:121 TEST 3 SPRING, 2019 SOLNS.

(1) (10 pts) $f(x) = (x-(5-2i))(x-(5+2i))(x+5)^4(x-2)^2$

(2) (10 pts) $P(x) = 3x^5 - 3x^3 + 8x^2 - 10x + 11$

$P(3)$:

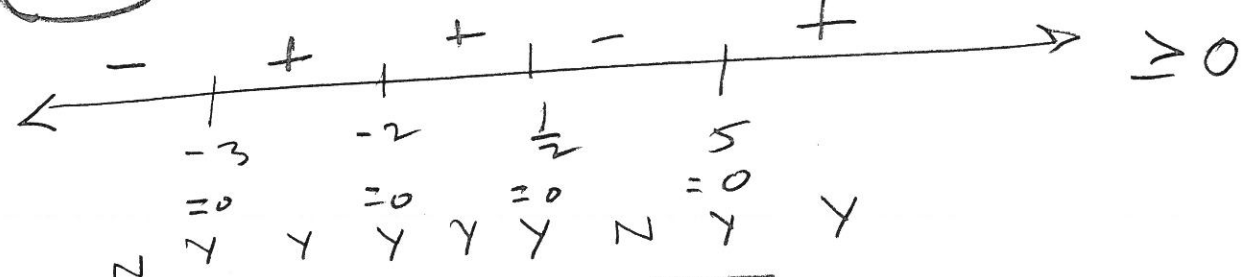
	3	0	−3	8	−10	11
3		9	27	72	240	690
	3	9	24	80	230	$701 = P(3)$

(3) (5 pts)

$P(x) = (x-3)(3x^4 + 9x^3 + 24x^2 + 80x + 230) + 701$

(4) $f(x) = (2x-1)(x+3)(x-5)(x+2)^2 = 2x^5 + 3x^4 - 46x^3 - 117x^2 - 52x + 60$

(2) (10 pts) Want $f(x) \geq 0$ $\quad \frac{1}{2}, -3, 5, -2$ ($m=2$)



$\Rightarrow \boxed{x \in [-3, \tfrac{1}{2}] \cup [5, \infty)}$

#1 w/o placeholder:

	3	−3	8	−10	11
3		9	18	78	204
	3	6	26	68	$215 = P(3)$

if you messed-up the $0x^4$ b.t.

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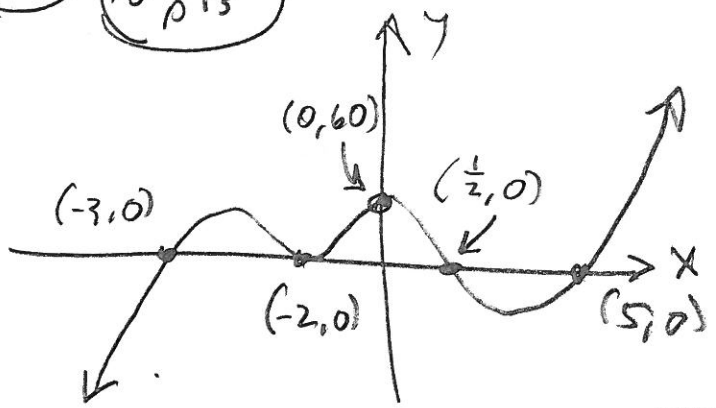
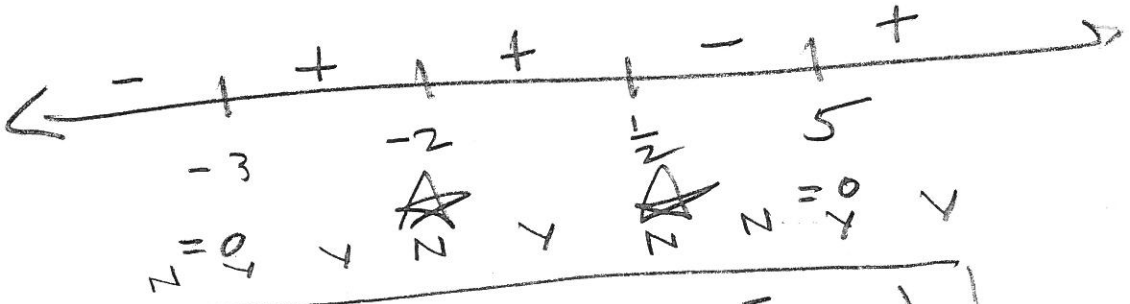
4b (10 pts)



4c (5 pts) $g(x) = \sqrt{\dfrac{(x+3)(x-5)}{(2x-1)(x+2)^2}} = \sqrt{\text{STUFF}}$.

