121 TEST 1 FALL '19

ALL (5pts)

(1) $7x+8 = 2x-5$

$9x = -13$

$\boxed{x = -\frac{13}{9}}$

(2) $\frac{1}{6}x + \frac{7}{3} = \frac{2}{21}x - \frac{5}{7}$

LCD work:

$6 = 2\cdot 3$
$3 = 3$
$21 = 3\cdot 7$
$7 = 7$
} LCD $= 2\cdot 3\cdot 7$

$\frac{x}{2\cdot 3}\cdot\frac{7}{7} + \frac{7}{3}\cdot\frac{2\cdot 7}{2\cdot 7} = \frac{2x}{3\cdot 7}\cdot\frac{3}{3} - \frac{5}{7}\cdot\frac{2\cdot 3}{2\cdot 3}$

$\frac{7x+98}{LCD} = \frac{4x-30}{LCD}$

$3x+98 = 4x-30$

$3x = -128$

$\boxed{x = -\frac{128}{3}}$

(3) $5x^2 = 7$

$x^2 = \frac{7}{5}$

$x = \pm\sqrt{\frac{7}{5}} = \pm\sqrt{\frac{7}{5}}\sqrt{\frac{5}{5}} = \pm\frac{\sqrt{35}}{\sqrt{5\cdot 5}}$ $\boxed{\pm\frac{\sqrt{35}}{5} = x}$

(4) $6x^2 - 4x + 5 = 0$

$a = 6, b = -4, c = 5$

$b^2 - 4ac = (-4)^2 - 4(6)(5)$

$= 16 - 120 = -104$

I got this wrong

$\sqrt{-104} =$

$= 2\sqrt{26}\, i$

$= 2i\sqrt{26}$

2 | 104
2 | 52
2 | 26
13

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm 2i\sqrt{26}}{2(6)} = \boxed{\frac{2 \pm i\sqrt{26}}{6} = x}$$

(5) $5x^2 = 7 \longrightarrow$

$5x^2 + 0x - 7 = 0$

$\longrightarrow a = 5, b = 0, c = -7$

$\longrightarrow b^2 - 4ac = 0^2 - 4(5)(-7)$

$= 140 > 0$ ∴ not a perfect square

2 | 140
2 | 70
5 | 35
3

$\longrightarrow$ 2 real (irrational) solns

(6) $6x^2 - 4x + 5 = 0$

See #4:

$b^2 - 4ac = 111 = -104 < 0$

$\Rightarrow$ 2 nonreal solns

(7) $132x^2 - 29x - 28 = 0$

$a = 132, b = -29, c = -28$

$\Rightarrow b^2 - 4ac = (-29)^2 - 4(132)(-28)$

$= 841 + 14784 = 15625$

$\sqrt{15625} = 125 \Rightarrow$

$b^2 - 4ac > 0$ & perfect square

$\Rightarrow$ 2 rational solns

(8) $x^2 - 12x - 1260 = 0$

$-12 = (-13) + 1$

$= -50 + 38$

$= -40 + 28$

$= -42 + 30$

-13

-1900

-1120

-1260 ! sweet!

$1260 = 2 \cdot 630 = 2 \cdot 2 \cdot 315 = 2 \cdot 2 \cdot 3 \cdot 105 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 35 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$

So $x^2 - 42x + 30x - 1260$

$= x(x-42) + 30(x-42)$

$= (x-42)(x+30) = 0$ $\Rightarrow$ $x \in \{-30, 42\}$

$\rightarrow$ I need to see thi

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(9) $132x^2 - 29x - 28 = 0$

Magic: $(132)(-28) = 3696$

MIDDLE:

$-29 = -30 + 1$ $\quad -30$

$= -50 + 21$ $\quad -1050$

$= -70 + 41$ $\quad -2870$

$= -90 + 61$ $\quad -5490$

$= -80 + 51$ $\quad -4080$

$= -75 + 46$ $\quad -3450$

$= -76 + 47$ $\quad -3572$

$= -77 + 48$ $\quad -3696$ Sweet!

So, $132x^2 - 77x + 48x - 28$

$= 11x(12x - 7) + 4(12x - 7)$

$= (12x - 7)(11x + 4) = 0$

$\Rightarrow \boxed{x \in \{-\frac{4}{11}, \frac{7}{12}\}}$

SLEDGEHAMMER: $x = -\frac{4}{11}, \frac{7}{12} \Rightarrow$

$132\left(x + \frac{4}{11}\right)\left(x - \frac{7}{12}\right)$

$= 12 \cdot 11\left(x + \frac{4}{11}\right)\left(x - \frac{7}{12}\right)$

$= \underline{(11x + 4)(12x - 7)}$ FACTORED!

