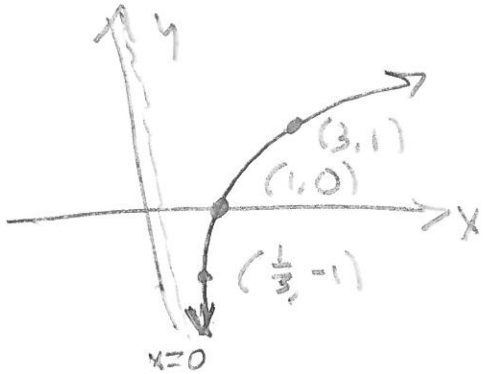
121 TEST 4 SPRING, 2019

(1) (20 pts) $f(x) = \log_3(x)$, $g(x) = 5\log_3(4x+24)+3$

$M_1 = 5f(4x+24)+3$

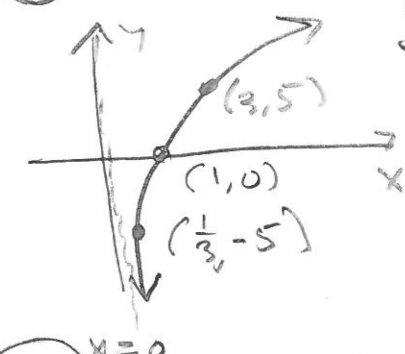
$M_2 = 5f(4(x+6))+3$

(0) $f(x) = \log_3(x)$



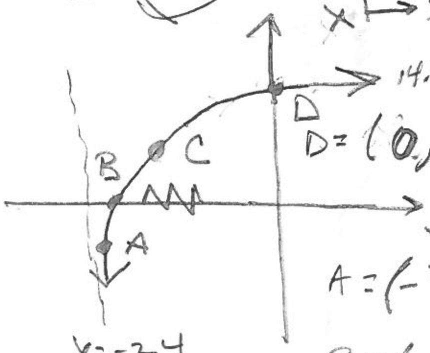
(1) $5f(x) = 5\log_3(x)$

$y \mapsto 5y$



(2) M1 $5f(x+24)$

$x \mapsto x-24$



14.4639463

$D = (0, 5\log_3(24))$

$A = (-\frac{71}{3}, -5)$

$B = (-23, 0)$

$C = (-21, 5)$

A: $\frac{1}{3} - 24 \cdot \frac{3}{3}$

$= \frac{1-72}{3} = -\frac{71}{3}$

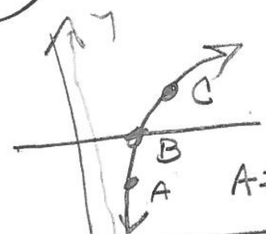
B: $1-24 = -23$

C: $3-24 = -21$

D: $5\log_3(0+24)+3$

M2 $5f(4x) = 5\log_3(4x)$

$x \mapsto \frac{1}{4}x$



$A = (\frac{1}{12}, -5)$

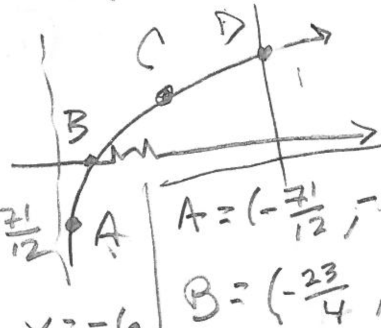
A: $\frac{1}{3} \div 4$ $= \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

