121 TEST 5 Spring, 2018

1 a 5pts $C(24,5) = \frac{24!}{5!\,19!} = \frac{\overset{4}{\cancel{24}}\cdot 23\cdot 22\cdot 21\cdot \cancel{20}}{\cancel{5\cdot 4\cdot 3\cdot 2}}$

$= 4\cdot 23\cdot 22\cdot 21$

b 5pts $P(24,5) = \frac{24!}{19!} = 24\cdot 23\cdot 22\cdot 21\cdot 20$

3 $4 + 3 + \frac{9}{4} + \frac{27}{16} + \dots + \frac{19683}{65536}$

a 5pts

$\boxed{a = 4,\ r = \frac{3}{4},\ n = 10}$

$\frac{19683}{65536} = 4\left(\frac{3}{4}\right)^{n-1}$

$4\cdot\frac{3^9}{4^9} = 4\left(\frac{3}{4}\right)^{n-1}$

$4\left(\frac{3}{4}\right)^9 = 4\left(\frac{3}{4}\right)^{n-1}$

$\Rightarrow 9 = n-1$

$\boxed{10 = n}$

3 | 19683
3 | 6561
3 | 2187
3 | 729
3 | 243
3 | 81
3 | 27
3 | 9
3

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2 b 5pts

$$4\left(\frac{1-\left(\frac{3}{4}\right)^{10}}{1-\frac{3}{4}}\right) = 4\left(\frac{1-\left(\frac{3}{4}\right)^{10}}{\frac{1}{4}}\right)$$

$$= 16\left(1-\left(\frac{3}{4}\right)^{10}\right) = 16\left[1-\frac{59049}{1{,}048{,}576}\right]$$

$$= 16\left[\frac{1048576-59049}{1048576}\right]$$

Full CREDIT

$$= 16\left[\frac{989527}{1048576}\right] = \boxed{\frac{989527}{65536}}$$

$65536 = 4^8$, 989527 is odd $\Rightarrow$ Lowest Terms

$$\approx \boxed{15.09898376}$$

3 5pts

$$\sum_{k=1}^{\infty} 3\left(\frac{2}{9}\right)^{k-1} = \frac{3}{1-\frac{2}{9}} = \frac{3}{\frac{7}{9}} = \boxed{\frac{27}{7}}$$

$$\approx 3.857142857$$

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4 $P(1+i)^n = R\left[\frac{(1+i)^n - 1}{i}\right]$ Set Solve

for P:

5pts $P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$= 1000\left[\frac{1-\left(1+\frac{.039}{12}\right)^{-(30)(12)}}{\frac{.039}{12}}\right]$

$\approx \boxed{\$212{,}013^{\underline{44}}}$

5 5pts $-2\log_4(5x+15)+7=0$

$-2\log_4(5x+15) = -7$

$\log_4(5x+15) = \frac{7}{2}$

$5x+15 = \log_4\left(\frac{7}{2}\right)$

$5x = \log_4\left(\frac{7}{2}\right) - 15$

$x = \frac{1}{5}\left[\log_4\left(\frac{7}{2}\right) - 15\right]$

$= \frac{1}{5}\left[\frac{\ln(7/2)}{\ln(4)} - 15\right] \approx -2.819264508$

$\approx \boxed{-2.8193}$

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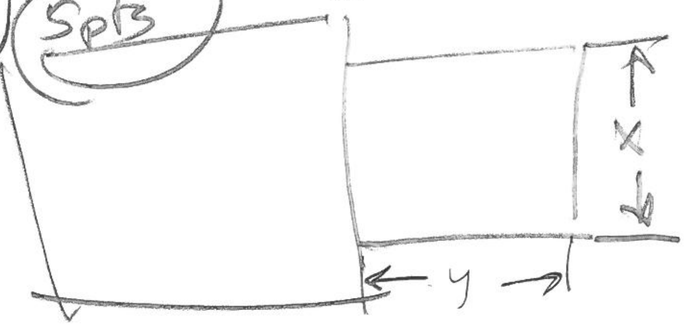
b

$x + 2y = 150$

a 5pts



A = Area as function of x.

