121 TEST 2 SPRING, 2019

1 5pts Kindness

2a $R = \{(-2,3), (1,5), (2,8), (3,-2)\}$

5pts $\boxed{\text{Yes,}}$ function

2b $\boxed{\mathcal{D} = \{-2, 1, 2, 3\}}$

5pts

2c 5pts $\boxed{\mathcal{R} = \{3, 5, 8, -2\}}$

2d 5pts Nothing stops R from being 1-to-1, because it __is__ 1-to-1!

3 $f(x) = \dfrac{x+2}{x-3}$ and $g(x) = \sqrt{x+4}$

a 5pts $\mathcal{D}(f)$: Need $x - 3 \neq 0$

$\Longrightarrow x \neq 3$

$\rightarrow \boxed{\mathcal{D}(f) = \{x \mid x \neq 3\} = (-\infty, 3) \cup (3, \infty)}$

b 5pts $\mathcal{D}(g)$: Need $x + 4 \geq 0$

$\Longrightarrow x \geq -4$

$\rightarrow \boxed{\mathcal{D}(g) = \{x \mid x \geq -4\} = [-4, \infty)}$

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2 c 5pts $\boxed{\dfrac{f}{g} = \dfrac{\left(\dfrac{x+2}{x-3}\right)}{\sqrt{x+4}}} = \dfrac{x+2}{(x-3)\sqrt{x+4}}$

2d 5pts $\mathcal{D}\left(\frac{f}{g}\right)$: Need $x \neq 3$ AND $x \geq -4$ AND $x \neq -4$

so $x > -4$

$= \boxed{(-4,3) \cup (3,\infty) = \mathcal{D}\left(\frac{f}{g}\right)}$

2e 5pts $f \circ g = f(g(x)) = \dfrac{g(x)+2}{g(x)-3} = \boxed{\dfrac{\sqrt{x+4}+2}{\sqrt{x+4}-3} = f \circ g}$

2f 5pts $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ AND } g(x) \in \mathcal{D}(f)\}$

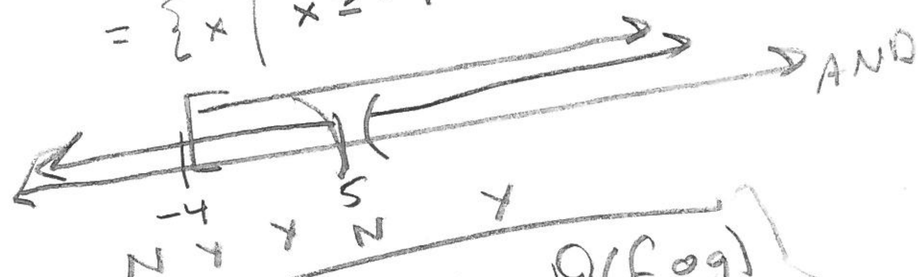
$= \{x \mid x \geq -4 \text{ and } \sqrt{x+4} \neq 3\}$

$= \{x \mid x \geq -4 \text{ and } x \neq 5\}$

$\sqrt{x+4} \neq 3 \Rightarrow$

$x+4 \neq 9 \Rightarrow$

$x \neq 5$



$= \boxed{[-4,5) \cup (5,\infty) = \mathcal{D}(f \circ g)}$

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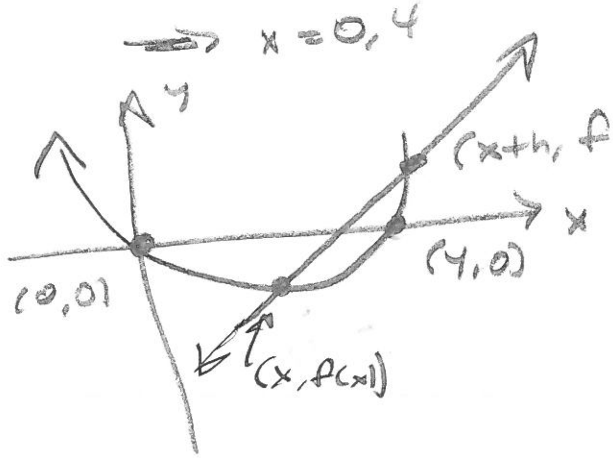
(4) (5 pts) $f(x) = x^2 - 4x = x(x-4) \overset{\text{SET}}{=} 0$

