121 TEST 5 Spring, 2019

1a 10pts $C(20,5) = \frac{20!}{5!15!}$ $20-5=15$

$= \frac{\cancel{20}^{1} \cdot 19 \cdot \cancel{18}^{3} \cdot 17 \cdot 16}{\cancel{5 \cdot 4 \cdot 3 \cdot 2}} = 19 \cdot 3 \cdot 17 \cdot 16 = 15{,}504$

"20 Choose 5"

1b 10pts $P(20,5) = \frac{20!}{15!} = 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16$

$\boxed{= 1860480}$ "20 choose & arrange 5"

2 10pts $9x^2 + 16y^2 - 90x + 96y = -225$

$9x^2 - 90x \quad + 16y^2 + 96y \quad = -225$

$9(x^2 - 10x \quad) + 16(y^2 + 6y \quad) = -225$

$9(x^2 - 10x + 5^2) + 16(y^2 + 6y + 3^2) = -225 + 9(25) + 16(9)$

$-225 + 225 + 144$

$9(x-5)^2 + 16(y+3)^2 = 144$

$\frac{9(x-5)^2 + 16(y+3)^2}{144} = \frac{144}{144}$

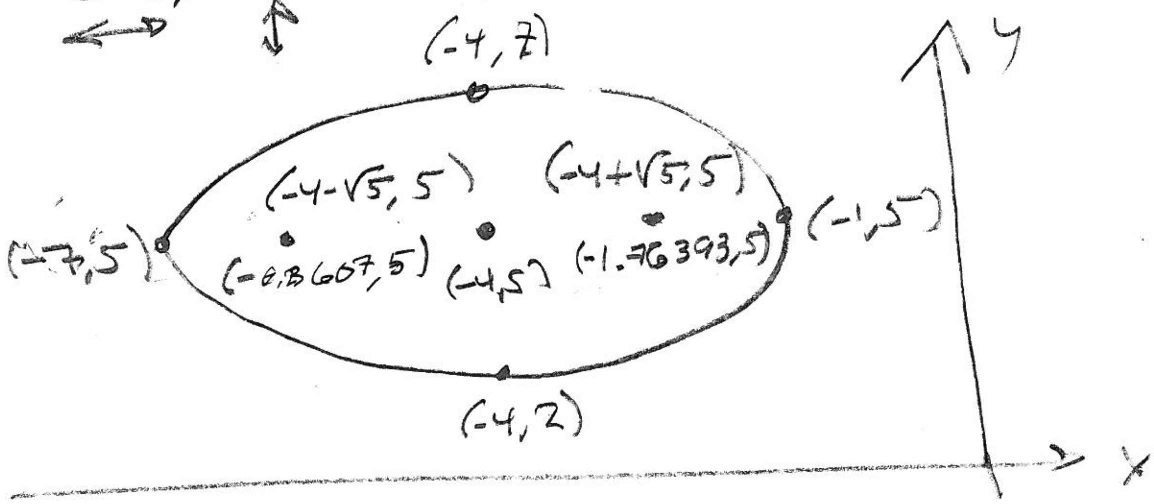
$$\boxed{\frac{(x-5)^2}{16} + \frac{(y+3)^2}{9} = 1}$$

121 T5

3 10 pts $\frac{(x+4)^2}{9} + \frac{(y-5)^2}{4} = 1$

$(h, k) = (-4, 5)$

$a = 3, \ b = 2$



$c^2 = a^2 - b^2 = 9 - 4 = 5$

$c = \sqrt{5}$

4 a 10 pts $\frac{1}{2}$-life is 3300 yrs

$\boxed{A(t) = A_0 e^{kt}}$

$A(3300) = A_0 e^{3300k} = \frac{1}{2} A_0$

$3300k = \ln\left(\frac{1}{2}\right) = -\ln(2)$

$\boxed{k = \frac{-\ln(2)}{3300}} \approx -.000210046 002$

b 5 pts 27% of A_0 remains.

$A_0 e^{kt} = .27 A_0 \qquad t = \frac{\ln .27}{k}$

$e^{kt} = .27$

$kt = \ln(.27) \qquad = \frac{\ln(.27)}{\frac{-\ln(2)}{3300}} \approx \boxed{6234 \text{ yrs}}$

121 TS

5a 5pts

x = # of cookies sold

y = $ in cash box

$(x_1, y_1) = (10, 67.5)$

$(x_2, y_2) = (30, 92.5)$

~~Let x = price of cookie~~ ~~y = amt of money in cash box~~ $/cookie

$\Longrightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{92.5 - 67.5}{30 - 10} = \frac{25}{20} = \frac{5}{4} = 1.25$

$\cancel{\frac{1.25 \$}{cookie}} = \frac{1.25 \$}{cookie}$

$y = m(x - x_1) + y_1$

$y = 1.25(x - 10) + 67.5$

$= 1.25x - 12.5 + 67.5$

$= 1.25x + 55$

Price per cookie

in cash box to start the day

(a) $1.25/cookie = m

(b) $55 to start

5pts (a) $1.25/cookie

5pts (b) $55 to start the day.

