121 Writing Project #3 Spring, 2018

(1) $f(x) = 2x^5 +$ smaller stuff $\longrightarrow$

End behavior is $2x^5$-like

(whatever happens in here)

(2) Descartes Rule of Signs

$$f(x) = 2\underbrace{x^5 - 19}_{1}\underbrace{x^4 + 166}_{2}\underbrace{x^3 - 457}_{3}\underbrace{x^2 + 482}_{4}\underbrace{x - 174}_{5}$$

So $\boxed{5, 3, \text{ or } 1 \text{ positive zero(s)}}$

$$f(-x) = -2x^5 - 19x^4 - 166x^3 - 457x^2 - 482x - 174$$

$\boxed{0 \text{ Negative zeros!}}$

(3) $a_n x^n = 2x^5 \longrightarrow q: 2$, $a_0 = -174 \longrightarrow p: -174$

$2 \,|\, 174$
$3 \,|\, 87$
29

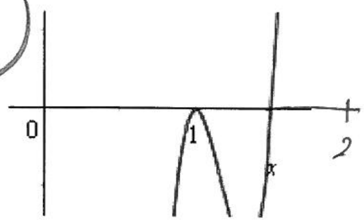
$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6, \pm 29, \pm 58, \pm 87, \pm 174,$$

$$\pm \frac{1}{2}, \pm \cancel{\frac{2}{2}}, \pm \frac{3}{2}, \pm \cancel{\frac{6}{2}}, \pm \frac{29}{2}, \pm \cancel{\frac{58}{2}}, \pm \frac{87}{2}, \pm \cancel{\frac{174}{2}}$$

$$\pm \frac{\text{factors of } a_0}{\text{factors of } a_n} = \pm \frac{p}{q}$$

121 WP 3

(4)



Looks like $x=1$ & $x=\frac{3}{2}$ are good guesses.

Looks like $x=1$ is repeated, because it doesn't cross the x-axis.

Divide by $x-1$:

1	2	-19	166	-457	482	-174	
		2	-17	149	-308	174	
1	2	-17	149	-308	174	0	Sweet!
		2	-15	134	-174		
1	2	-15	134	-174	0		Nice!
		2	-13	121			
	2	-13	121	Nope. Done with one.			

Always guess same thing again if it worked

$\frac{3}{2}$	2	-15	134	-174	NOTE: I'M USING
		3	-18	174	THE DEPRESSED
	2	-12	116	0	POLYNOMIAL!

63
3

Scratch: $2x^2-12x+116=0$

$x^2-6x+58=0$

$b^2-4ac=(-6)^2-4(1)(58)=36-232<0$

$\longrightarrow$ No more real zeros! So...

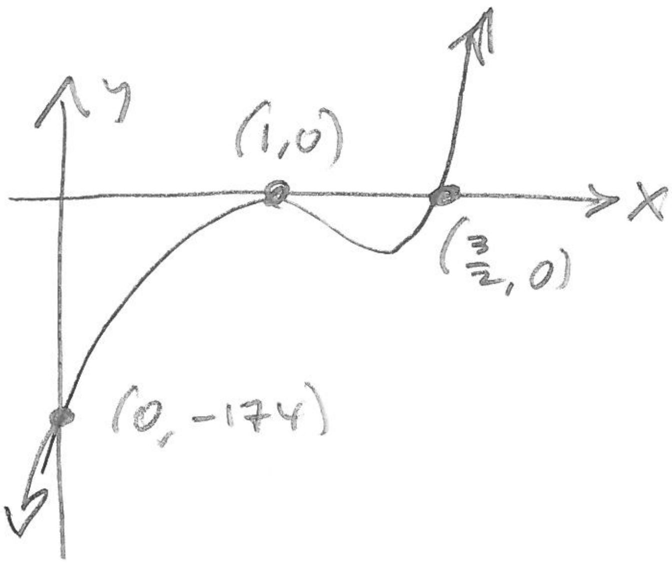
#5

$$\boxed{f(x)=(x-1)^2\left(x-\tfrac{3}{2}\right)\left(2x^2-12x+116\right) \text{ with } x=1,\ m=2\ \&\ x=3,\ m=1}$$

