MAT 121-G11 Test 4 – Spring, 2019 Name_____
100 Points Covers Chapter 4 and 5 NO GRAPHING CALCULATORS!!!

1. (20 pts) Starting with $f(x)=\log_3(x)$, sketch the graph of $g(x)=5\cdot\log_3(4x+24)+3$ in 5 steps (counting $f(x)=\log_2(x)$ as the first step). Use $x=\frac{1}{3}, x=1$, and $x=3$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$.

2. Find the *exact* *x*- and *y*-intercepts for $g(x)$ from #1. That means no decimal approximations. I'm looking for the symbolic manipulations that you will need at the next level.

   a. (4 pts) *x*-intercept: *A* =

   b. (4 pts) *y*-intercept: *B* =

   c. (2 pts) Label your final graph for #1 with the intercepts labeled with *A* and *B*.

3. (5 pts) Find the inverse, $g^{-1}(x)$, for $g(x)$ in #1. The moves are very similar to what you did in #2a.

4. Let $f(x)=\sqrt{x-24}$ and $g(x)=x^2+7x-18$.

   a. (5 pts) What is the domain of f ?

   b. (5 pts) What is the domain of g ?

   c. (5 pts) Determine $\left(\frac{f}{g}\right)(x)$. (Sometimes this is just called $\frac{f}{g}$ in the text.)

   d. (5 pts) What is the domain of $\left(\frac{f}{g}\right)(x)$?

   e. (5 pts) Determine $(f\circ g)(x)$ (Again, sometimes just called $f\circ g$).

   f. (5 pts) What is the domain of $f\circ g$?

5. The next two domain questions are very closely related.

   a. (5 pts) What is the domain of $\sqrt{\frac{(x-2)^4(x+8)}{(x+4)^3(x-5)^3}}$?

   b. (5 pts) What is the domain of $\log_{11}\left(\frac{(x-2)^4(x+8)}{(x+4)^3(x-5)^3}\right)$?

6. (10 pts) Solve $\ln(x+9)+\ln(x-2)=\ln(24)$. Give the exact solution(s).

7. Suppose the half-life of C-14 is 4900 years. (It isn't, quite, but just suppose…).

   a. (10 pts) Derive the exponential decay model, $A(t)=A_0e^{kt}$. The trick is to use the half-life to find the relative decay rate, *k*.

   b. (5 pts) How old is a sample of charcoal from a prehistoric fire pit, if 66% of the C-14 has decayed (i.e., 34% is left.) ? Round to the nearest year in your final answer. If it makes it easier for you, use an initial mass of 100 g of radioactive C-14 and a final mass of 53 g of the radioactive material. It's the same thing.

Bonus Answer up to four (4) 5-pointers. That's a total of 20 bonus points possible. Points to be had. Standards are high.

B 1 (5 pts) Solve the absolute value inequality: $|-5x+2|-7\geq -5$.

B 2 (5 pts) Re-write $f(x)=5x^2-2x-11$ in the form $a(x-h)^2+k$.

B 3 (5 pts) Solve the exponential equation $3\cdot 5^x=10\cdot \pi^x$. Give the exact answer in terms of natural logarithms.

B 4 (5 pts) Sketch the graph of $R(x)=\dfrac{(x-2)^4(x+8)}{(x+4)^3(x-5)^3}$. Your sign pattern from #5a is a great assist.

B 5 (5 pts) Sketch the graph of $Q(x)=\sqrt{\dfrac{(x-2)^4(x+8)}{(x+4)^3(x-5)^3}}$. Your graph from **B4** is a great assist.

B 6 (5 pts) Sketch the graph of $g(x)=-5\cdot 3^{3x+9}+11$.

B 7 (Super-Bonus) A hard-working teacher borrows $80,000 to buy a house on a 30-year mortgage, which they're amortizing with monthly payments (at the end of each month) at 4.5% annual percentage rate, compounded daily. I want you to figure out their monthly payments, by answering the following:

a) (5 pts) Given the Future Value of an annuity is given by $S=R\left[\dfrac{(1+i)^n-1}{i}\right]$, and the Future Value of a lump sum drawing interest is given by $A=P(1+i)^n$. Solve the equation $A = S$ for R. This will give the payment in symbolic form.

b) (5 pts) Supply the values *i*, *n*, and *P* from the word problem and give me the monthly payment, to the nearest penny.