MAT 121-G81-G82 Test 3 – Spring, 2019 Name________________

70 Points Covers Chapter 3 NO GRAPHING CALCULATORS!!!

Remember to use separate paper for everything except your name. Leave a margin in the top left corner. Spread out your work. Use only one column for your work. *Submit problems in order!!!*

1. (10 pts) Form a polynomial of *minimal degree* in *factored form* that has real coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

$x = 5 - 2i$, multiplicity 1; $x = -5$, multiplicity 4; $x = 2$, multiplicity 2.

2. (10 pts) Use synthetic division to find $P(3)$ if $P(x) = 3x^5 - 3x^3 + 8x^2 - 10x + 11$
.

3. (5 pts) Represent the work you just did on the previous problem by writing $P(x)$ in the form $Dividend = Divisor \bullet Quotient + Remainder$.

4. Suppose $f(x) = (2x-1)(x+3)(x-5)(x+2)^2 = 2x^5 + 3x^4 - 40x^3 - 117x^2 - 52x + 60$. I'm showing you both factored and expanded form to help you answer the following:

   a. (10 pts) Solve the inequality $f(x) \geq 0$. Your sign pattern for this one will be helpful in the next two. You just have to interpret what you're seeing.

   b. (10 pts) Provide a rough sketch of *f*, using its zeros, their respective multiplicities and its end behavior. Include *x*- and *y*-intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better quality graph.

   c. (5 pts) What is the domain of $g(x) = \sqrt{\dfrac{(x+3)(x-5)}{(2x-1)(x+2)^2}}$?

5. Let $f(x) = 9x^5 + 6x^4 - 53x^3 - 10x^2 + 68x - 56$

   a. (5 pts) Use Descartes' Rule of Signs to determine the possible number of positive and negative zeros of *f*.

   b. (5 pts) List all possible rational zeros of *f*.

   c. (Bonus 5 pts) Find the smallest possible integer bounds on positive and negative zeros.

6. (10 pts) Find the *real* zeros of $f(x) = 9x^5 + 6x^4 - 53x^3 - 10x^2 + 68x - 56$. Then factor *f* over the set of **real numbers**. This should involve an irreducible quadratic factor.

(If things go haywire, come up with a *plausible*-looking polynomial, in factored form, with the right number of real roots and 2 nonreal roots. The 2 nonreal roots will still be living inside the irreducible quadratic factor, so you'll have to make up a quadratic factor with nonreal zeros).

7. (5 pts) Find the remaining (nonreal) zeros of f and factor f over the set of **complex numbers**. This step requires breaking down the quadratic piece that's irreducible over the real numbers, and incorporating the result into your answer for the #6. The fundamental theorem tells us that *nothing* is irreducible over the complex numbers.

(You can still get full credit for this one, even if things went haywire, in #6, if you solve the depressed equation, correctly, and display your *plausible*-looking follow-up to your *plausible*-looking answer to #6. The more you know about what you're pointing towards, the more points you'll earn.)

8. (5 pts) You don't need to graph $R(x) = \dfrac{3x^3 - 14x^2 - 7x + 10}{x^2 + x - 6}$, here, but I do want to see its asymptotes.

9. (10 pts) Sketch the graph of $F(x) = \dfrac{3x^2 - 17x + 10}{x^2 + x - 6}$ Show all asymptotes and intercepts.

ANSWER ANY TWO (2) OF THE FOLLOWING, FOR UP TO 20 BONUS POINTS!!!

B1 (10 pts) Form a polynomial of *minimal degree* in *factored form* that has ***rational*** coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong.

Zeros: $x = 3 - \sqrt{2}$, multiplicity 2;
$x = 2 + 3i$, multiplicity 1;
$x = 3$, multiplicity 4.

B2 Solve both of the following absolute value inequalities.

a. (5 pts) $|2x - 7| + 8 > 9$

b. (5 pts) $|3x + 1| + 13 < 8$

B3 (10 pts) Sketch the graph of $R(x) = \dfrac{3x^3 - 14x^2 - 7x + 10}{x^2 + x - 6}$.

a. You already found $R(x)$'s asymptotes in #8.

b. One of $R(x)$'s x-intercepts is $(5, 0)$.

B4 (10 pts) Sketch the graph of $G(x) = \dfrac{3x^3 - 14x^2 - 7x + 10}{x^3 + 2x^2 - 5x - 6}$. Hint: $G(x)$ looks exactly like $F(x)$, from #9, except it has a hole. For this one, you may simply add the hole for G's graph to your graph of F in #9.

B5 (10 pts) If $f(x) = \sqrt{x + 11}$ and $g(x) = \dfrac{2}{x - 6}$, what is the domain of $f \circ g$?