For Domain, need stuff ≥ 0. Same sign pattern, with $x = \frac{1}{2}, -2$ thrown out:



$\Rightarrow$ $\boxed{D(g) = [-3, -2) \cup (-2, \frac{1}{2}) \cup [5, \infty)}$

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(5) $f(x) = 9x^5 + 6x^4 - 53x^3 - 10x^2 + 68x - 56$

(a) (5 pts) Descartes' | 3 or 1 pos. zeros |

$f(-x) = -9x^5 + 6x^4 + 53x^3 - 10x^2 - 68x - 56$

| 2 or 0 negative zeros |

(b) (5 pts) p's: 56 Want all $\frac{p}{q}$'s:

q's: 9

$3 \mid 9$
$\quad 3$

$2 \mid 56$
$2 \mid 28$
$2 \mid 14$
$\quad 7$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 7, \pm 14, \pm 28, \pm 56,$
$\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}, \pm \frac{7}{3}, \pm \frac{14}{3}, \pm \frac{28}{3}, \pm \frac{56}{3},$
$\pm \frac{1}{9}, \pm \frac{2}{9}, \pm \frac{4}{9}, \pm \frac{8}{9}, \pm \frac{7}{9}, \pm \frac{14}{9}, \pm \frac{28}{9}, \pm \frac{56}{9}$

(c) Bonus (5 pts)

LUB = 3 by Theorem
$x = 2$ by #6

3 | 9 6 -53 -10 68 -56
27 99 138 BIG GIANT
9 33 46 128 HUGE

-3 | 9 6 -53 -10 68 -56
-27 63 -30 120 -BIG
9 -21 10 -40 108 -HUGE

$x = -3$ by Thm.
$x = -2$ by #6

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(6) (10 pts)

1 | 9 6 -53 -10 68 -56
9 15 -38 -48
9 15 -38 -48 20

-1 | 9 6 -53 -10 68 -56
-9 3 50 -40
9 -3 -50 40 28

2 | 9 6 -53 -10 68 -56 (x=2)
18 48 -10 -70 56
2 | 9 24 -5 -20 28 0 sweet!
18 84 158
9 42 79

4 | 9 24 -5 -20 28
36 240
9 60 235

-2 | 9 24 -5 -20 28
-18 -12 34 -28
-2 | 9 6 -17 14 0 sweet!
-18 24 -14
9 -12 7 0 sweet!

(x=-2, x=-2!)

$a=9, b=-12, c=7 \rightarrow b^2-4ac = 144-4(9)(7) = 144-252 = -108 < 0$

No more real zeros

$4 \cdot 36 = 144$, $144+108 = 252$

So $f(x) = (x-2)(x+2)^2(9x^2-12x+7)$ is factored over the reals.

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(7) $9x^2 - 12x + 7 = 0$

$\Delta = b^2 - 4ac = -108$

$\sqrt{108} = 2 \cdot 3\sqrt{3} = 6\sqrt{3}$, so

2 | 108
2 | 54
3 | 27
3 | 9
3

$$x = \frac{12 \pm 6\sqrt{3}\,i}{2(9)} = \frac{6\left[2 \pm \sqrt{3}\,i\right]}{18}$$

$$= \frac{2 \pm \sqrt{3}\,i}{3} = x \text{ et } s_0$$

$$f(x) = 9(x-2)(x+2)^2\left(x - \frac{2+\sqrt{3}\,i}{2}\right)\left(x - \frac{2-\sqrt{3}\,i}{2}\right)$$

(8) (5pts) $R(x) = \dfrac{3x^3 - 14x^2 - 7x + 10}{x^2 + x - 6} = \dfrac{3x^3 - 14x^2 - 7x + 10}{(x+3)(x-2)}$

