121 TEST 2 FALL 2019

(2) $R = \{(-2,3), (1,5), (2,3), (3,-2)\}$

(a) 5pts R is a function.

(b) 5pts $\mathcal{D}(R) = \{-2, 1, 2, 3\}$

(c) 5pts $\mathcal{R}(R) = \{3, 5, -2\}$

(d) 5pts R is not 1-to-1 because

$f(-2) = f(2) = 3$.

(3) $f(x) = \sqrt{x-9}$, $g(x) = \dfrac{x+3}{x-5}$

(a) 5pts $\mathcal{D}(f) = [9, \infty)$, from $x - 9 \geq 0$ & $\sqrt{\ }$

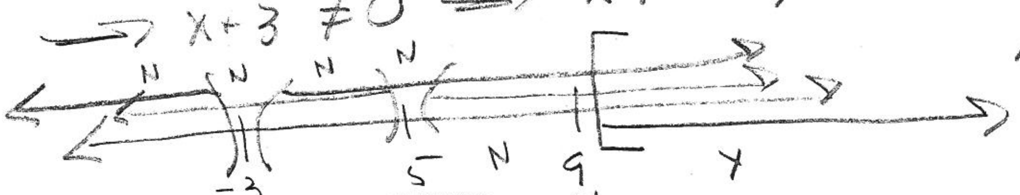
(b) 5pts $\mathcal{D}(g) = \mathbb{R} \setminus \{5\}$, from $x - 5 = 0$ & $\dfrac{\text{stuff}}{0}$

(c) 5pts $\dfrac{f}{g} = \dfrac{\sqrt{x-9}}{\left(\dfrac{x+3}{x-5}\right)}$

(d) 5pts Need $x \geq 9$, $x \neq 5$ and $\dfrac{x+3}{x-5} \neq 0$

$\Rightarrow x + 3 \neq 0 \Rightarrow x \neq -3$, so

AND



$\Rightarrow \boxed{[9, \infty)}$

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(e) (5pts) $f \circ g = \sqrt{\dfrac{x+3}{x-5} - 9}$

(f) (5pts) $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

So $x \neq 5$ AND

$$\frac{x+3}{x-5} \geq 9$$

$$\longrightarrow \frac{x+3}{x-5} - \frac{9(x-5)}{x-5} = \frac{x+3-9x+45}{x-5} = \frac{-8x+48}{x-5} \geq 0$$

