$.

$g(x) = x^2 - 1$, $f(x) = \sqrt{x}$

9. (5 pts) Evaluate $\log_2(96) - \log_3(3)$ *without a calculator* !!

$= \log_2\left(\frac{96}{3}\right) = \log_2(32) = \log_2(2^5) = 5$

10. A fast-growing city is growing exponentially (obeying the law of uninhibited growth) with a growth rate of 6%. The population was 30,000 in 2005.

a. (5 pts) Write an exponential function to model the situation. Tell what each variable represents.

$P(t) = P_0 e^{kt} = 30000 e^{.06t}$

t = time, in years after 2005

P = Population as a function of t.

b. (5 pts) In what year will the population reach 90,000?

$P(t) = 90000$

$30000 e^{.06t} = 90000$

$e^{.06t} = \frac{90000}{30000} = 3$

$2005 + 18 = 2023$

$e^{.06t} = 3$

$.06t = \ln 3$

$t = \frac{\ln 3}{.06} \approx 18.3102\,0481 \approx \boxed{18 \approx t}$

So, in $\boxed{2023}$

11. (5 pts) Solve *without a calculator*: $\pi^{x+1} = e^x$. All I want is a symbolic answer and symbolic manipulations you perform to *get* there. For full credit, your answer should have a symbolic π in it.

$$\ln(\pi^{x+1}) = \ln(e^x) = x$$

$$\ln(\pi^{x+1}) = x$$

$$(x+1)\ln(\pi) = x$$

$$x\ln(\pi) + 1\ln(\pi) = x$$

$$-1\ln(\pi) = \quad -\ln\pi$$

$$x\ln(\pi) = x - \ln(\pi)$$

$$-x \quad = -x$$

$$x\ln(\pi) - x = -\ln(\pi)$$

$$\to x\ln(\pi) - x = -\ln(\pi)$$

$$x(\ln(\pi) - 1) = -\ln(\pi)$$

$$x = \frac{-\ln(\pi)}{\ln(\pi) - 1}$$

OR

$$\frac{\ln\pi}{1 - \ln\pi}$$

12. (5 pts) Write the following as the logarithm of a single expression. Assume that variables represent positive numbers. $3\log_5(x+7) - 2\log_5(x-7) + \log_5 9$

$$= \log_5\left(\frac{(x+7)^3(9)}{(x-7)^2}\right)$$

13. (10 pts) Solve: $\log_5(x-4) + \log_5(x+2) = \log_5 7$

$$\log_5((x-4)(x+2)) = \log_5 7$$

$$(x-4)(x+2) = 7$$

$$x^2 - 2x - 8 = 7$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$x = 5$ OR $x = -3$ → Nope

CHECK:

$x = -3 \notin \mathcal{D}$

$x = 5$:

$$\log_5(5-4) + \log_5(5+2)$$

$$= \log_5(1) + \log_5(7)$$

$$= 0 + \log_5(7) = \log_5 7 \checkmark$$

FINAL ANS:

$$\boxed{\{5\}}$$

14. (5 pts) The half-life of carbon-14 is (approximately) 5800 years. (I think it's 5600 years in the textbook, but let's roll with 5800.) Using this half-life, we obtain an exponential decay function

$$A(t) = A_0 e^{-kt} = A_0 e^{-\frac{\ln 2}{5800}t} \approx A_0 e^{-0.00011950813t}.$$

How old is a sample from a neolithic fire pit if it is found that 12% of naturally-occurring carbon-14 is present in the sample? For ease of solving this problem, you may want to just use a symbolic k until the last step. Round your final answer to the nearest year.

$A(t) = .12A_0$

$A_0 e^{-kt} = .12 A_0$

$e^{-kt} = .12$

$-kt = \ln(.12)$

$t = \dfrac{\ln(.12)}{-k} = \dfrac{\ln(.12)}{-\left(\frac{\ln(2)}{5800}\right)} \approx 17,741.5834$

$\approx \boxed{17,742 \text{ yrs old}}$

15. Find the geometric sums:

a. (5 pts) $\frac{3}{4}+\frac{1}{2}+\frac{1}{3}+.....+\frac{32}{729}$ (Be careful finding your a, r, and n in $a \cdot r^{n-1}$)

$a_n = ar^{n-1}$

$a_1 = a = \frac{3}{4}$

$r = \dfrac{a_2}{a_1} = \dfrac{\frac{1}{2}}{\frac{3}{4}} = \left(\frac{1}{2}\right)\left(\frac{4}{3}\right) = \frac{2}{3} = r$

Not enough room, Steve!

2|32
2|16
2|8
2|4
2

$\frac{32}{729} = ar^{n-1} = \frac{3}{4} \cdot \left(\frac{2}{3}\right)^{n-1}$

Need n:

$\frac{2^{n-1}}{4} = 32$

$2^{n-1} = 128 = 2^7$

$n-1 = 7 \Rightarrow n = 8$

$\dfrac{a(1-r^8)}{1-r} = \dfrac{\frac{3}{4}\left(1-\left(\frac{2}{3}\right)^8\right)}{1-\frac{2}{3}}$

$= \dfrac{\frac{3}{4}\left(\frac{6305}{6561}\right)}{\frac{1}{3}} = \dfrac{6305}{2916}$

≈ 2.162208505

b. (5 pts) $\sum_{k=1}^{\infty} \frac{2}{3} \cdot \left(\frac{3}{5}\right)^{k-1}$

$= \dfrac{a}{1-r} = \dfrac{\frac{2}{3}}{1-\frac{3}{5}} = \dfrac{\frac{2}{3}}{\frac{2}{5}}$

$= \left(\frac{2}{3}\right)\left(\frac{5}{2}\right) = \boxed{\frac{5}{3} = 1.\overline{66}}$