$\mathcal{D} = \mathbb{R} \setminus \{-3, 2\}$ | M.A : $x = -3$, $x = 2$

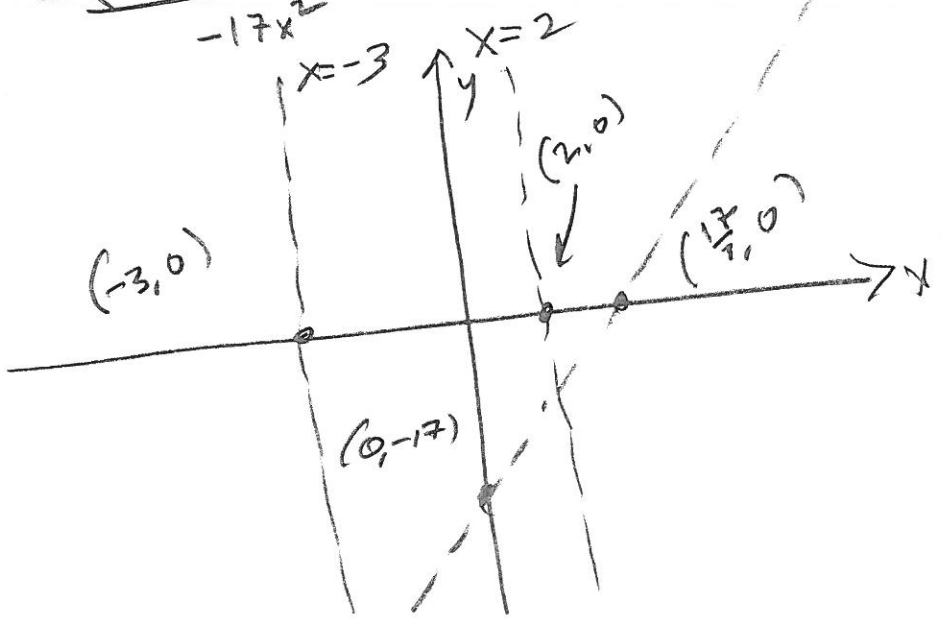
O.A. :

$$\begin{array}{r|l} 3x^3 - 14x^2 - 7x + 10 & x^2 + x - 6 \\ -(3x^3 + 3x^2 - 18x) & 3x - 17 \\ \hline -17x^2 & \end{array}$$

O.A. : $y = 3x - 17$

x	y
0	17
$\frac{17}{3}$	0

$3x - 17 = 0$
$3x = 17$
$x = \frac{17}{3}$



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(9) (10 pts) $f(x) = \dfrac{3x^2-17x+10}{x^2+x-6} = \dfrac{(3x-2)(x-5)}{(x+3)(x-2)}$

$3x^2-17x+10 = 3x^2-15x-2x+10 = 3x(x-5) - 2(x-5)$

$= (x-5)(3x-2) \rightsquigarrow \boxed{\left(\frac{2}{3},0\right),\ (5,0)\ x\text{-int}}$

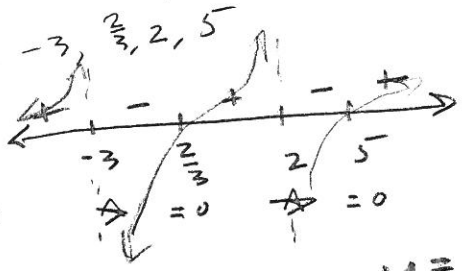
$x^2+x-6 = (x+3)(x-2) \Longrightarrow$

$\mathcal{D} = \mathbb{R} \setminus \{-3, 2\} \Rightarrow$ V.A.: $x=-3,\ x=2$

$y\text{-int}: \left(0, -\frac{5}{3}\right)$

$\frac{10}{-6} = -\frac{5}{3}$

H.A.: $\frac{3x^2}{x^2} \neq 3 = y$ = H.A.