$-8x+48=0$

$-8x=-48$

$x=+6$ is critical

$x-5=0$

$x=5$ is critical

$- \quad + \quad - \quad \geq 0 \qquad \dfrac{-8x+48}{x-5}$

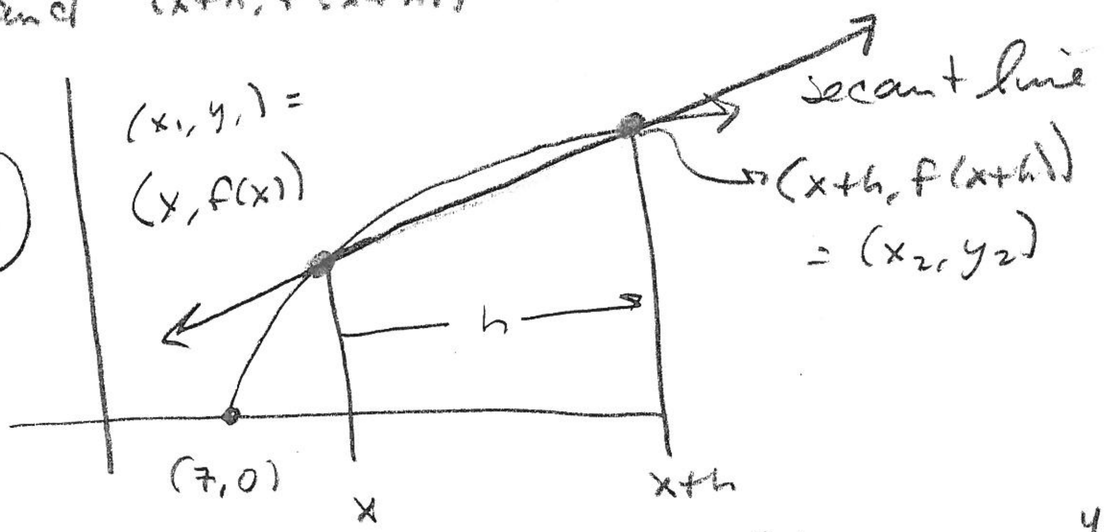
$5 \quad 6$

$\star \quad =0 \longrightarrow$

N N Y Y N

$$\boxed{\mathcal{D}(f \circ g) = (5, 6]}$$

4) $f(x) = 5\sqrt{x-7}$. We illustrate the slope of a secant line to $f(x)$, between $(x, f(x))$ and $(x+h, f(x+h))$



5pts

The slope of the secant line is $m = \frac{y_2 - y_1}{x_2 - x_1} = \cancel{f(x+h) - f}$

$= \frac{f(x+h) - f(x)}{h}$

5) 5pts $f(x) = 3x^2 - 5x$

$\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 5(x+h) - (3x^2 - 5x)}{h}$

$= \frac{3(x^2 + 2xh + h^2) - 5x - 5h - 3x^2 + 5x}{h}$

$= \frac{3x^2 + 6xh + 3h^2 - 5h - 3x^2}{h}$

$= \frac{6xh + 3h^2 - 5h}{h} = \frac{h(6x + 3h - 5)}{h}$

$= \boxed{6x + 3h - 5}$ $\xrightarrow{h \to 0}$ $\boxed{6x - 5 = f'(x)}$

$h \neq 0$

Bonus 2pts

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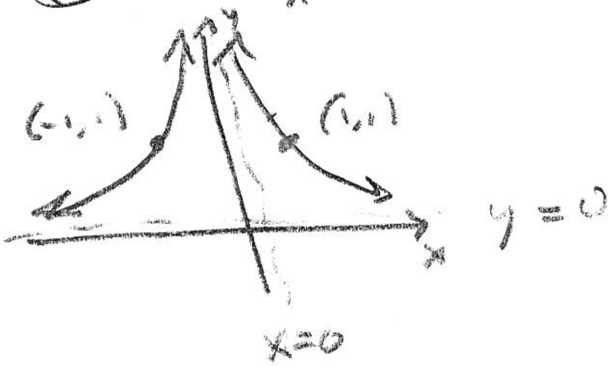
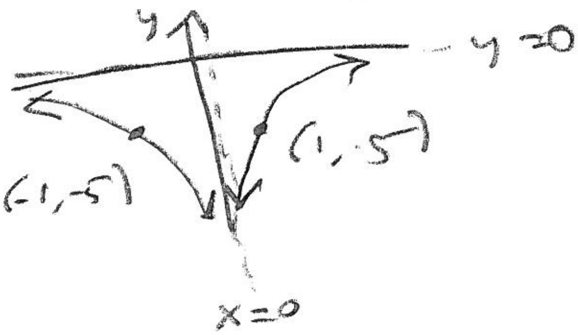
(6) $g(x) = \dfrac{-5}{(2x+4)^2} + 7$ version.