Sledgehammer:

$b^2 - 4ac$

$= (-29)^2 - 4(132)(-28)$

$= 15625$ (previous work)

$= 125^2$ (also previous)

$\Rightarrow$

$x = \frac{29 \pm 125}{2(132)}$

$= \begin{cases} \frac{29+125}{264} = \frac{154}{264} \\ \frac{29-125}{264} = -\frac{96}{264} \end{cases}$

$\frac{154}{264}$:

$2 \mid 154$

$7 \mid 77$

11

$2 \mid 264$

$2 \mid 132$

$2 \mid 66$

$3 \mid 33$

11

$\frac{2 \cdot 7 \cdot 11}{2^3 \cdot 3 \cdot 11} = \frac{7}{2^3 \cdot 3} = \frac{7}{12}$

$\frac{96}{264}$:

$2 \mid 96$

$2 \mid 48$

$2 \mid 24$

$2 \mid 12$

$2 \mid 6$

3

$\frac{2^5 \cdot 3}{2^3 \cdot 3 \cdot 11} = \frac{2^2}{11} = \frac{4}{11}$

(12) T1

$\frac{12}{2} = 6 \rightsquigarrow 6^2 = 36$ (5)

(10) $x^2 - 12x - 1260$

$= x^2 - 12x + 6^2 - 36 - 1260$

$= (x-6)^2 - 1296$ SET 0

$\Rightarrow (x-6)^2 = 1296$

$x - 6 = \pm\sqrt{1296}$

$= \pm\sqrt{2^4 \cdot 3^4}$

$= \pm 2^2 \cdot 3^2 = 36$

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
	3

So,

$x - 6 = \pm 36$

$x = 6 \pm 36$ → 42, → −30

$x \in \{-30, 42\}$

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(11) $7x^2+3x-10$ (M1)

$(\frac{3}{7})(\frac{1}{2}) = \frac{3}{14} \rightsquigarrow (\frac{3}{14})^2 = \frac{9}{196}$

$= 7(x^2+\frac{3}{7}x) - 10$

$= 7(x^2+\frac{3}{7}x+(\frac{3}{14})^2) - 10 - 7(\frac{3}{14})^2$

$= 7(x+\frac{3}{14})^2 - \frac{10}{1}\cdot\frac{28}{28} - \frac{9}{28}$

$= 7(x+\frac{3}{14})^2 - \frac{289}{28} = 0$

$7(\frac{9}{14^2}) = 7(\frac{9}{2^2\cdot 7^2}) = \frac{9}{2^2\cdot 7} = \frac{9}{28}$

$\Rightarrow 7(x+\frac{3}{14})^2 = \frac{289}{28} = \frac{17^2}{4\cdot 7}$

$(x+\frac{3}{14})^2 = \frac{289}{7\cdot 7\cdot 4}$

$x+\frac{3}{14} = \pm\sqrt{\frac{17^2}{4\cdot 7^2}} = \pm\frac{17}{2\cdot 7} = \frac{17}{14}$ $\quad \frac{-280-9}{28} = -\frac{289}{28}$

$x = \frac{-3\pm 17}{14}$ $\quad \frac{14}{14}$ $\quad \frac{-20}{14} = \frac{-10}{7}$

$x \in \{-\frac{10}{7}, 1\}$

(11) (12)

$7x^2 + 3x - 10 = 0$

$x^2 + \frac{3}{7}x - \frac{10}{7} = 0$

$x^2 + \frac{3}{7}x = \frac{10}{7}$

$x^2 + \frac{3}{7}x + \left(\frac{3}{14}\right)^2 = \frac{10}{7} + \frac{9}{196}$

$= \frac{10}{7} \cdot \frac{2^2 \cdot 7}{4 \cdot 7} + \frac{9}{196}$

$= \frac{280 + 9}{196} = \frac{289}{196}$

2|196
2|98
7|49
7

$\Rightarrow \left(x + \frac{3}{14}\right)^2 = \frac{289}{196}$

$\Rightarrow x + \frac{3}{14} = \pm\sqrt{\frac{289}{196}} = \pm\frac{17}{14}$

$\Rightarrow x = \frac{-3 \pm 17}{14}$, etc.

(12) $(x_1, y_1) = (-4, 2)$, $m = \frac{2}{5}$

(a) $m_{\parallel} = m \Rightarrow$ $y = m(x - x_1) + y_1$

$y = \frac{2}{5}(x + 4) + 2$ STOP!