A = (-3, 0)

B = $(0, -\frac{5}{3})$

C = $(\frac{2}{3}, 0)$

D = (2, 0)

E = (5, 0)

F = $(\frac{7}{5}, 3)$ BONUS

$3x^2-17x+10 = 3x^2+3x-18$

$-17x+10 = 3x-18$

$-20x = -28$

$x = \frac{7}{5}$

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B1 (10pts) $P(x) = (x-3-\sqrt{2})(x-3+\sqrt{2})(x-2+3i)(x-2-3i)(x-3)^4$

B2 (10pts) (a) $|2x-7|+8 > 4$

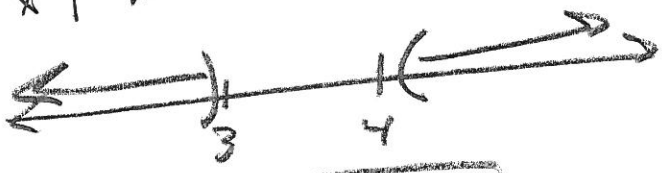
$|2x-7| > 1$

$2x-7 > 1$ OR $2x-7 < -1$

$2x > 8$ $\quad 2x < 6$

$x > \frac{8}{2}$ $\quad x < \frac{6}{2}$

$\{x \mid x > 4$ OR $x < 3\}$

$=$ 

$= (-\infty, 3) \cup (4, \infty)$

(b) $|3x+1| + 13 < 8$

$|3x+1| < -5$ Never! $\quad \emptyset$

12.1 E3

10 pts B3 $R(x) = \dfrac{3x^3 - 14x^2 - 7x + 10}{x^2 + x - 6}$

(5,0) is x-int:

$$\begin{array}{r|rrrr} 5 & 3 & -14 & -7 & 10 \\ & & 15 & 5 & -10 \\ \hline & 3 & 1 & -2 & 0 \end{array}$$

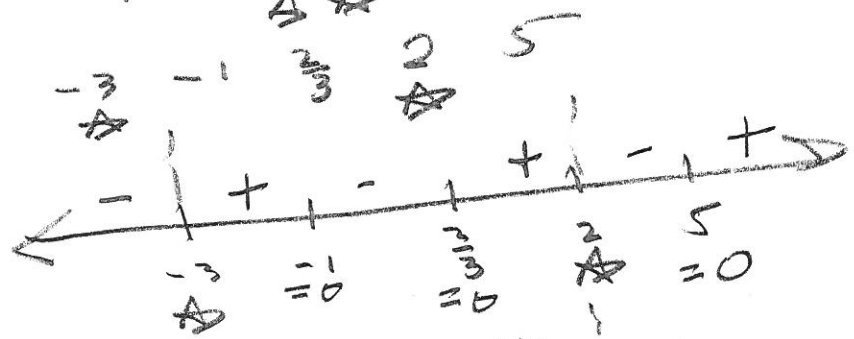
$3x^2 + x - 2 = (3x - 2)(x + 1)$

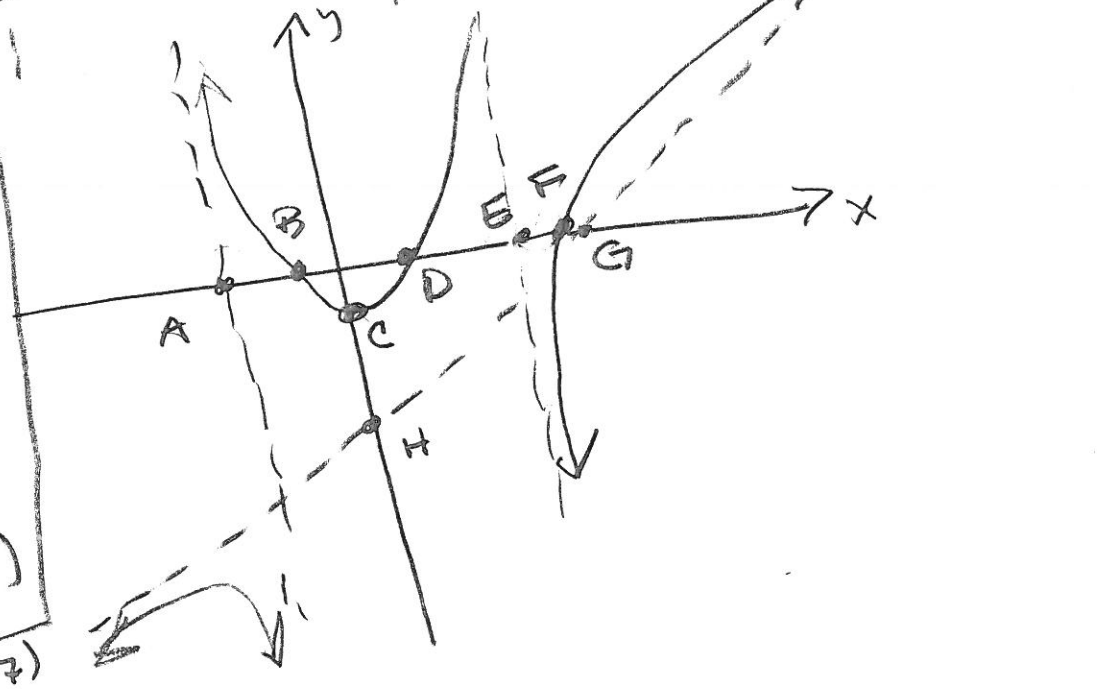
$R(x) = \dfrac{(3x-2)(x+1)(x-5)}{(x+3)(x-2)}$