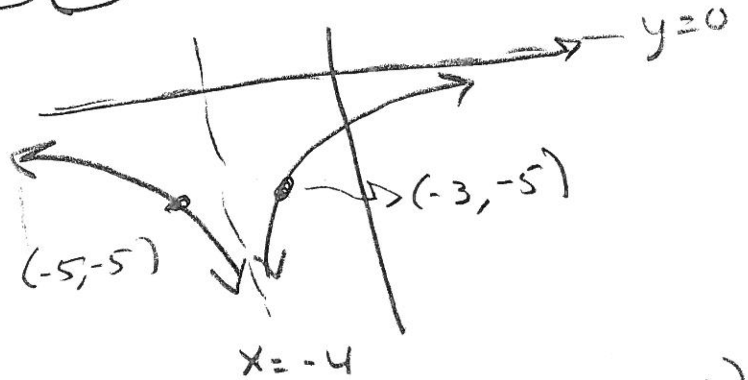
(0) $f(x) = \dfrac{1}{x^2}$



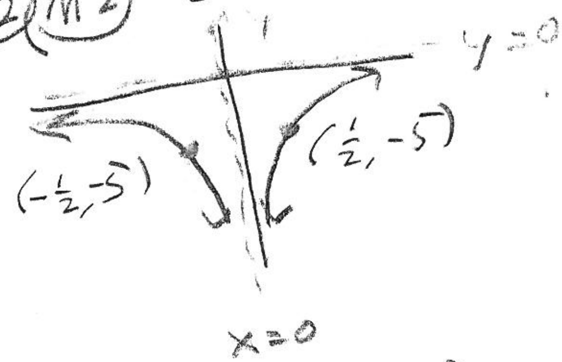
(1) $-5f(x) = -\dfrac{5}{x^2}$



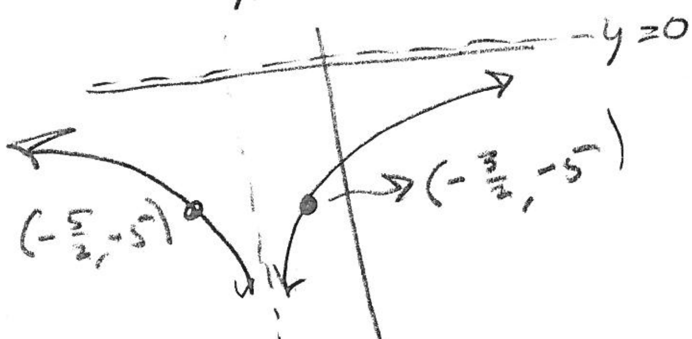
(2) (M1) $-5f(x+4) = \dfrac{-5}{(x+4)^2}$



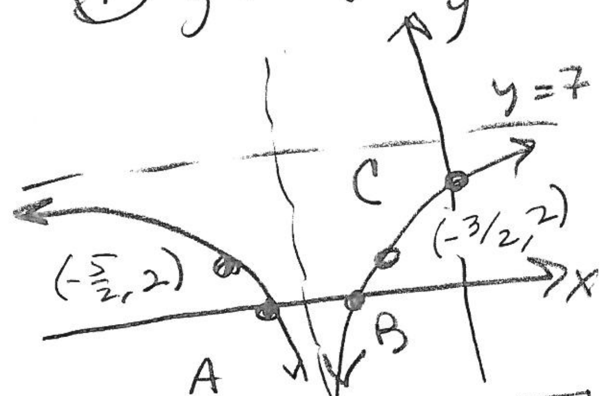
(2) (M2) $-5f(2x)$



(3) $-5f(2x+4) = -5f(2(x+2))$
M1 M2



(4) $g(x)$ (up 7)



(M2) $-\frac{1}{2} \longmapsto -\frac{1}{2} - 2 = -\frac{5}{2}$

$\frac{1}{2} \longmapsto \frac{1}{2} - 2 = -\frac{3}{2}$

(M1) $-5 \longmapsto -\frac{5}{2}$

$-3 \longmapsto -\frac{3}{2}$

$A = \left(-2 - \frac{\sqrt{35}}{14}, 0\right)$

$B = \left(-2 + \frac{\sqrt{35}}{14}, 0\right)$

$C = \left(0, \frac{107}{16}\right)$

6b 5pts $\mathcal{D}(g) = \mathbb{R} \setminus \{-2\}$

$R(g) = (-\infty, 7)$

6c

x-int:

$g(x) = 0$

$\frac{-5}{(2x+4)^2} + 7 = 0$

$-\frac{5}{(2x+4)^2} = -7$

$\frac{1}{(2x+4)^2} = \frac{7}{5}$

$(2x+4)^2 = \frac{5}{7}$

$2x+4 = \pm\sqrt{\frac{5}{7}} = \pm\frac{\sqrt{35}}{7}$

$2x = -4 \pm \frac{\sqrt{35}}{7}$

$x = -2 \pm \frac{\sqrt{35}}{14}$ is fine

OR $\frac{-28 \pm \sqrt{35}}{14}$

y-int:

$g(0) = \frac{-5}{4^2} + 7$

$= \frac{-5 + 7(16)}{16} = \frac{-5 + 112}{16}$

$= \frac{107}{16}$

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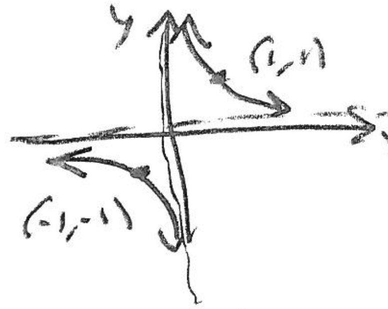
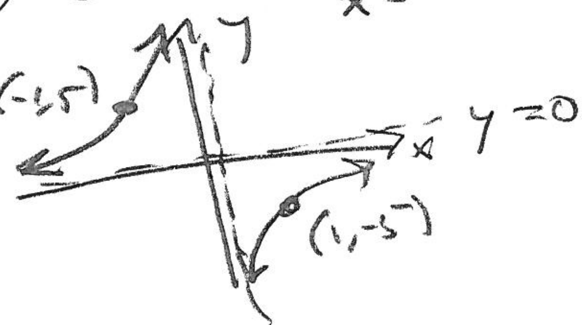
6) $g(x) = \frac{-5}{(2x+4)^3} + 7$ version