$B = (\frac{1}{4}, 0)$

$C = (\frac{3}{4}, 5)$

(3) $5f(4x+24)$ M1 $x \mapsto \frac{1}{4}x$ $5\log_3(4x+24)$

$= 5f(4(x+6))$ M2 $x \mapsto x-6$



$5\log_3(4(x+6))$

M2: A: $\frac{1}{12} - 6 = \frac{1}{12} - \frac{72}{12} = -\frac{71}{12}$

B: $\frac{1}{4} - 6 = \frac{1-24}{4} = -\frac{23}{4}$

C: $\frac{3}{4} - 6 = \frac{3-24}{4} = -\frac{21}{4}$

$A = (-\frac{71}{12}, -5)$

$B = (-\frac{23}{4}, 0)$

$C = (-\frac{21}{4}, 5)$

M1 A: $-\frac{71}{3} \div 4 = -\frac{71}{12}$

B: $-\frac{23}{12}$

C: $-\frac{21}{4}$

D: $5\log_3(0+24)$

$D = (0, 5\log_3(24))$

14.4639463

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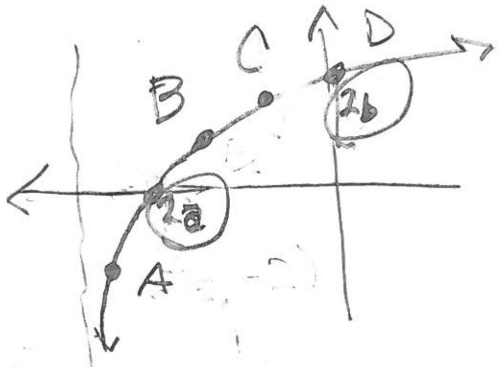
4 #1 Cont'd

$g(x) = 5f(4x+24)+3 = 5f(4(x+6))+3$

M1 & M2 : $y \longmapsto y+3$



$x=-6$

$A = (-\frac{71}{12}, -2)$

$B = (-\frac{23}{4}, 3)$

$C = (-\frac{21}{4}, 8)$

$D = (0, 5\log_3(24)+3)$

A: $-5+3=-2$

B: $0+3=3$

C: $5+3=8$

By transformations:

$2a = (\frac{1}{4}(3^{-3/5})-6, 0)$

-5.870679535

$2b = (0, 5\log_3(24)+3)$

17.4639468

2a 4pts x-int: $y=0$

$5f(4x+24)+3=0$

$5\log_3(4x+24) = -3$

$\log_3(4x+24) = -\frac{3}{5}$

$3^{\log_3(4x+24)} = 3^{(-\frac{3}{5})}$

$4x+24 = 3^{-3/5}$

$4x = 3^{-3/5}-24$

$x = \frac{3^{-3/5}-24}{4}$

1a $(\frac{1}{4}(3^{-3/5}-24), 0)$

OR $(\frac{1}{4}\cdot 3^{-3/5}-6, 0)$

2b 4pts $g(0) =$

$= 5(\log_3(4(0)+24))+3$

$= 5\log_3(24)+3$

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3 5pts $x = 5\log_3(4y+24)+3 = x$

$$5\log_3(4y+24) = x-3$$

$$\log_3(4y+24) = \frac{x-3}{5}$$

$$3^{\log_3(4y+24)} = 3^{\frac{x-3}{5}}$$

$$4y+24 = 3^{\frac{x-3}{5}}$$

$$4y = 3^{\frac{x-3}{5}} - 24$$

$$y = \frac{3^{\frac{x-3}{5}}-24}{4} = \frac{1}{4}\left(3^{\frac{x-3}{5}}\right) - \frac{24}{4}$$

$$= \boxed{\frac{1}{4}\cdot 3^{\frac{x-3}{5}} - 6 = g^{-1}(x)}$$

4 $f(x) = \sqrt{x-24}$, $g(x) = x^2+7x-18$

a 5pts $\mathcal{D}(f)$: Need $x-24 \geq 0$

$$\Rightarrow x \geq 24$$

$$\Rightarrow \mathcal{D}(f) = \{x \mid x \geq 24\} =$$ 

$$= \boxed{[24, \infty) = \mathcal{D}(f)}$$

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4 b 5 pts $\mathcal{D}(g)$: g(x) is a polynomial with real coefficients $\Rightarrow$

$$\boxed{\mathcal{D}(g) = \mathbb{R}} = (-\infty, \infty)$$

c 5 pts

$$\frac{f}{g} = \boxed{\frac{f(x)}{g(x)} = \frac{\sqrt{x-24}}{x^2+7x-18}}$$

d 5 pts

$$\mathcal{D}\left(\frac{f}{g}\right) = \mathcal{D}(f) \underset{\text{AND}}{\cap} \mathcal{D}(g) \underset{\text{AND}}{\cap} \{x \mid g(x) \neq 0\}$$

g(x) $\neq 0$ is only piece we don't have.