$\Longrightarrow x = 0, 4$



(5) (5 pts) $f(x) = 3x^2 - 5x \Longrightarrow$

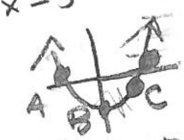
$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2 - 5(x+h) - [3x^2 - 5x]}{h}$$

$$= \frac{3(x^2+2xh+h^2) - 5x - 5h - 3x^2 + 5x}{h}$$

$$= \frac{3x^2 + 6xh + 3h^2 - 5x - 5h - 3x^2 + 5x}{h}$$

$$= \frac{6xh + 3h^2 - 5h}{h} = \frac{h(6x+3h-5)}{h} = \boxed{6x+3h-5}$$

(B6) (5 pts) $h(x) = \begin{cases} 2x+4 & \text{if } x<2 \quad \text{open dot} \\ x^2-5 & \text{if } x \geq 2 \quad \text{closed do} \end{cases}$



$x^2-5,\ x \geq 2$

$y = 2x+1,\ x<2$

$2x+4=0$

$2x = -4$

$x = -2$

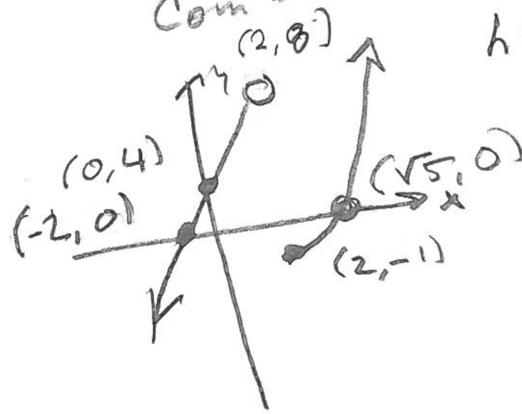
$2x+1=0$

$2x=-1$

$x = -\frac{1}{2}$

Combine

$h(x)$



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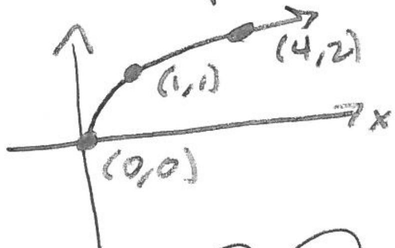
6 20 pts