x-int: $(\frac{2}{3}, 0), (-1, 0), (5, 0)$

VA: $x = -3, x = 2$

$\frac{2}{3}, -1, 5, -3, 2$





A = (-3, 0)
B = (-1, 0)
C = (0, -5/3)
D = (2/3, 0)
E = (2, 0)
F = (5, 0)
G = (17/3, 0)
H = (0, -17)

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B4 10 pts Using B3:

$$G(x) = \frac{3x^3 - 14x^2 - 7x + 10}{x^3 - 2x^2 - 5x - 6} = \frac{(3x-2)(x+1)(x-5)}{(\quad)(\quad)(\quad)}$$

x+1 is the extra factor upstairs

$$\begin{array}{r|cccc} -2 & 1 & -2 & -5 & -6 \\ & & -2 & 8 & -6 \\ \hline & 1 & -4 & 3 & -12 \end{array}$$ Bad Question

$$\begin{array}{r|cccc} -3 & 1 & -2 & -5 & -6 \\ & & -3 & 15 & \\ \hline & 1 & -5 & 10 & \end{array}$$

$$\begin{array}{r|cccc} 3 & 1 & -2 & -5 & -6 \\ & & 3 & 3 & \\ \hline & 1 & 1 & -2 & \end{array}$$ Nope

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B5 10pts $f(x) = \sqrt{x+11}$, $g(x) = \frac{2}{x-6}$

$D(f) = \{x \mid x \geq -11\}$

$D(g) = \{x \mid x \neq 6\}$

$D(f \circ g) = \{x \mid x \in D(g) \text{ and } g(x) \in D(f)\}$

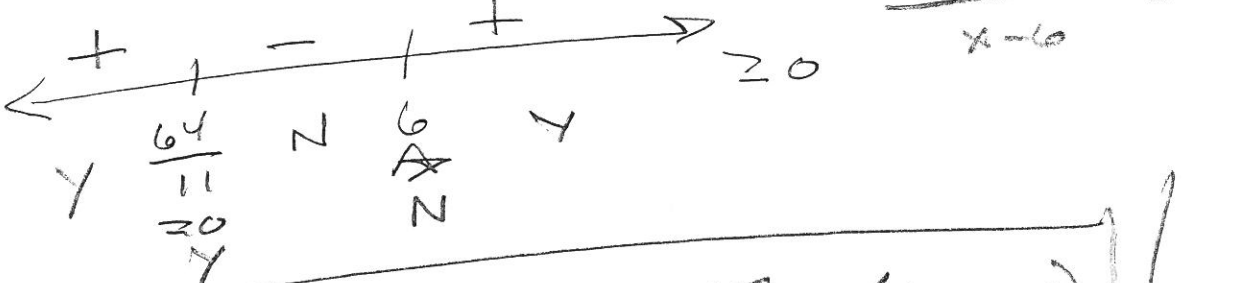
$= \{x \mid x \neq 6 \text{ and } \frac{2}{x-6} \geq -11\}$

$\frac{2}{x-6} \geq -11 \longrightarrow$

$\frac{2}{x-6} + \left(\frac{11}{11}\right)\left(\frac{x-6}{x-6}\right) \geq 0 \longrightarrow$

$\frac{2+11x-66}{x-6} \geq 0 \longrightarrow \frac{11x-64}{x-6} \geq 0$

$\frac{11\left(x-\frac{64}{11}\right)}{x-6} \geq 0$



$\rightarrow D(f \circ g) = (-\infty, -\frac{64}{11}] \cup (6, \infty)$!