$x^2 + 7x - 18 \neq 0$

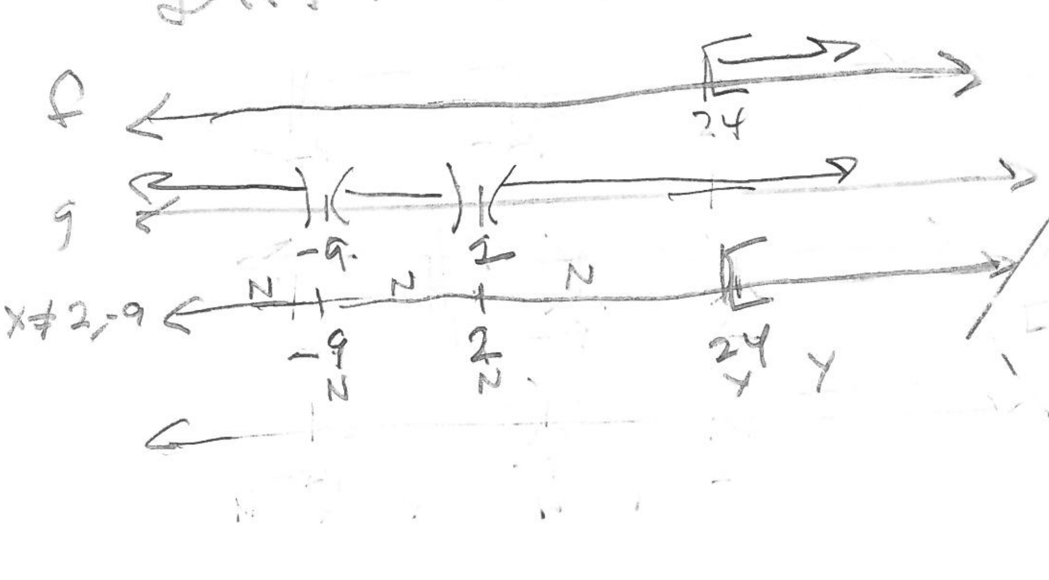
$(x+9)(x-2) \neq 0$

$\Rightarrow$ $x+9 \neq 0$ AND $x-2 \neq 0$

$x \neq -9$ AND $x \neq 2$

SO

$\mathcal{D}(f)$ AND $\mathcal{D}(g)$ AND $\{x \mid x \neq 2 \text{ AND } x \neq -9\}$



AND

$$\boxed{\mathcal{D} = [24, \infty)}$$

12.1 J4

4e 5pts $f\circ g = (f\circ g)(x) = f(g(x))$

$$= \sqrt{g(x)-24} = \boxed{\sqrt{(x^2+7x-18)-24} = f\circ g}$$

4f 5pts $\mathcal{D}(f\circ g)$

in $= \sqrt{x^2+7x-42}$

Need $x^2+7x-42 \geq 0$

$a=1$
$b=7$
$c=-42$

$\therefore b^2-4ac = 7^2-4(1)(-42)$

$= 49+168 = 217$

$2\,|\,42$
$3\,|\,21$
7

$7\,|\,217$
31 is prime

is not a perfect-square integer $\Longrightarrow$ irrational sol'ns

So, quad formula or complete the square

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-7 \pm \sqrt{217}}{2(1)}$$

$+ \qquad - \qquad +$

$\dfrac{-7-\sqrt{217}}{2} \qquad \dfrac{-7+\sqrt{217}}{2}$

≥ 0

parabola
$a>0$
2 real roots

$+ \quad - \quad +$

$$\boxed{\mathcal{D} = \left(-\infty, \frac{-7-\sqrt{217}}{2}\right] \cup \left[\frac{-7+\sqrt{217}}{2}, \infty\right)}$$

