121 TEST 3

Spring, 2018

(1) $\boxed{f(x) = (x-(5-3i))(x-(5+3i))(x-3)^4(x+5)^2}$ 10pts

(2)

2	5	-3	0	7	-20	44
		10	14	28	70	100
	5	7	14	35	50	$\boxed{144 = P(2)}$

10pts

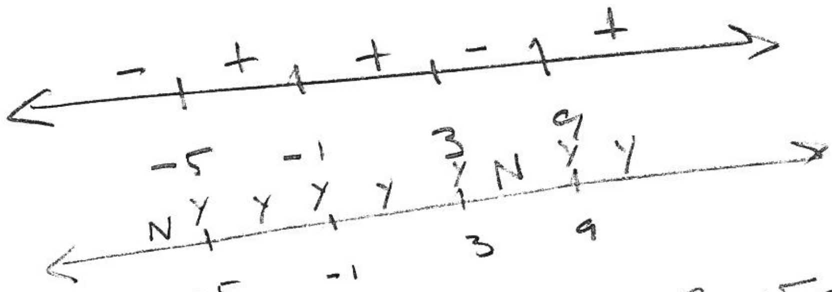
(3) $\boxed{P(x) = (x-2)(5x^4+7x^3+14x^2+35x+50)+144}$ 5pts

(1) $(x+5)(x+1)^2(x-3)(x-9) = x^5 + \cdots + 135 = f(x)$

(2) $f(x) \geq 0$ $\quad -5, -1, 3, 9$

x^5

≥ 0



$\Rightarrow x \in [-5,-1] \cup [-1,3] \cup [9,\infty)$

$\boxed{= [-5,3] \cup [9,\infty)}$ 10pts

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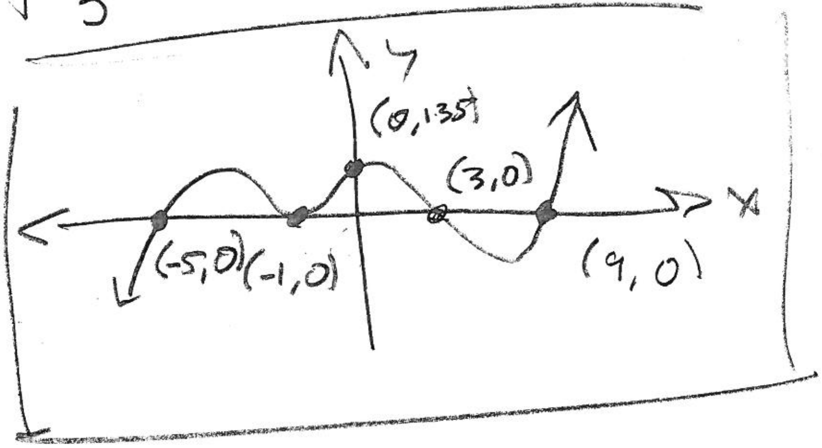
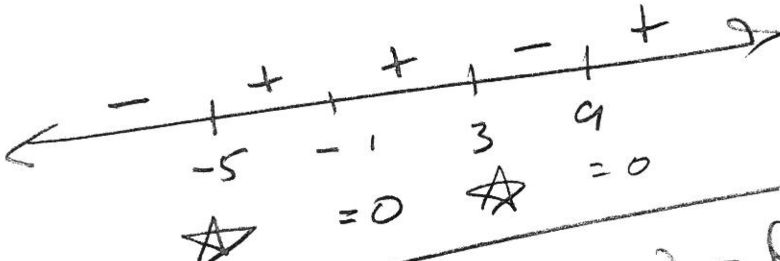
4b



10 pts

4c $g(x) = \sqrt{\dfrac{(x+1)^2(x-9)}{(x+5)(x-3)}}$ domain?