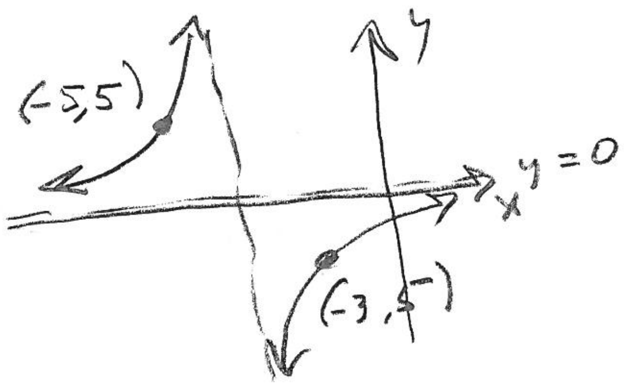
0) $f(x) = \frac{1}{x^3}$



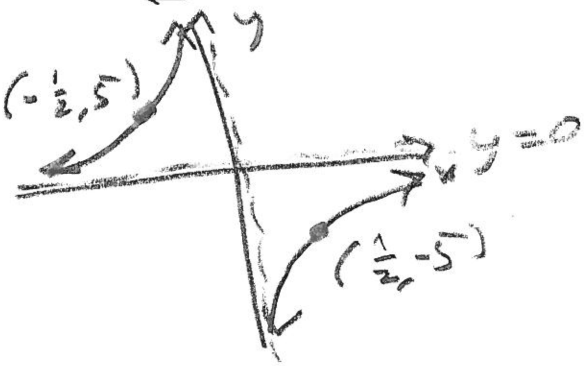
1) $-5f(x) = -\frac{5}{x^3}$



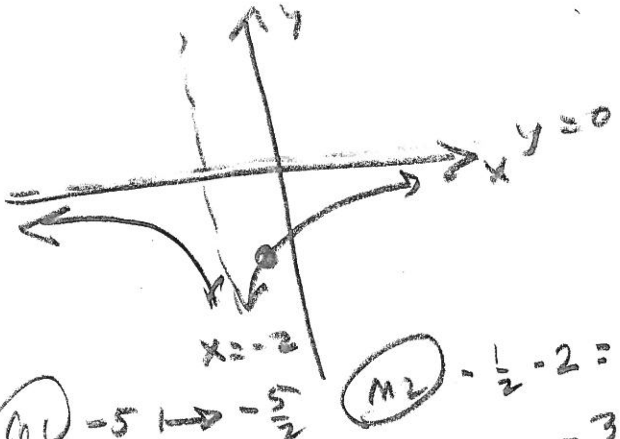
2) M1 $-5f(x+4) = \frac{-5}{(x+4)^2}$



2) M2 $-5f(2x) = \frac{-5}{(2x)^2}$



3) $-5f(\underset{M1}{2x+4}) = -5f(\underset{M2}{2(x+2)})$



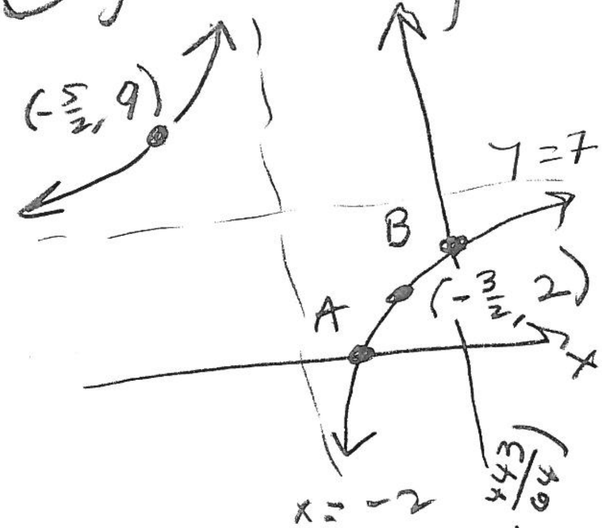
M1 $-5 \longmapsto -\frac{5}{2}$

$-3 \longmapsto -\frac{3}{2}$

M2 $-\frac{1}{2} - 2 = -\frac{5}{2}$

$\frac{1}{2} - 2 = -\frac{3}{2}$

4) $g(x)$ (up 7)



$A = \left(\frac{\sqrt[3]{245}}{7} - 4, 0\right)$

$B = \left(0, \frac{443}{64}\right)$

6b Sols

x-int

$$\frac{-5}{(2x+4)^3}+7=0$$

$$\frac{-5}{(2x+4)^3}=-7$$

$$\frac{1}{(2x+4)^3}=\frac{7}{5}$$

$$(2x+4)^3=\frac{5}{7}$$

$$2x+4=\sqrt[3]{\frac{5}{7}}=\sqrt[3]{\frac{5\cdot 7^2}{7^3}}=\frac{\sqrt[3]{245}}{7}$$

$$2x=\frac{\sqrt[3]{245}}{7}-4$$

$$x=\frac{\frac{\sqrt[3]{245}}{7}-4}{2}$$ is fine

$$=\frac{\sqrt[3]{245}-28}{14}$$ is, too.

y-int

$$g(0)=\frac{-5}{4^3}+7$$

$$=\frac{-5+448}{64}=\frac{443}{64}$$

$$\begin{array}{r} {}^{2}64 \\ \times\ 7 \\ \hline 448 \end{array}$$

6c

$$\boxed{\begin{array}{l} \mathcal{D}(g)=\mathbb{R}\setminus\{-2\} \\ \mathcal{R}(g)=\mathbb{R}\setminus\{7\} \end{array}}$$

7 5 pts $f(x) = 6x+5$ is 1-to-1.

<u>Proof</u> Suppose $f(x_1) = f(x_2)$. Then

$$6x_1 + 5 = 6x_2 + 5$$

$$\Rightarrow 6x_1 = 6x_2$$

$$\Rightarrow x_1 = x_2 \Rightarrow f \text{ is 1-to-1.}$$

8 y is jointly proportional to the square of x and the cube of z.

$$\boxed{y = k x^2 z^3}$$

B1 $f(x) = \frac{1}{\sqrt{x}} \Rightarrow \frac{f(x+h) - f(x)}{h} = \frac{1}{h}\left[\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right]$

$$= \frac{1}{h}\left[\frac{1}{\sqrt{x+h}} \cdot \frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x+h}}{\sqrt{x+h}}\right] = \frac{1}{h}\left[\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right]$$

$$= \frac{1}{h}\left[\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right]\left[\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}\right] = \frac{1}{h}\left[\frac{x - (x+h)}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}\right]$$

$$= \frac{1}{h}\left[\frac{-h}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}\right] = \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \quad (h \neq 0)$$

$$\xrightarrow{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x} + \sqrt{x})} = \frac{-1}{x(2\sqrt{x})} = \boxed{\frac{-1}{2x\sqrt{x}} = f'(x)}$$