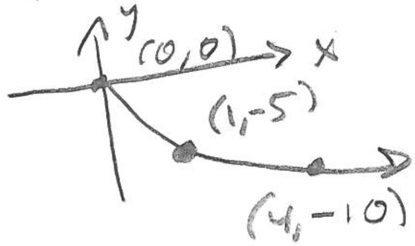
$g(x) = -5\sqrt{-7x-21} + 11$

$= -5\sqrt{-7(x+3)} + 11$

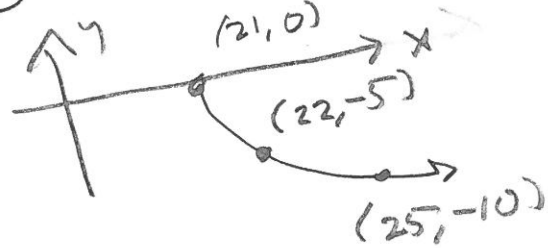
0 $f(x) = \sqrt{x}$



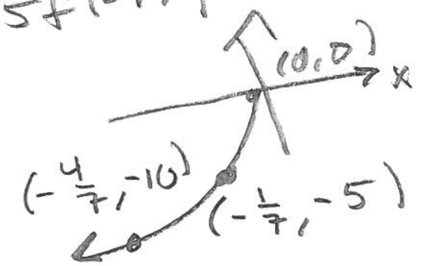
1 $-5f(x) = -5\sqrt{x}$



2 M1 $-5f(x-21) = -5\sqrt{x-21}$

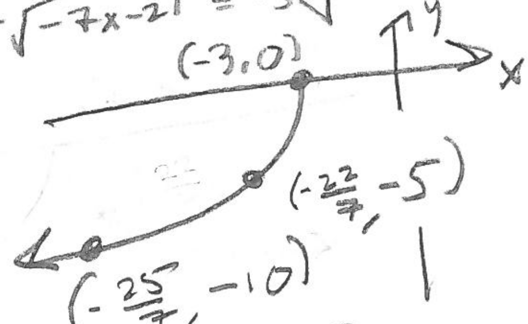


2 M2 $-5f(-7x) = -5\sqrt{-7x}$

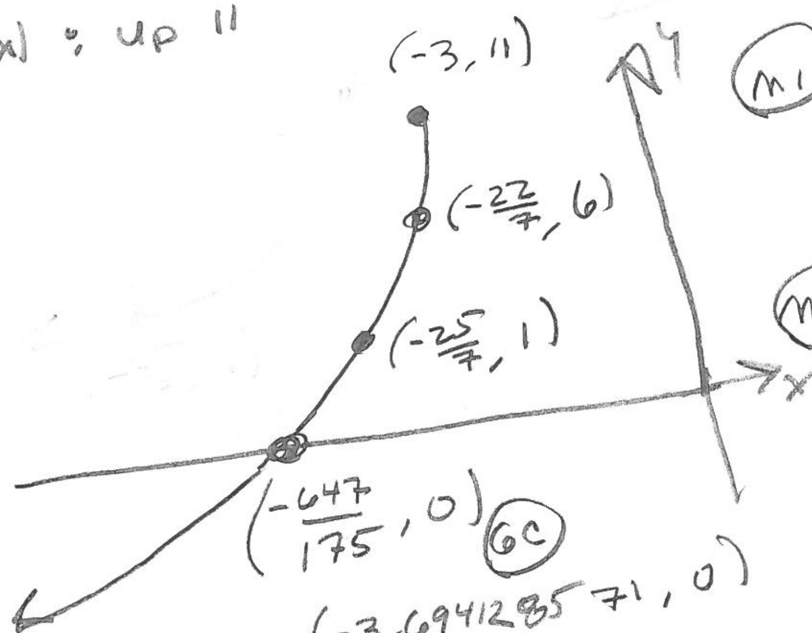


3 $-5f(-7x-21) = -5f(-7(x+3))$

$= -5\sqrt{-7x-21} = -5\sqrt{-7(x+3)}$



4 $g(x)$: up 11



M1 $-\frac{1}{7}(21) = -3$

$-\frac{1}{7}(22) = -\frac{22}{7}$

$-\frac{1}{7}(25) = -\frac{25}{7}$

M2 $0-3 = -3$

$-\frac{4}{7}-3 = \frac{-4-21}{7} = -\frac{25}{7}$

$-\frac{1}{7}-3 = \frac{-1-21}{7} = -\frac{22}{7}$

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6b 5pts

$$\boxed{\begin{aligned} \mathcal{D}(g) &= (-\infty, 3] \\ \mathcal{R}(g) &= (-\infty, 21] \end{aligned}}$$

6c 5pts $\nexists$ y-intercept

x-intercept: $-5\sqrt{-7x-21} + 11 = 0$

$$-5\sqrt{-7x-21} = -11$$

$$\sqrt{-7x-21} = \frac{-11}{-5} = \frac{11}{5}$$

$$-7x-21 = \left(\frac{11}{5}\right)^2 = \frac{121}{25}$$

$$-7x = 21 + \frac{121}{25} = \frac{525+121}{25} = \frac{646}{25}$$

$$x = -\frac{1}{7}\left(\frac{646}{25}\right) = -\frac{646}{175} \approx -3.691428571$$

$\boxed{\left(-\frac{646}{175}, 0\right) \text{ is x-intercept}}$

$(-3.691428571, 0)$

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7 5pts $f(x) = \frac{x+7}{x-11}$ is 1-to-1.

<u>Proof</u> Suppose $f(x_1) = f(x_2)$. Then

$$\frac{x_1+7}{x_1-11} = \frac{x_2+7}{x_2-11}$$

$$\Rightarrow (x_1+7)(x_2-11) = (x_2+7)(x_1-11)$$

$$\Rightarrow x_1x_2 - 11x_1 + 7x_2 - \underline{77} = x_2x_1 - 11x_2 + 7x_1 \underline{-77}$$

$$\Rightarrow -11x_1 + 7x_2 = -11x_2 + 7x_1$$

$$\Rightarrow -18x_1 = -18x_2$$

$$\Rightarrow x_1 = x_2 \Longrightarrow f(x) \text{ is 1-to-1 } ! \blacksquare$$

8 5pts $y = \frac{k\sqrt[3]{x}\, z^3}{\sqrt{u}\, w^2} = k\frac{z^3\sqrt[3]{x}}{w^2\sqrt{u}} = y$ for some $k \in \mathbb{R}$.

B1 5pts $f(x) = \frac{1}{\sqrt{x}} \Longrightarrow \frac{1}{h}\left[f(x+h) - f(x)\right]$

$$= \frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right]$$

$$= \frac{1}{h}\left[\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x+h}\sqrt{x}}\right] = \frac{1}{h}\left[\frac{(\sqrt{x}-\sqrt{x+h})}{\sqrt{x}\sqrt{x+h}}\right]\left(\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}\right)$$

$$= \frac{1}{h}\left[\frac{x-(x+h)}{(\sqrt{x}\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}\right]$$

$$= \frac{1}{h}\left[\frac{h}{\sqrt{x}\sqrt{x+h}\,(\sqrt{x}+\sqrt{x+h})}\right] = \frac{1}{\sqrt{x}\sqrt{x+h}\,(\sqrt{x}+\sqrt{x+h})}$$

$$\xrightarrow{h\to 0} \frac{1}{\sqrt{x}\sqrt{x}\,(2\sqrt{x})} = \boxed{\frac{1}{2x\sqrt{x}} = f'(x)}$$