$= \frac{2}{5}x + \frac{8}{5} + \frac{10}{5}$

$= \frac{2}{5}x + \frac{18}{5}$

$= .4x + 3.6$

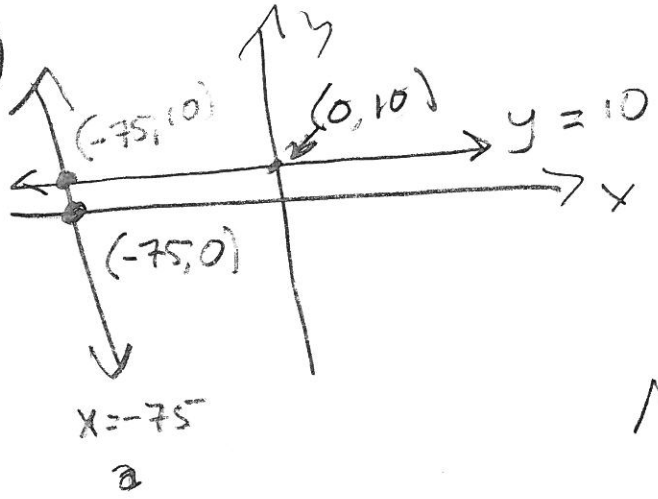
(b) $m_{\perp} = -\frac{1}{m} = -\frac{5}{2} \Rightarrow$

$y = -\frac{5}{2}(x + 4) + 2$

$= -\frac{5}{2}x - \frac{20}{2} + \frac{4}{2}$

$= -\frac{5}{2}x - \frac{16}{2} = -\frac{5}{2}x - 8$

(13)

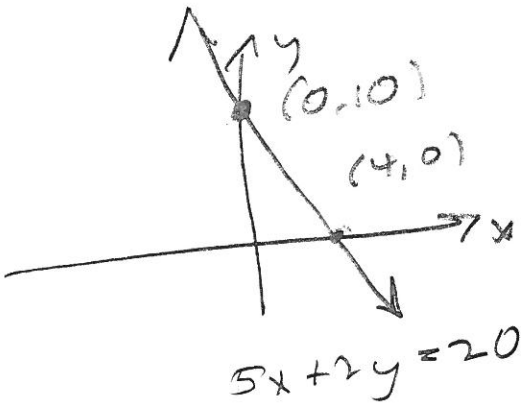


$x = -75$

(b)

(14) $5x + 2y = 20$

x	y
0	10
4	0



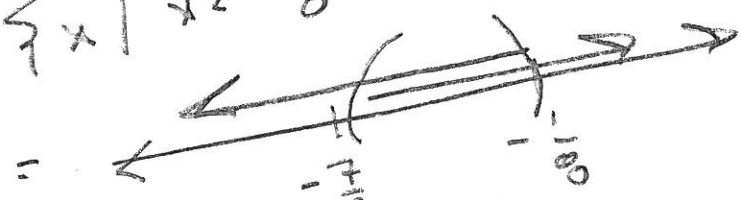
(15) $|8x+4| < 3$

$8x+4 < 3$ and $8x+4 > -3$

$8x < -1$ $\quad 8x > -7$

$\left\{ x \mid x < -\frac{1}{8} \text{ and } x > -\frac{7}{8} \right\}$

AND



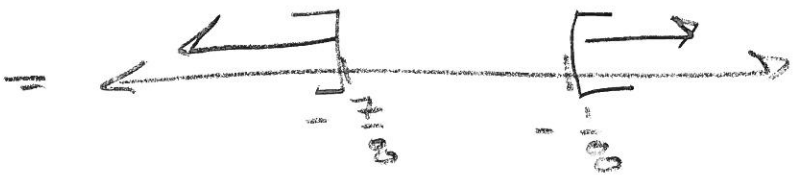
$= \left(-\frac{7}{8}, -\frac{1}{8}\right)$