$\approx (-\infty, -10.86545993] \cup [3.86545993, \infty)$

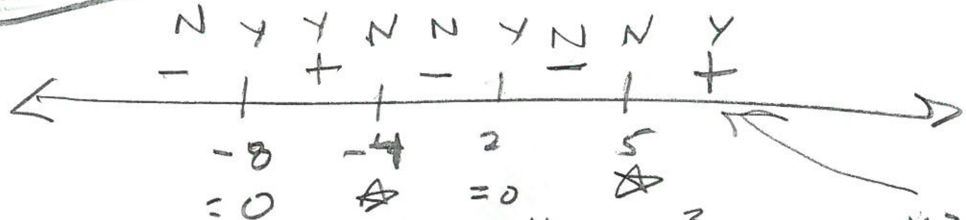
121 J4

(5a) (5 pts) $f(x) = \sqrt{\dfrac{(x-2)^4(x+8)}{(x+4)^3(x-5)^3}}$

$2, -8, -4, 5$

$-8, -4, 2, 5$

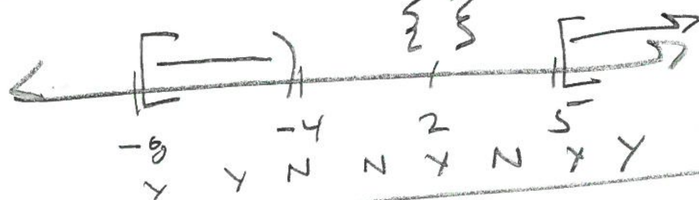
DOMAIN



≥ 0

$x > 5 \Rightarrow$ All factors positive $\Rightarrow$ positive +

MANAGE SIGN CHANGES $\rightarrow$ $(x-8)^1$ ODD, $(x+4)^3$ ODD, $(x-2)^4$ EVEN, $(x-5)^3$ ODD



$= \boxed{[-8,-4) \cup \{2\} \cup [5,\infty) = \mathcal{D}(f)}$

(b) (5 pts) $g(x) = \log_n \left(\dfrac{(x-2)^4(x+8)}{(x+4)^3(x-5)^3} \right)$

Same inside as $f(x)$, so same sign pattern, only ">0" rather than "≥ 0"

> 0

$\Rightarrow$ 

$\Rightarrow \boxed{\mathcal{D}(g) = (-8,-4) \cup (5,\infty)}$

12 T4

6 10pts $\ln(x+9) + \ln(x-2) = \ln(24)$

$\mathcal{D}$: Need $x > -9$ & $x > 2$ $\Longrightarrow x > 2$ needed

AND

-9 2

$\Longrightarrow \ln((x+9)(x-2)) = \ln(24)$

$e^{\ln(x^2+7x-18)} = e^{\ln(24)}$

$x^2+7x-18 = 24$

$x^2+7x-42 = 0$

$2 \mid 42$, $3 \mid 21$, 7

$7 \mid 217$, 31 is prime

See #4f: $x = \dfrac{-7 \pm \sqrt{217}}{2}$

$\swarrow$ $\searrow$ 2.86545993

$\approx -10.86545993 \notin \mathcal{D} \Longrightarrow$

$x \in \left\{ \dfrac{-7+\sqrt{217}}{2} \right\} \approx \{2.865459931\}$

7 $\frac{1}{2}$-life is 4900 yrs

2 10pts $A_0 e^{k(4900)} = \frac{1}{2} A_0$

$A_0 e^{kt}$, A_0 = initial mass of C_{14}

$\Longrightarrow e^{4900k} = \frac{1}{2}$

$\ln(e^{4900k}) = \ln\left(\frac{1}{2}\right) = -\ln(2)$

$4900k = -\ln(2)$

$\boxed{k = -\dfrac{\ln(2)}{4900}} \Longrightarrow \boxed{A(t) = A_0 e^{kt}}$

$k \approx -1.414586083 \times 10^{-4}$

$\boxed{\approx -.0001414586083 \approx k}$

0.00001.4

10^{-4}

12.1 T4

#6 5pts 66 % has decayed

34 % remains.

So $A_0 e^{kt} = .34 A_0$ Solve for t

$$\Rightarrow e^{kt} = .34$$

$$\Rightarrow \ln(e^{kt}) = \ln(.34)$$

$$\Rightarrow kt = \ln(.34)$$

$$\Rightarrow t = \frac{\ln(.34)}{k} = \frac{\ln(.34)}{\left(-\frac{\ln(2)}{4900}\right)}$$