$A = xy$ & $x + 2y = 150$

$2y = 150 - x$

$y = 75 - \frac{1}{2}x$

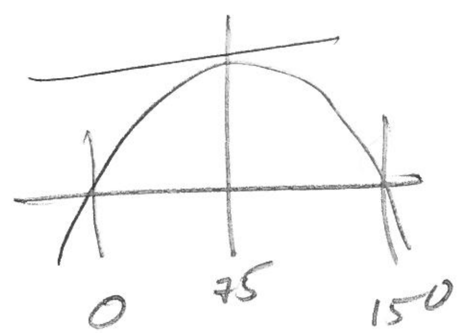
$$\boxed{A(x) = x\left(75 - \tfrac{1}{2}x\right)}$$

b 5pts

$75 - \frac{1}{2}x = 0$

$-\frac{1}{2}x = -75$

$x = 150$



$\boxed{x = 75}$ maximizes $A(x)$

c 5pts $y = 75 - \frac{1}{2}(75) = \frac{75}{2}$ or 37.5

So $\boxed{75 \text{ ft by } 37.5 \text{ ft}}$ are the dimensions

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7 a 5pts t = Time in minutes

D = difference between temperature of the milk and the water bath, in degrees Farenheit.

Given $(t_0, D_0) = (0, 180^\circ - 32^\circ) = (0, 148^\circ)$

& $(t_1, D_1) = (10, 140^\circ - 32^\circ) = (10, 108^\circ)$

b 5pts $\boxed{D_0 = 148^\circ}$

c 5pts $D(t) = D_0 e^{kt} = 148\, e^{kt} \longrightarrow$

$D(10) = 148 e^{10k} = 140 - 32 = 108$

$e^{10k} = \frac{108}{148} = \frac{27}{37}$

$10k = \ln(27/37)$

$\boxed{k = \frac{1}{10} \ln(27/37)} \approx -.031508\,1047$ is FINE

d 5pts $\boxed{D(t) = 148\, e^{kt}}$

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7e 5pts Want Temp = 110°,

i.e. $D(t) = 110 - 32 = 78$

$$148e^{kt} = 78$$

$$e^{kt} = \frac{78}{148}$$

$$kt = \ln(78/148)$$

$$t = \frac{1}{k}\ln(78/148)$$

$$= \frac{-10}{\ln(27/37)}\ln(78/148)$$

≈ 20.3282125 min.

So, about $\boxed{\text{10 more minutes}}$

8 a 5pts (10, 50), (30, 90)

x = # of cookies sold

y = Money in the cash box ($)

$$m = \frac{90-50}{30-10} = \frac{40}{20} = \boxed{2\ \frac{\text{dollars}}{\text{cookie}}}$$

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8b 5pts $y = 2(x-10) + 50$

$= 2x - 20 + 50$

$\boxed{y = 2x + 30}$

c $\boxed{y(0) = 30}$

MAT 121-G11 Test 5 – Spring, 2018 Name KEY
100 Points NO GRAPHING CALCULATORS!!!

Do your work and circle final answers on separate paper, provided. Remember, to write big and bold, but mostly bold, or your teacher can't read it and give you any points for it. Leave a margin for the staple!

1. 5 people are going to be chosen out of 24 people to sit on stage at an awards ceremony.

   a. (5 pts) How many ways can the 5 people be chosen?

   b. (5 pts) How many ways could you choose the 5 people and arrange their seating on stage?

2. Consider the finite geometric sum $4+3+\frac{9}{4}+\frac{27}{16}+\dots+\frac{19683}{65536}$.

   a. (5 pts) Determine a, r and n for the sum.

   b. (5 pts) Use a, r, and n to determine the *exact* value of the sum $\sum_{k=1}^{n} a\cdot r^{k-1}=a\left(\frac{1-r^n}{1-r}\right)$. For full credit, submit your answer as a (possibly improper) fraction, in lowest terms. Decimal answers are worth at most 9 points.

3. (5 pts) Find the sum of the infinite geometric series $\sum_{k=1}^{\infty} 3\cdot\left(\frac{2}{9}\right)^{k-1}$.

Future Value of Annuity: $FV=R\left(\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\left(\frac{r}{m}\right)}\right)=R\left(\frac{(1+i)^n-1}{i}\right)$ **Future Value of Savings:** $A=P\left(1+\frac{r}{m}\right)^{mt}=P(1+i)^n$

4. (5 pts) **(Present Value of Annuity)** If your monthly house payments are $1,000, and the loan is for 30 years, at 3.9% annual percentage rate, compounded monthly, how much did you borrow in the first place?

5. (5 pts) Solve $-2\log_4(5x+15)+7=0$. Give an exact answer and an answer rounded to 4 decimal places.

6. A homeowner wants to enclose a rectangular garden next to her house. She has a total of 150 feet of fencing. She will use the house for one side of the garden and will not need fencing on that side.

   a. (5 pts) Let x be the length (in feet) of the side of the garden next to the house. Write an equation for the area of the garden as a function of x, assuming she uses all the fencing for the other three sides.

   b. (5 pts) What value of x will maximize the area?

   c. (5 pts) What will the dimensions be for the garden with the maximum area?