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B2 5pts $h(x) = 3x^2+5x-5$

$= 3\left(x^2+\frac{5}{3}x \quad \right)-5$

$= 3\left(x^2+\frac{5}{3}x+\left(\frac{5}{6}\right)^2\right)-5-3\left(\frac{25}{36}\right)$

$-5-\frac{25}{12}$

$\boxed{= 3\left(x+\frac{5}{6}\right)^2-\frac{85}{12}}$

$= -\frac{60-25}{12} = -\frac{85}{12}$

$\boxed{\Rightarrow (h,k) = \left(-\frac{5}{6}, -\frac{85}{12}\right)}$

B3 5pts $f(x) = \dfrac{x-5}{x^2-5x-7}$

$\mathcal{D}(f)$: Need $x^2-5x-7 \neq 0$

$\Rightarrow x^2-5x \neq 7$

$\Rightarrow x^2-5x+\left(\frac{5}{2}\right)^2 \neq 7+\frac{25}{4} = \frac{28+25}{4}$

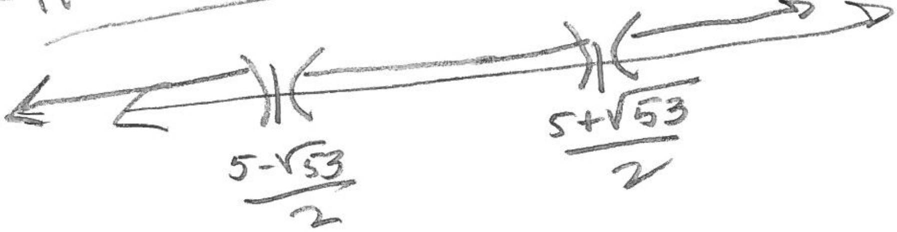
$\Rightarrow \left(x-\frac{5}{2}\right)^2 \neq \frac{53}{4}$

$\Rightarrow x-\frac{5}{2} \neq \pm\sqrt{\frac{53}{4}} = \pm\frac{\sqrt{53}}{2}$

$\Rightarrow x \neq \frac{5\pm\sqrt{53}}{2}$

$\boxed{\mathcal{D}(f) = \left\{x \mid x \neq \frac{5\pm\sqrt{53}}{2}\right\}} = \left(-\infty, \frac{5-\sqrt{}}{}\right.$

$= \boxed{\left(-\infty, \frac{5-\sqrt{53}}{2}\right) \cup \left(\frac{5-\sqrt{53}}{2}, \frac{5+\sqrt{53}}{2}\right) \cup \left(\frac{5+\sqrt{53}}{2}, \infty\right)}$



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B4 $f(x) = \dfrac{x^{77} - 5x^{12} + 17x}{\sqrt{-7x-21}}$

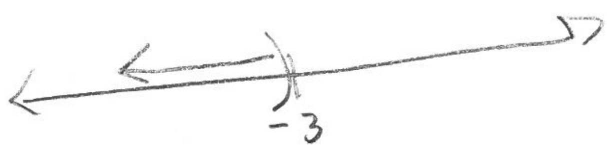
$\mathcal{D}(f)$: Need $-7x-21 \geq 0$ AND $\sqrt{-7x-21} \neq 0$

i.e., Need $-7x-21 > 0$

$\Longrightarrow -7x > 21$

$\Longrightarrow x < \frac{21}{-7} = -3$

$\Longrightarrow \mathcal{D}(f) = \{x \mid x < -3\} = (-\infty, -3)$

= 

B5 5pts $|5-2x| > 17$

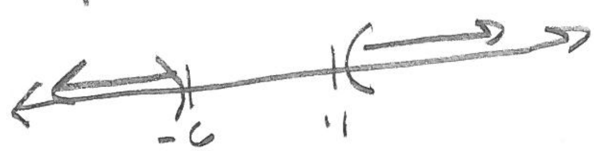
$\Longrightarrow 5-2x > 17$ OR $5-2x < -17$

$-2x < -22$

$\Longrightarrow -2x > 12$

$\Longrightarrow x < \frac{12}{-2} = -6$ OR $x > \frac{-22}{-2} = 11$

$\Longrightarrow \mathcal{D}(f) = \{x \mid x < -6 \text{ OR } x > 11\}$

= 

$= \boxed{(-\infty, -6) \cup (11, \infty)}$