$$= \ln(.34)\left(-\frac{4900}{\ln(2)}\right) = -\frac{4900\ln(.34)}{\ln(2)}$$

$$\approx +7626.327408 \text{ yrs}$$

$$\approx \boxed{7626 \text{ yrs old}}$$

121 T4

B1 5pts

$$|-5x+2|-7 \ge -5$$

$$|-5x+2| \ge 2$$

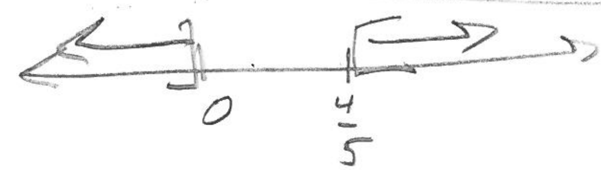
$-5x+2 \ge 2$ OR $-5x+2 \le -2$

$-5x \ge 0$ $\quad$ $-5x \le -4$

$x \le \frac{0}{-5} = 0$ $\quad$ $x \ge \frac{-4}{-5} = \frac{4}{5}$

$$\boxed{x \in \{x \mid x \le 0 \text{ OR } x \ge \tfrac{4}{5}\}}$$

=



OR

U

$$= \boxed{(-\infty, 0] \cup [\tfrac{4}{5}, \infty)}$$

B2 5pts $f(x) = 5x^2 - 2x - 11$

$$= 5\left(x^2 - \frac{2}{5}x \quad\right) - 11$$

$$= 5\left(x^2 - \frac{2}{5}x + \left(\frac{1}{5}\right)^2\right) - 11 - 5\left(\frac{1}{25}\right)$$

$$= \boxed{5\left(x + \frac{1}{5}\right)^2 - \frac{56}{5} = f(x)}$$

Scratch: $-11 - \frac{1}{5} = \frac{-55-1}{5} = -\frac{56}{5}$

121 T4

B3 5pts $3 \cdot 5^x = 10 \cdot \pi^x$

M1 $\ln(3 \cdot 5^x) = \ln(10 \cdot \pi^x)$

$\ln(3) + \ln(5^x) = \ln(10) + \ln(\pi^x)$

$\ln(3) + x\ln(5) = \ln(10) + x\ln(\pi)$

$\ln(3) + (\ln(5))x = \ln(10) + (\ln(\pi))x$

$a = \ln(3),\ b = \ln(5),\ c = \ln(10),\ d = \ln(\pi)$

$\Rightarrow a + bx = c + dx$

$\Rightarrow bx - dx = c - a$

$\Rightarrow x(b-d) = c - a$

$\Rightarrow x = \dfrac{c-a}{b-d} = \boxed{\dfrac{\ln(10) - \ln(3)}{\ln(5) - \ln(\pi)} = x} \approx$

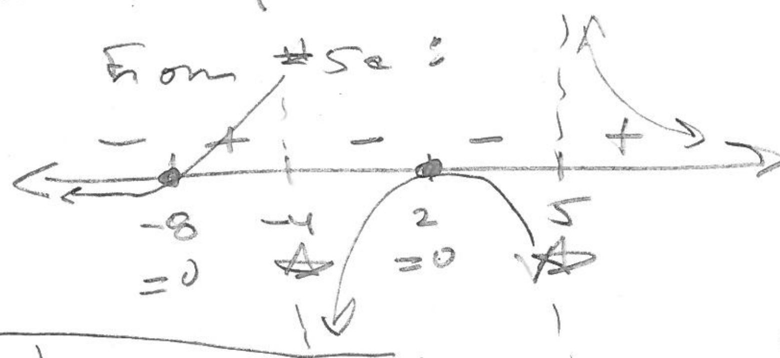
B4 5pts $R(x) = \dfrac{(x-2)^4(x+8)}{(x+4)^3(x-5)^3} = \dfrac{x^5 + \cdots}{x^6 + \cdots}$

$\mathcal{D}: \mathbb{R} \setminus \{-4, 5\}$

is proper: $y = 0$ is H.A.

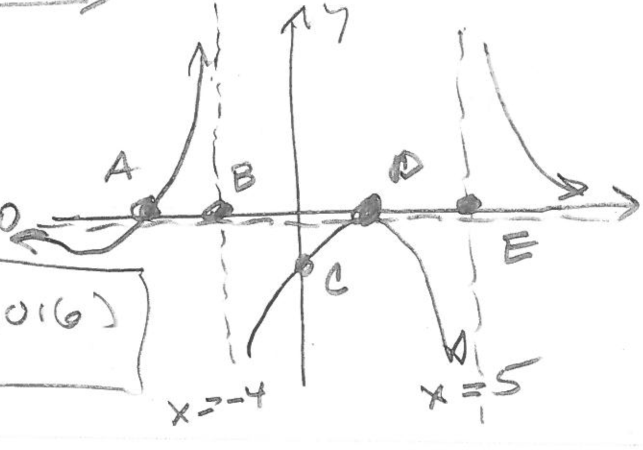
From #5a:

