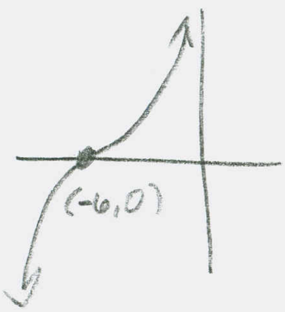
121 ONLINE TEST 3 FALL 2010 KEY

① Polynomial? If not, why not?

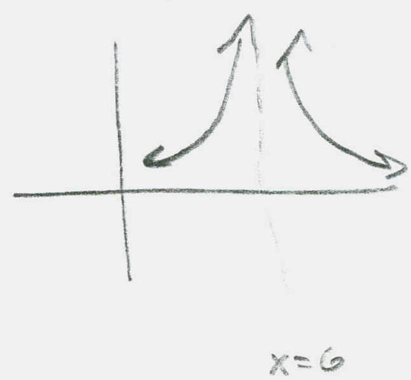
(a) $\sqrt{x^2-3}+2x$ No. The x^2-3 is enclosed in a radical

(b) $x^4-3x^2+\frac{2}{3}$ Yes

② (a) $(x+6)^3$



(b) $\frac{1}{(x-6)^2}$



③ (a) $\boxed{(x+4)^2(x-2)^3}$ $-4=x, m=2$; $x=2, m=3$

(b) $x=2, m=1$; $x=5, m=2$; $x=7-8i, m=1$

$\boxed{(x-2)(x-5)^2(x-(7-8i))(x-(7+8i))}$

④ $(x-(5+3i))(x-(5-3i))$

$=x^2-(5-3i)x-(5+3i)x+(5+3i)(5-3i)$

$=x^2-5x+3ix-5x-3ix+5^2+3^2$

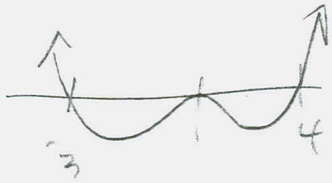
$=\boxed{x^2-10x+34}$

5 $f(x) = (x-1)^2(x+3)(x-4)^3$

(a) $x = 1$, $m = 2$ ~~t~~ouch

$x = -3$, $m = 1$ cross

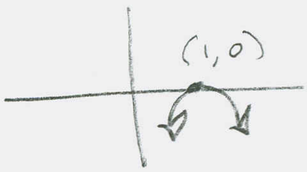
$x = 4$, $m = 3$ cross



(b) @ $x = 1$: $(x-1)^2(1+3)(1-4)^3$

$= (4)(-27)(x-1)^2$

$= -108(x-1)^2$

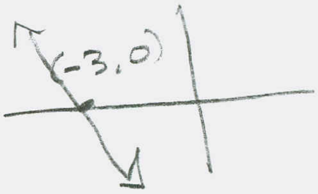


@ $x = -3$ $(-3-1)^2(x+3)(-3-4)^3$

$= (-4)^2(-7)^3(x+3)$

$= -(4)^2(7)^3(x+3)$

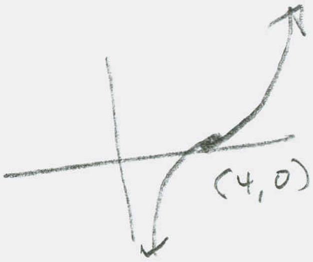
$-5488(x+3)^2$



@ $x = 4$ $(4-1)^2(4+3)(x-4)^3$

$= (3^2)(7)(x-4)^3$

$= 63(x-4)^3$



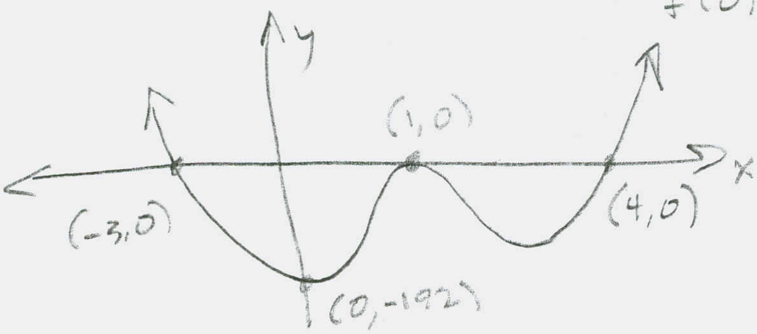
(c) $x \to \pm\infty$: $(x)^2(x)(x)^3 = x^6$

$f(0) = (-1)^2(3)(-4)^3$

$= -(3)(64)$

$= -192$

(d)



6 (a) $(x-1)^2(x+3)(x-4)^3 \geq 0$

$x \in \boxed{(-\infty,-3] \cup [4,\infty) \cup \{1\}}$

(b) $\dfrac{(x-1)^2(x-4)^3}{x+3} \geq 0$ Same sign pattern.