B2 $h(x) = 5x^2 - 2x + 7$ M1

$= 5\left(x^2 - \frac{2}{5}x \qquad\right) + 7$

$= \underbrace{5\left(x^2 - \frac{2}{5}x + \left(\frac{1}{5}\right)^2\right)} + 7 - 5\left(\frac{1}{25}\right)$

$= 5\left(x - \frac{1}{5}\right)^2 + 7 - \frac{1}{5}$ $\qquad \frac{35-1}{5} = \frac{34}{5}$

$= \boxed{5\left(x - \frac{1}{5}\right)^2 + \frac{34}{5} = h(x)}$

$h(x) = 5x^2 - 2x + 7$

$\Rightarrow \frac{h(x)}{5} = x^2 - \frac{2}{5}x + \frac{7}{5}$

$= x^2 - \frac{2}{5}x + \left(\frac{1}{5}\right)^2 - \frac{1}{25} + \frac{7}{5} \cdot \frac{5}{5}$

$-1 + 35$

$= \left(x - \frac{1}{5}\right)^2 + \frac{34}{25}$

$\Rightarrow \boxed{h(x) = 5\left(x - \frac{1}{5}\right)^2 + \frac{34}{5}}$

(B3) $f(x) = \dfrac{x-5}{5x^2-2x+7}$

$a=5,\ b=-2,\ c=7$

$b^2-4ac = 4-4(5)(7) < 0 \Rightarrow$ No real sol'ns

$\mathcal{D}(f)$: Need $5x^2-2x+7 \neq 0$

$\Rightarrow$ by previous work, that

$x - \frac{1}{5} = \pm\sqrt{-\frac{34}{5}} \notin \mathbb{R}$

$\Rightarrow 5\left(x-\frac{1}{5}\right)^2 + \frac{34}{5} = 0$

$\Rightarrow 5\left(x-\frac{1}{5}\right)^2 = -\frac{34}{5}$

This eq'n has no real sol'ns $\Rightarrow$

$\Rightarrow \mathcal{D} = \mathbb{R}$

$\boxed{\mathcal{D}(f) = \mathbb{R}}$

(B4) $\mathcal{D}(f) = \mathbb{R}$ by previous work.

$f(x) = \dfrac{x^{77}-5x^{12}+17x}{5x^2-2x+7}$

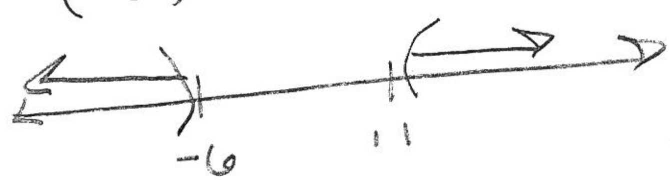
(B5) $|5-2x| > 17$

$5-2x > 17$ OR $5-2x < -17$

$-2x > 12$ $-2x < -22$

$x < -6$ OR $x > 11$

$\Rightarrow x \in (-\infty,-6) \cup (11,\infty)$ OR

from 

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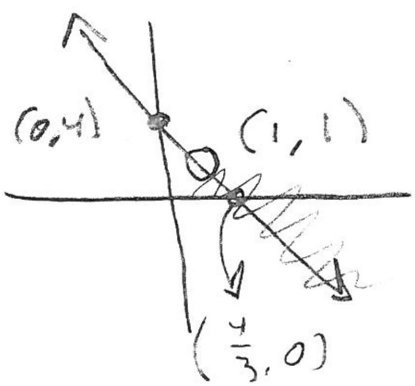
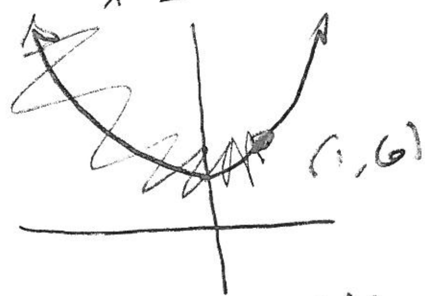
B6 $h(x) = \begin{cases} -3x+4 & \text{if } x<1 \\ x^2+5 & \text{if } x \geq 1 \end{cases}$

$x<1$:

$-3x+4$



$x \geq 1$



$h(x) =$