$R(0) = \dfrac{(-2)^4(8)}{4^3(-5)^3}$



$D = (2,0)$
$E = (5,0)$

$A = (-8,0)$
$B = (-4,0)$
$C = (0, -\frac{2}{125}) = (0, -.016)$



12.1 J4

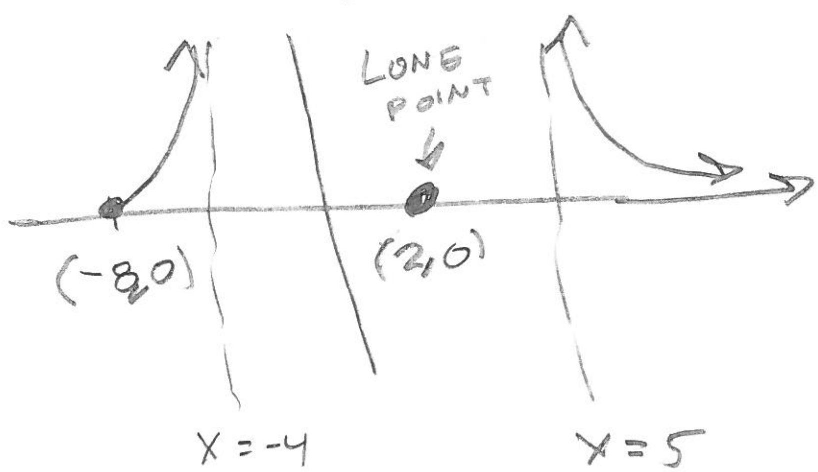
Scratch for B4

y-intercept : $R(0) = \dfrac{(-2)^4(8)}{4^3(-5)^3}$

$= \dfrac{-2^4 \cdot 2^3}{2^6 \cdot 5^3} = \dfrac{-2^7}{2^6 \cdot 125} = \dfrac{-2}{125} = -.016$

