Final Test, Fall, 2019 Name Comprehensive

Do all work and **put all your work and (circled) answers** on the white paper provided. Do not write on the backs of the white pages. Leave a margin at the top left corner on every page. A BIG "121" works really well for the top left corner of every page. Show only ONE COLUMN of work. Make sure your work is in proper order before turning in the stack.

Leave room between problems. Do not squeeze work in to fit a page. Start a fresh page. When in doubt about fitting a problem onto a page, start a fresh page. THE ONLY WRITING ON THIS SHEET SHOULD BE YOUR NAME.

If you can't make DARK WRITING with a pencil, borrow a pen. I won't deal with faint writing. I don't have time.

- 1. Solve $10x^2 x 21 = 0$ in three (3) ways:
 - a. (10 pts) Use quadratic formula.
 - b. (10 pts) Complete the square.
 - c. (10 pts) Factoring.
- 2. (20 pts) Sketch the graph of $g(x) = -2\sqrt{5x-20} + 10$. Sketch the graph of g(x) by starting with the graph of $f(x) = \sqrt{x}$ and transforming it in 4 steps (so 5 total graphs) to g(x). Label the points (0, 0) and (1, 1) in your first graph, and show where they get moved around (flipped, stretched, shifted up or down) by each transformation.
- 3. (20 pts) Find an equation in point-slope form through the point (-3,8) of the line that is perpendicular to $y = \frac{7}{3}x - 1,256.4827$
- 4. (20 pts) Solve the absolute value inequality: $|-17x+5|+13 \ge 27$. Give answers in set-builder, line-graph and interval notation forms.
- 5. Suppose I told you that $f(x) = x^6 + 10x^5 178x^4 + 472x^3 + 545x^2 1850x + 1000$ has factored form $f(x) = (x+20)(x-1)^2(x+2)(x-5)^2$.
 - a. (10 pts) Solve $f(x) \le 0$. I expect to see solid analysis of a number-line sign pattern. Show your decision-making process, for as many partial-credit points as possible on this one, especially, as it is work that informs parts 'b' and 'c,' below.
 - b. (10 pts) Find the domain of $g(x) = \ln(f(x)) = \ln((x+20)(x-1)^2(x+2)(x-5)^2)$. Adapt the sign pattern from part 'a' to this question. Much of your work is already done. Not all.
 - c. (10 pts) Provide a rough sketch of f(x), based on your work in part 'a.'
- 6. Let $f(x) = 4x^3 20x^2 + 37x 39$
 - a. (10 pts) Use synthetic division to show that x = 3 is a zero of f(x).

- b. (10 pts) Use your work from part 'a' to help you factor f(x) over the field of complex numbers.
- 7. There are 13 finalists for 4 scholarships.
 - a. (10 pts) If all scholarships are the same, how many ways can can the 4 scholarships be awarded?
 - b. (10 pts) Suppose the 4 scholarships are ranked, so that the top student earns the biggest scholarship, the #2 student earns the next-biggest, and so on down to the 4th student earning the smallest award (which is still very good, as is #4 student). How many different ways can the 4 scholarships be awarded, then?
- 8. (20 pts) Find the equation of the parabola with vertex at (3, 4) and focus at (3, 5).
- 9. (20 pts) Write the equation of the ellipse $25x^2 + 9y^2 + 200x 72y + 544 = 225$ in standard form and sketch its graph.

BONUS SECTION: Work any 2 bonus questions for up to 40 bonus points.

- 1. Let $f(x) = \sqrt{x-8}$ and $g(x) = \frac{x+4}{x-10}$.
 - a. (5 pts) What is the domain of f?
 - b. (5 pts) What is the domain of g?
 - c. (5 pts) Write the function $f \circ g$. Do not simplify.
 - d. (5 pts) What is the domain of $f \circ g$?
- 2. (20 pts) Sketch the graph of $F(x) = \frac{2x^2 x 10}{x^2 + 3x 4}$.
- 3. (20 pts) A woman borrowed money to buy a car for \$40,000. She put \$10,000 down and is financing the other \$30,000 with a loan at 9.2% a.p.r. compounded monthly for 4 years. Assuming there are no fees, what are her monthly payments?
- 4. Evaluate the following geometric sums. Give final answers as fractions in lowest terms.

a. (10 pts)
$$\sum_{k=1}^{10} 2 \cdot \left(\frac{1}{3}\right)^{k-1}$$
.
b. (10 pts) $\sum_{k=1}^{\infty} 2 \cdot \left(\frac{1}{3}\right)^{k-1}$

5. (20 pts) Find the foci and sketch the graph of the ellipse given by the equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$