#4

121 ~~R~~ WP #3

(6)



(7) It remains only to find the zeros of $2x^2-12x+116$ & we're already partway there. (See previous.)

$$2x^2-12x+116=0 \longrightarrow x^2-6x+58=0$$

$a=1, b=-6, c=58$

$$\begin{array}{r|l} 2 & 196 \\ 2 & 98 \\ 7 & 49 \\ & 7 \end{array}$$

$$\longrightarrow b^2-4ac = 36-232 = -196$$

$$\longrightarrow x = \frac{6 \pm \sqrt{-196}}{2(1)} = \frac{6 \pm 14i}{2} = \boxed{3 \pm 7i}$$

This gives

$$2(x-1)^2\left(x-\tfrac{3}{2}\right)\left(x-(3+7i)\right)\left(x-(3-7i)\right) = f(x)$$

121 WP #3

(8) $R(x) = \dfrac{2x^2-x-15}{x^2-4x-21} = \dfrac{(2x+5)(x-3)}{(x-7)(x+3)}$

$D = \mathbb{R} \setminus \{-3, 7\}$

$\boxed{\text{V.A.: } x=-3,\ x=7}$

H.A.: $\dfrac{2x^2 + \cdots}{x^2 + \cdots} \xrightarrow{x \to \infty} \dfrac{2x^2}{x^2} = 2 = y$