B5 5pts $Q(x) = \sqrt{\dfrac{(x-2)^4(x+8)}{(x+4)^3(x-5)^3}} = \sqrt{R(x)}$

So, take $\sqrt{\text{graph of B4}}$

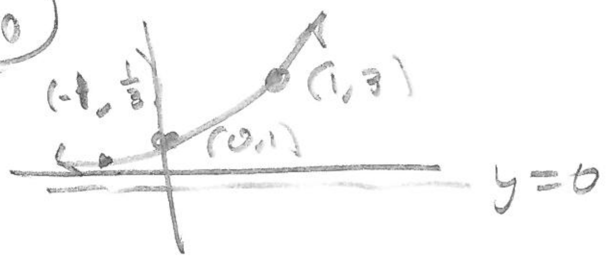


121 T4

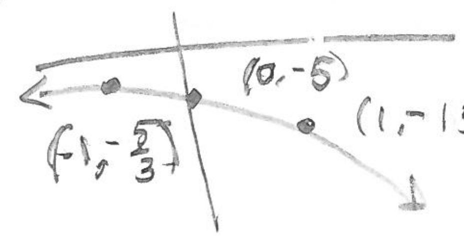
B6 5pts $g(x) = -5 \cdot 3^{3x+9} + 11$

$f(x) = 3^x$

0

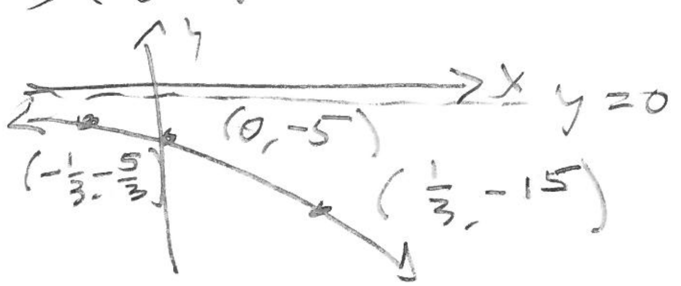


1 $-5f(x) = -5 \cdot 3^x$



$y \mapsto -5y$

2 $-5(3^{3x}) = -5f(3x)$ $\quad x \mapsto \frac{1}{3}x$

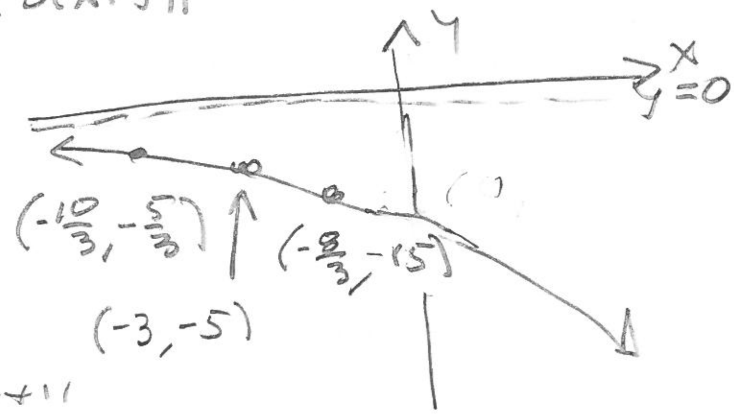


3 $-5(3^{3(x+3)}) = -5f(3(x+3))$ $\quad x \mapsto x - 3$

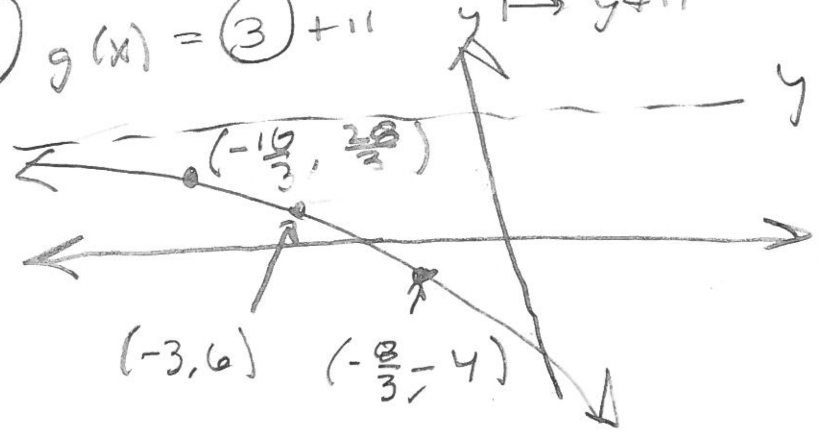
$-\frac{1}{3} - 3 = \frac{-1-9}{3} = -\frac{10}{3}$

$0 - 3 = -3$

$\frac{1}{3} - 3 = \frac{1-9}{3} = -\frac{8}{3}$



4 $g(x) = 3 + 11$ $\quad y \mapsto y + 11$



$-\frac{5}{3} + 11 = \frac{-5+33}{3}$

$= \frac{28}{3}$

$-5 + 11 = 6$

$-15 + 11 = -4$

121 JY

B7 $P = \$80{,}000 \quad t=30,\ m=12,\ r=.045$

a) 5pts $\Rightarrow mt = n = 12(30) = 360$

$i = \frac{r}{m} = \frac{.045}{12}$ END-OF-MONTH

Banker doesn't care if you pay back with interest at the end or in payments. His future value will be the same.

$S = A$

$$R\left[\frac{(1+i)^n - 1}{i}\right] = P(1+i)^n$$

We know P, i, n. Want R

$$\Rightarrow R = P(1+i)^n\left[\frac{i}{(1+i)^n - 1}\right]$$

$$= P(1+i)^n\left[\frac{i}{(1+i)^n\left(1 - \frac{1}{(1+i)^n}\right)}\right]$$

$$= \frac{Pi}{1-(1+i)^{-n}} = R$$

b) 5pts

$$= \frac{(80000)\left(\frac{.045}{12}\right)}{1 - \left(1 + \frac{.045}{12}\right)^{-360}} \approx 405.3482479$$

$$\boxed{\approx \$405^{\underline{35}} \approx R}$$

Tricky manipulation

$$(1+i)^n - 1 = (1+i)^n\left[\frac{(1+i)^n}{(1+i)^n} - \frac{1}{(1+i)^n}\right]$$

$$= (1+i)^n\left[1 - (1+i)^{-n}\right]$$