6 10pts $(x - 3y)^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$= x^5 + 5x^4(-3y) + 10x^3(-3y)^2 + 10x^2(-3y)^3$

$+ 5x(-3y)^4 + (-3y)^5$

$= \boxed{x^4 - 15x^4y + 90x^3y^2 - 270x^2y^3 + 405xy^4 - 243y^5}$

121 T5

7 10pts Let $\boxed{\begin{array}{l} x = \text{amt of time John works (hrs)} \\ y = \text{" " " Bill " (hrs)} \end{array}}$

5 hrs for John solo
8 " " Bill solo

$\frac{1}{5}x + \frac{1}{8}y = 1$

Now, John starts 2 hours later, or $x = y - 2$
Bill " " " earlier $y = x + 2$

So $\frac{1}{5}x + \frac{1}{8}(x+2) = 1$

LCD $= 5 \cdot 8 = 40$

$\frac{x}{5} \cdot \frac{8}{8} + \frac{1}{8} \cdot \frac{5}{5}(x+2) = \frac{1}{1} \cdot \frac{40}{40}$

$\frac{8x + 5x + 10}{LCD} = \frac{40}{LCD}$

$13x + 10 = 40$

$13x = 30$

$x = \frac{30}{13}$

$\boxed{x = \frac{30}{13}}$ $\Rightarrow y = x + 2 = \frac{30}{13} + \frac{26}{13}$

$= \boxed{\frac{56}{13}}$

$\boxed{x = \frac{30}{13}, \ y = \frac{56}{13}}$

$x \approx 2.3076923, \ y \approx 4.3076923$

121 J5

8 $\left[\begin{array}{ccc|c} 1 & 2 & -4 & -23 \\ -2 & -3 & 5 & 26 \\ 3 & 4 & -5 & -22 \end{array}\right] \begin{array}{l} R1 \\ 2R1+R2 \\ -3R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -4 & -23 \\ 0 & 1 & -3 & -20 \\ 0 & -2 & 7 & 47 \end{array}\right]$

$\begin{array}{l} R1 \\ R2 \\ 2R2+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -4 & -23 \\ 0 & 1 & -3 & -20 \\ 0 & 0 & 1 & 7 \end{array}\right]$

$x+2y-4z=-23$

$y-3z=-20$

$\boxed{z=7}$

$y-3z=y-3(7)=y-21=-20$

$\rightarrow \boxed{y=1}$

$x+2y-4z=x+2(1)-4(7)=x+2-28=x-26$

$x-26=-23$

$\boxed{x=3}$ $\boxed{(x,y,z)=(3,1,7)}$

B1 SPB $f(x)=14\log_4(-4x-8)+7$

$14\log_4(-4y-8)+7=x$

$14\log_4(-4y-8)=x-7$

$\log_4(-4y-8)=\frac{x-7}{14}$

$-4y-8=4^{\frac{x-7}{14}}$

$-4y=4^{\frac{x-7}{14}}+8$

$y=\frac{4^{\frac{x-7}{14}}+8}{-4}=\boxed{\frac{4^{\frac{x-7}{14}}}{-4}-2=f^{-1}(x)}$

121 T5

B2 5pts $|-3x-5|-4<8$

$|-3x-5|<4$

$-3x-5<4$ and $-3x-5>-4$

$-3x<9$ and $-3x>1$

$\{x \mid x>-3$ and $x<-\frac{1}{3}\}$

AND

$= \boxed{\left(-3, -\frac{1}{3}\right)}$

B3 $3+\frac{3}{4}+\frac{3}{16}+\cdots+\frac{3}{16{,}384}$

a 5pts $a+\cdots+ar^{n-1}$

$\boxed{a=3,\ r=\frac{1}{4},\ n=8}$

$4 \overline{)16384}$
$4 \overline{)4096}$
$4 \overline{)1024}$
$4 \overline{)256}$
$4 \overline{)64}$
$4 \overline{)16}$
4

$16384=4^7$

$7=n-1$

$\boxed{8=n}$

b 5pts $3\left(\frac{1-\left(\frac{1}{4}\right)^8}{1-\frac{1}{4}}\right)=3\left(\frac{1-\frac{1}{65336}}{\left(\frac{3}{4}\right)}\right)$

$$\begin{array}{r} 16384 \\ \times\ 4 \\ \hline 65536 \end{array}$$

$=\left(\frac{4}{3}\right)(3)\left(\cdots\right)$

$=\frac{4(65335)}{65536}=\boxed{\frac{65335}{16384}} \approx 3.999938965$

.121 T5

B4 5 pts $\sum_{k=1}^{\infty} 5 \cdot \left(\frac{2}{3}\right)^{k-1}$ $a=5,\ r=\frac{2}{3}$

$\Longrightarrow S = 5\left(\frac{1}{1-r}\right) = 5\left(\frac{1}{1-2/3}\right)$

$= 5\left(\frac{1}{\frac{1}{3}}\right) = 5 \cdot \frac{3}{1} = \boxed{15}$

B5 5 pts Present Value Banker doesn't care if you give him payments or one lumpsum of the same amount.

$$P(1+i)^n = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Want Present Value $P \Longrightarrow$

$$P = (1+i)^{-n} R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= R\left[\frac{(1+i)^{-n}\left((1+i)^n - 1\right)}{i}\right]$$

$$= R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$R = \$700,\ r = .0425,\ t = 30,\ m = 12$

$i = \frac{r}{m} = \frac{.045}{12},\ n = mt = 12(30) = 360$

$$P = 700\left[\frac{1-\left(1+\frac{.045}{12}\right)^{-360}}{\frac{.045}{12}}\right] \approx \boxed{\$138,152.81}$$

121 TS

B6 5pts $f(x) = 4x^2 - 3x + 1$

$= 4\left(x^2 - \frac{3}{4}x \quad\right) + 1$

$= \underbrace{4\left(x^2 - \frac{3}{4}x + \left(\frac{3}{8}\right)^2\right) + 1 - 4\left(\frac{9}{64}\right)}$

$= \boxed{4\left(x - \frac{3}{8}\right)^2 + \frac{7}{16} = f(x)}$

$1 - \frac{9}{16} = \frac{16-9}{16} = \frac{7}{16}$