$\boxed{y = 2 \text{ is H.A.}}$

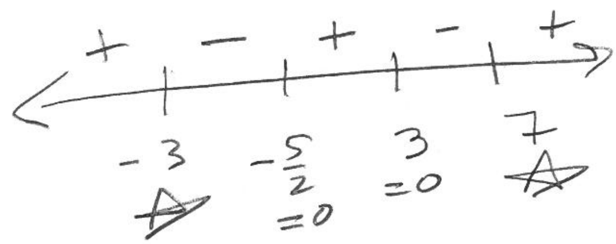
x-int: $x = -\frac{5}{2}$, $x = 3$

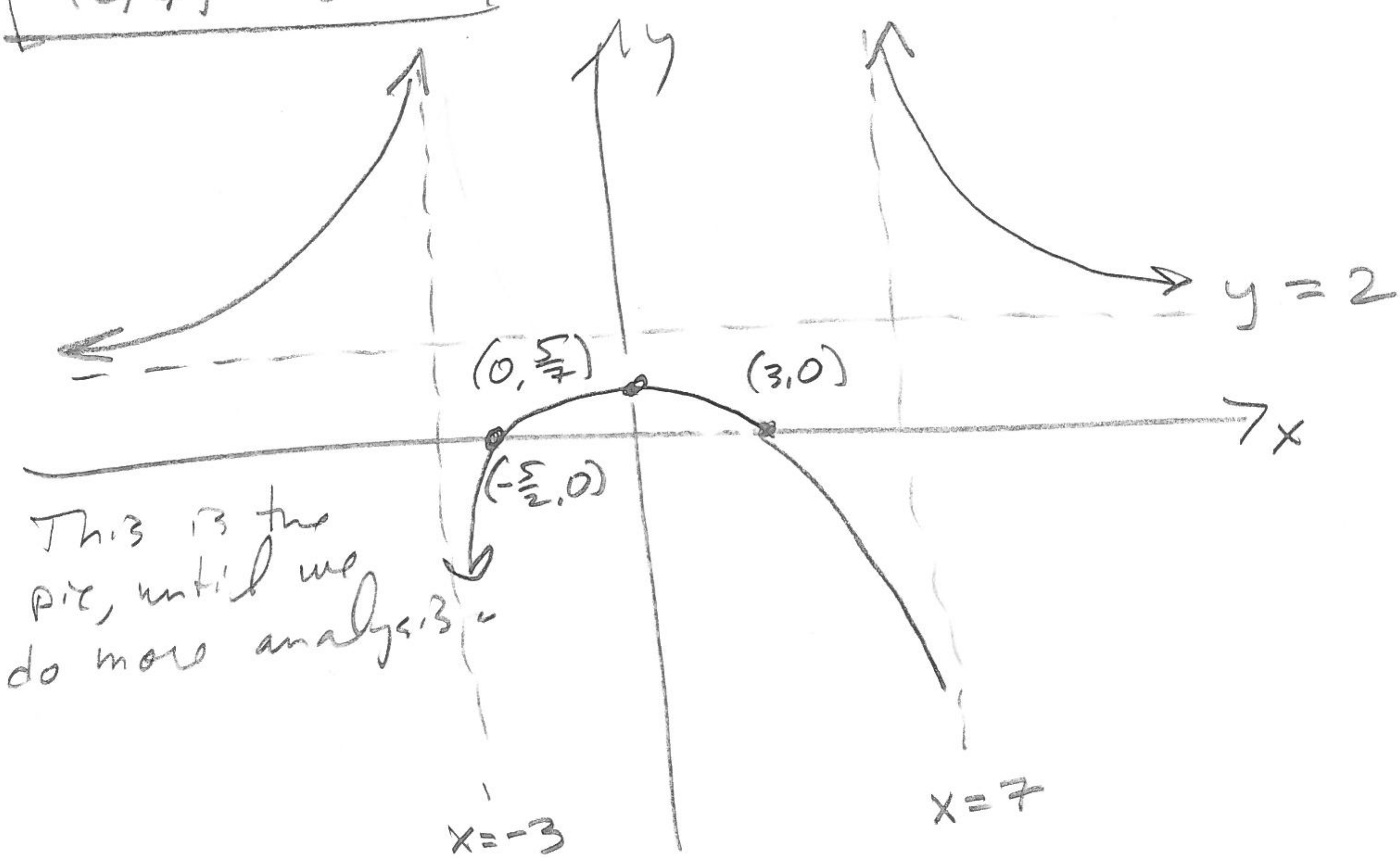
$\boxed{(-\tfrac{5}{2}, 0),\ (3,0) \text{ are x-ints}}$

y-int: $\dfrac{-15}{-21} = \dfrac{5}{7}$

$\boxed{(0, \tfrac{5}{7}) \text{ is y-int}}$

All factors are $(x-c)^1$
1 is odd. Sign changes





This is the pic, until we do more analysis.

121 WP 1

For BONUS 5 pts

$R(x) \overset{\text{SET}}{=} 2 \longrightarrow$

$$\frac{2x^2-x-15}{(x-7)(x+3)} = \frac{2}{1}\cdot\frac{x^2-4x-21}{x^2-4x-21} = \frac{2x^2-8x-42}{\text{LCD}}$$

$$\Longrightarrow \frac{2x^2-x-15-(2x^2-8x-42)}{\text{LCD}} = 0 \longrightarrow$$