$\boxed{(-\infty,-3) \cup \{1\} \cup [4,\infty)}$ $x \neq -3$

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$$
\begin{array}{r l}
 & 2x^2-3x+2 \quad r\;-2x-1 \\
x^2-1 & \overline{)\,2x^4-3x^3+0x^2+x-3} \\
 & -(2x^4 \qquad -2x^2) \\
\hline
 & -3x^3+2x^2+x-3 \\
 & -(-3x^3 \qquad +3x) \\
\hline
 & 2x^2-2x-3 \\
 & -(2x^2 \qquad -2) \\
\hline
 & -2x-1
\end{array}
$$

(8) $x^4 - 11x^3 + 42x^2 - 14x - 68 = f(x)$

3 or 1 positive zeros

$f(-x) = x^4 + 11x^3 + 42x^2 + 14x - 68$

1 negative zero

$2 \,|\, 68$, $2 \,|\, 34$, 17

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 4, \pm 17, \pm 34, \pm 68$

$\underline{-1}$	1	-11	42	-14	-68
		-1	12	-54	68
$\underline{1}$	1	-12	54	-68	0
		1	-11	43	
	1	-11	43	No	

$(x+1)(x^3 - 12x^2 + 54x - 68)$

$\underline{2}$	1	-12	54	-68
		2	-20	68
	1	-10	34	0

$\boxed{(x+1)(x-2)(x^2 - 10x + 34)}$

(9) $x^2 - 10x + 34 = 0$

$x^2 - 10x = -34$

$x^2 - 10x + 5^2 = -34 + 25$

$(x-5)^2 = -9$

$x - 5 = \pm 3i$

$x = 5 \pm 3i$

$\boxed{(x+1)(x-2)(x-(5+3i))(x-(5-3i))}$

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$\boxed{B}$ $\mathcal{D}$ of $\sqrt{\dfrac{(x-1)^2(x-4)^3}{x+3}}$ is

$$\left\{ x \;\middle|\; \frac{(x-1)^2(x-4)^3}{x+3} \geq 0 \right\}$$

$$= \boxed{(-\infty,-3) \cup \{1\} \cup [4,\infty)}$$

Same as $\boxed{6b}$

10 DONE, Darn it.

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$$R(x) = \frac{3x^3-6x^2-27x+54}{x^3-x^2-5x-3} = \frac{3(x-2)(x^2-9)}{(x+1)^2(x-3)}$$

$\mathcal{D} = \{ x \mid x \neq -1 \text{ and } x \neq 3 \}$

$x^2-9=(x-3)(x+3)$

$$R^*(x) = \frac{3(x-2)\cancel{(x-3)}(x+3)}{(x+1)^2\cancel{(x-3)}} = \frac{3(x-2)(x+3)}{(x+1)^2}, \ x \neq +3$$

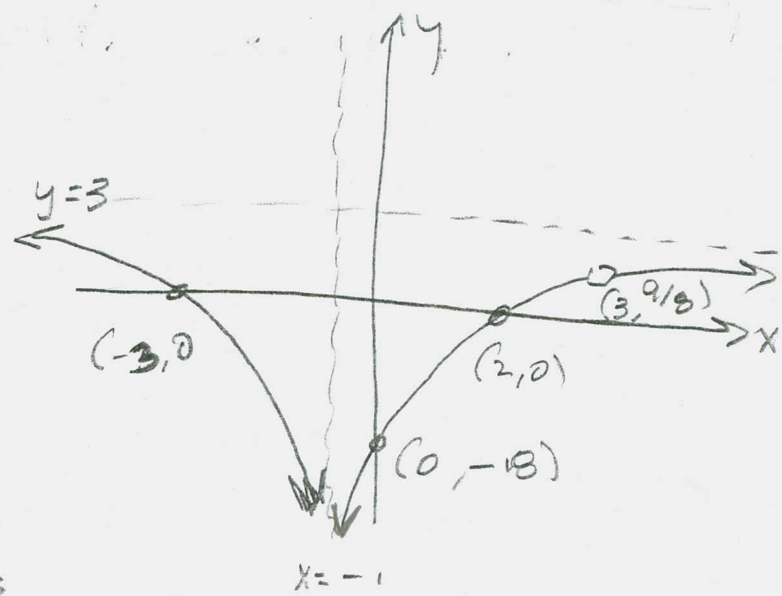
HOLE AT $x=3$:

$$R^*(3) = \frac{3(3-2)(3+3)}{(3+1)^2}$$

$$= \frac{3(1)(6)}{16} = \frac{18}{16} = \frac{9}{8}$$

$\boxed{(3, 9/8) = \text{HOLE}}$

$\boxed{\text{V.A.: } x=-1}$



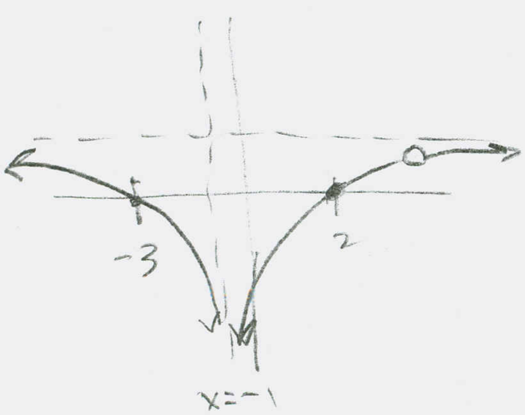
ZEROS: $x=-3, 2$ CROSS

$\boxed{\text{x-int: } (-3,0), (2,0)}$ CROSS

y-int:

$R(0) = \frac{54}{-3} = -18$

$\boxed{\text{y-int: } (0,-18)}$



NEAR $x=-1$: $\dfrac{3(-1-2)(-1+3)}{(x+1)^2}$

$$= \frac{3(-3)(2)}{(x+1)^2} = \frac{-18}{(x+1)^2}$$