(15b) $|8x+4| \geq 3$

$8x+4 \geq 3$ OR $8x+4 \leq -3$

$8x \geq -1$ OR $8x \leq -7$

$\{x \mid x \geq -\frac{1}{8}$ OR $x \leq -\frac{7}{8}\}$

$=$ 

OR

$= (-\infty, -\frac{7}{8}] \cup [-\frac{1}{8}, \infty)$

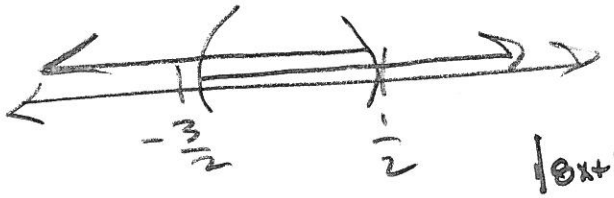
(15c) $|8x+4| - 5 < 3$

$|8x+4| < 8$

$8x+4 < 8$ AND $8x+4 > -8$

$8x < 4$ $8x > -12$

$\{x \mid x < \frac{4}{8} = \frac{1}{2}$ and $x > -\frac{12}{8} = -\frac{3}{2}\}$

$=$ 

AND

$= (-\frac{3}{2}, \frac{1}{2})$

(15d) $|8x+4| + 5 \geq 3$

$|8x+4| \geq -2$

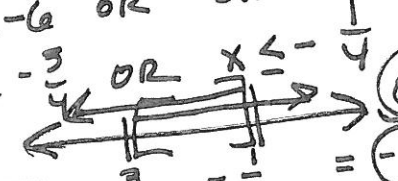
ALWAYS $\Rightarrow$ $\mathbb{R}$

$|8x+4| \geq -2$

$8x+4 \geq -2$ OR $8x+4 \leq 2$

$8x \geq -6$ OR $8x \leq -2$

$x \geq -\frac{3}{4}$ OR $x \leq -\frac{1}{4}$



OR

$= (-\infty, \infty)$!

$= \mathbb{R}$

$= \{x \mid x \in \mathbb{R}\}$

(17) Let x = the # of hours they work

John = 20 hrs, Tracy = 15 hrs. So

$$\frac{1}{20}x + \frac{1}{15}x = 1$$

(16) Let x = the amt of 30% alcohol (liters)

y = " " " 60% " (liters)

	Vol	Pure Alc
	x	$.3x$
	y	$.6y$
total	$x+y$ $=100$	$.44(100)$ $=44$

So $x+y=100$

& $.3x+.6y=44$

OR $.3x+.6(100-x)=44$

(B1) Solve #17:

$$\frac{x}{20} + \frac{x}{15} = 1$$

$$\frac{x}{2\cdot2\cdot5}\cdot\frac{3}{3} + \frac{x}{3\cdot5}\cdot\frac{2\cdot2}{2\cdot2} = \frac{60}{60}$$

$$3x + 4x = 60$$

$$7x = 60$$

$$x = \frac{60}{7} \text{ hrs}$$

2|20 3|15
2|10 5
5

$60 = 2\cdot2\cdot3\cdot5$
$= 60$

~~B2~~ 121

T1

B5 $x^3 - 8 = 0$

$(x-2)(x^2+2x+4)$

$x=2$ or $x^2+2x+4=0$

$x^2+2x+1^2 = -4+1$

$(x+1)^2 = -3$

$x+1 = \pm\sqrt{3} = \pm i\sqrt{3}$

$x = -1 \pm i\sqrt{3}$

$x \in \{2, -1 \pm i\sqrt{3}\}$

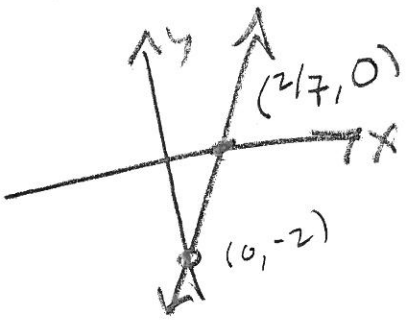
B3 $y = 7x-2$

x	y
0	-2
$\frac{2}{7}$	0

$7x-2=0$

$7x=2$

$x=\frac{2}{7}$



B4 $f(x) = x^2+12x-1260$

$= x^2+12x+6^2-36-1260$

$= (x-6)-1296$

B2 John's 2 hours late

Let x = # of hours Tracy works. Then $x-2$ = " " " John works, and

$\frac{1}{20}(x-2) + \frac{1}{15}x = 1$

2 | 20
2 | 10
5

3 | 15
5

LCD = 2·2·3·5 = 60

$\frac{x-2}{2\cdot2\cdot5}\cdot\frac{3}{3} + \frac{x}{3\cdot5}\cdot\frac{2\cdot2}{2\cdot2} = \frac{LCD}{LCD} = \frac{60}{60}$

$3x-6+4x = 60$

$7x = 66$

$x = \frac{66}{7}$

$x-2 = \frac{66-14}{7}$

$= \frac{52}{7} = x-2$