$$2x^2-x-15-2x^2+8x+42=0$$

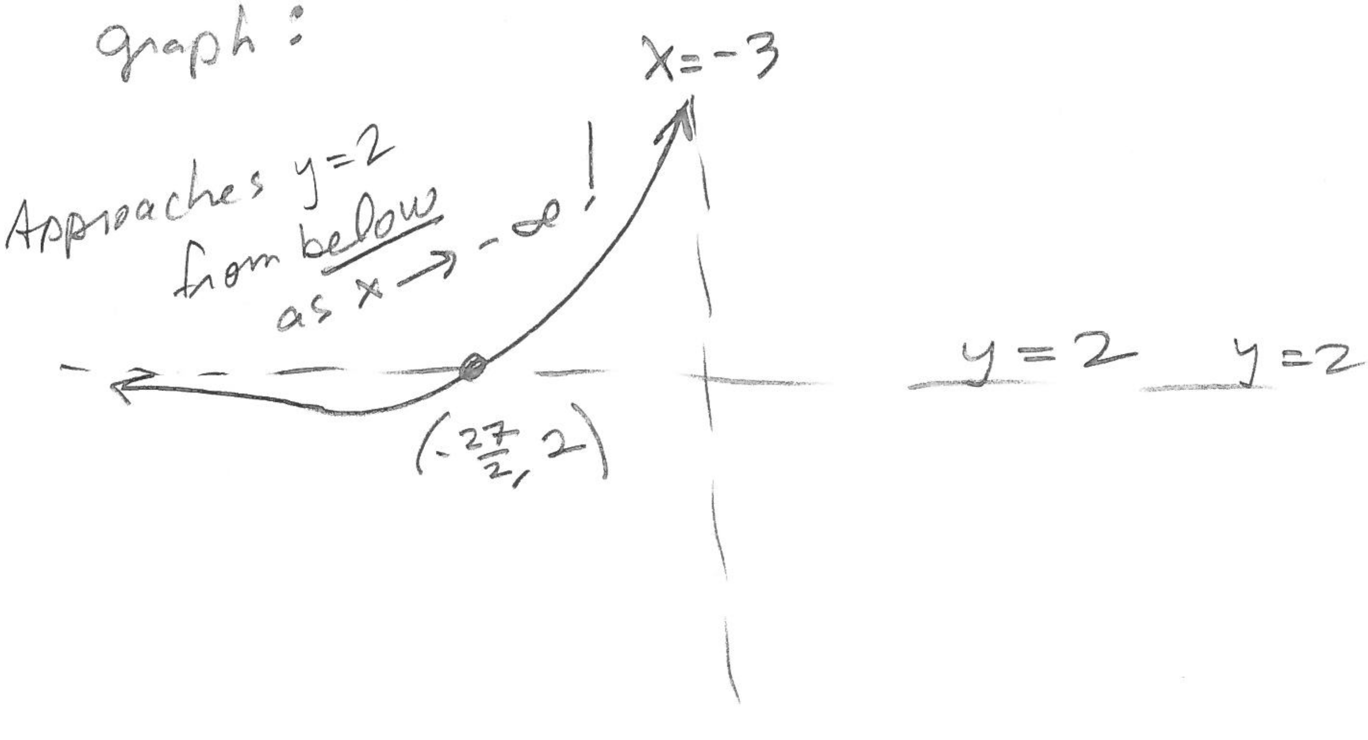
$\cancel{4}$ 15
27
42

$\longrightarrow 7x+27=0$

$\Longrightarrow 7x=-27$

$\Longrightarrow x=-\frac{27}{7} \rightsquigarrow \boxed{\left(-\frac{27}{7}, 2\right)}$

This changes the left part of the graph:



'21 WP #3

(9) $Q(x) = \dfrac{2x^3-23x^2-4x+165}{x^3-15x^2+23x+231}$

$= \dfrac{(2x+5)(x-3)(x-c)}{(x-7)(x+3)(x-c)}$ Need to find $x-c$. Don't like the $(2x+5)(x-3)$ as much as $(x-7)(x+3)$ for this chore:

Divide $x^3-15x^2+23x+231$ by $x-7$:

7	1	-15	23	231
		7	-56	-231
-3	1	-8	-33	0
		-3	33	
	1	-11	0	

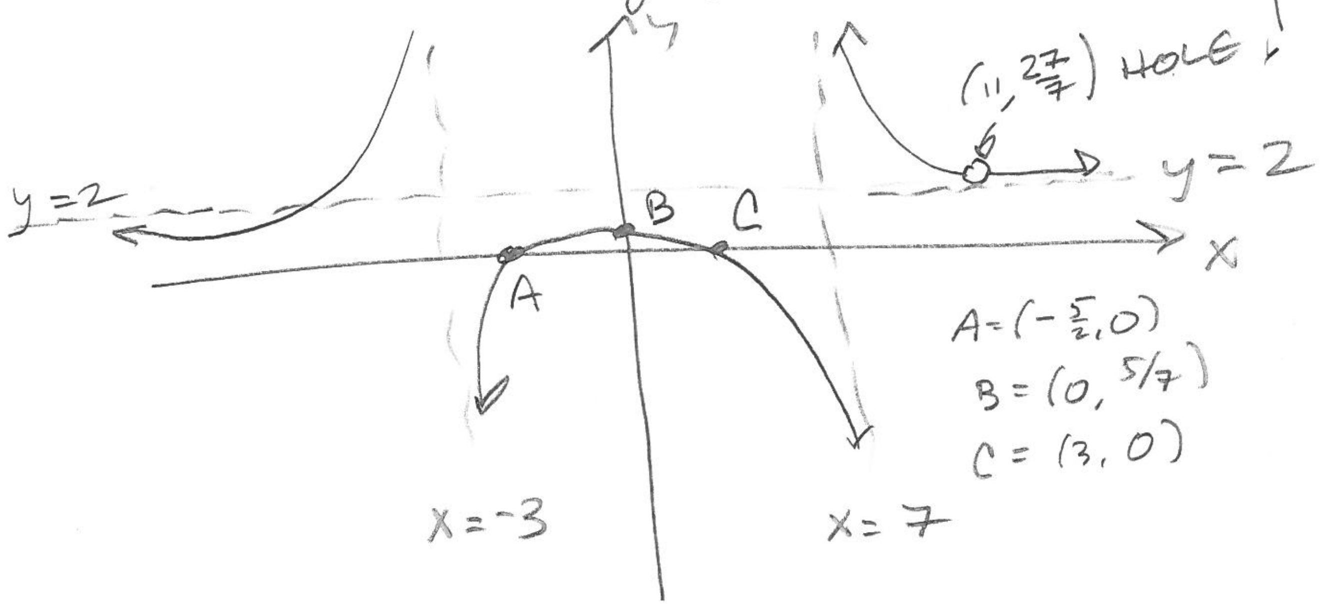
Now by $x+3$:

So, $x-c = x-11$ ⟶ Hole @ $x=11$ on graph of $R(x)$ in #8!

$R(11) = \dfrac{(2(11)+5)(11-3)}{(11-7)(11+3)}$

$= \dfrac{(22+5)(8)}{(4)(14)}$

$= \dfrac{(27)(8)}{(4)(14)} = \dfrac{27}{7}$



121 WP #3

(10) $T(x) = \dfrac{x^3 - 13x^2 + 55x - 91}{x^2 - x - 2} = \dfrac{(x-7)(x^2-6x+13)}{(x-2)(x+1)}$

$\mathcal{D}: \mathbb{R} \setminus \{-1, 2\}$

$$\begin{array}{r|rrrr} 1 & 1 & -13 & 55 & -91 \\ & & 1 & -12 & \\ \hline & 1 & -12 & 43 & \end{array}$$

$\boxed{\text{V.A. } x=-1, x=2}$

$\boxed{\text{H.A. NONE}}$

$$\begin{array}{r|rrrr} -1 & 1 & -13 & 55 & -91 \\ & & -1 & 14 & \\ \hline & 1 & -14 & 69 & \end{array}$$

O.A.: Need to do long division

$$\begin{array}{r|rrrr} 7 & 1 & -13 & 55 & -91 \\ & & 7 & -42 & 91 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

! Sweet!

11 - 3

$x^2 - 6x + 13 = 0$

$x^2 - 6x + 3^2 = -13 + 9$

$(x-3)^2 = -4$ No real soln

$\boxed{(7,0) \text{ is only x-int.}}$

y-int: $\dfrac{-91}{-2} = \dfrac{91}{2} \rightarrow \boxed{\left(0, \tfrac{91}{2}\right) \text{ is y-int}}$

121 WP #3

10 cnt'd We find the oblique asymptote:

$$\begin{array}{r l} & x - 12 \\ x^2 - x - 2 \,\big)\! & x^3 - 13x^2 + 55x + 165 \\ & -(x^2 - x^2 - 2x) \\ \hline & -12x^2 - 53x \end{array}$$

$y = x - 12$ is O. A.