Need $\dfrac{(x+1)^2(x-9)}{(x+5)(x-3)} \geq 0$



$\Rightarrow x \in \boxed{(-5, 3) \cup [9, \infty) = D(g)}$ 5 pts

$(-5, 1] \cup [1, 3)$

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(5) $f(x) = 2x^5 + 4x^4 - 9x^3 - 43x^2 - 53x - 21$

(a) Descartes:

1 positive root

$f(-x) = \underbrace{-2x^5 + 4x^4}_{1} \underbrace{+ 9x^3}_{2} \underbrace{- 43x^2}_{3} \underbrace{+ 53x}_{4} - 21$

5 pts

4, 2 or 0 negative roots

(b) $\frac{p}{q}$: $\frac{21}{2} \rightarrow \pm 1, \pm 3, \pm 7, \pm 21$

$\pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{7}{2}, \pm \frac{21}{2}$

(6) out of sequence

1	2	4	-9	-43	-53	-21
		2	6	-3	-46	
	2	6	-3	-46	NO	

(5c) $x = -4$ by Theorem
$x = -1$ in fact.
$x = 3$ by Theorem
9 in fact.

-1	2	4	-9	-43	-53	-21
		-2	-2	11	32	21
-1	2	2	-11	-32	-21	0 sweet!
		-2	0	11	21	
-1	2	0	-11	-21	0 sweet	
		-2	2	NO		
	2	-2	-9			

$x = -1, m = 2$
$x = 3, m = 1$

10 pts

$\boxed{f(x) = (x+1)^2(x-3)(2x^2+6x+7)}$

3	2	0	-11	-21
		6	18	21
	2	6	7	0 sweet!

$b^2 - 4ac = (-2)^2 - 4(6)(7) < 0$

$= 4$ No more real roots.

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5c

-1	2	4	-9	-43	-53	-21
		-2	-2			
	2	2	-11			

BONUS

I Know $x=-1$ is the lowest/least of the roots, but this test fails.

-2	2	4	-9	-43	-53	-21
		-4	0			
	2	0	-9			

Lower Bound

$x=-1$ in fact
$x=-4$ by theorem

-3	2	4	-9	-43	-53	-21
		-6	6			
	2	-2	-3 NO			

upper Bound
$x=3$ by Thm & in fa[ct]

-4	2	4	-9	-43	-53	-21
		-8	16	-28	284	$-$BIG
	2	-4	7	-71	233	$-$BIG

By the Theorem on Bounds, $x=-4$ is the best bound obtainable, even though we KNOW $x=-1$ is a lower bound and it's the GREATEST lower bound.

$x=-4$ using theorem

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5.C cont'd. We KNOW $x=3$ is the least upper bound on real roots, but let's check the theorem:

$$
\begin{array}{r|rrrrrr}
3 & 2 & 4 & -9 & -43 & -53 & -21 \\
  &   & 6 & 30 & 63 & 60 & 21 \\
\hline
  & 2 & 10 & 21 & 20 & 7 & 0
\end{array}
$$

All ≥ 0, so the Theorem gives us the LEAST upper bound $\boxed{x=3.}$

(7) Continuing with $f(x) = (x+1)^2(x-3)(2x^2+6x+7)$

$b^2-4ac = 6^2-4(2)(7) = 36-56 = -20$

$x = \dfrac{-6 \pm 2i\sqrt{5}}{2(2)} = \dfrac{3 \pm i\sqrt{5}}{2} = x$ $\quad 2\,\underline{|20}$ $\quad 2\,\underline{|10}$ $\quad 5$

$$\boxed{f(x) = 2(x+1)^2(x-3)\left(x-\left(\frac{3+i\sqrt{5}}{2}\right)\right)\left(x-\left(\frac{3-i\sqrt{5}}{2}\right)\right)}$$

5 pts

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8 $R(x) = \dfrac{3x^3+14x^2-15x-50}{2x^2-7x-15} = \dfrac{\sim\sim}{(2x+3)(x-5)}$

$\mathcal{D} = \mathbb{R} \setminus \{-\frac{3}{2}, 5\}$

5 pts

$\boxed{\text{V.A.: } x = -\frac{3}{2},\ x = 5}$

O.A.:

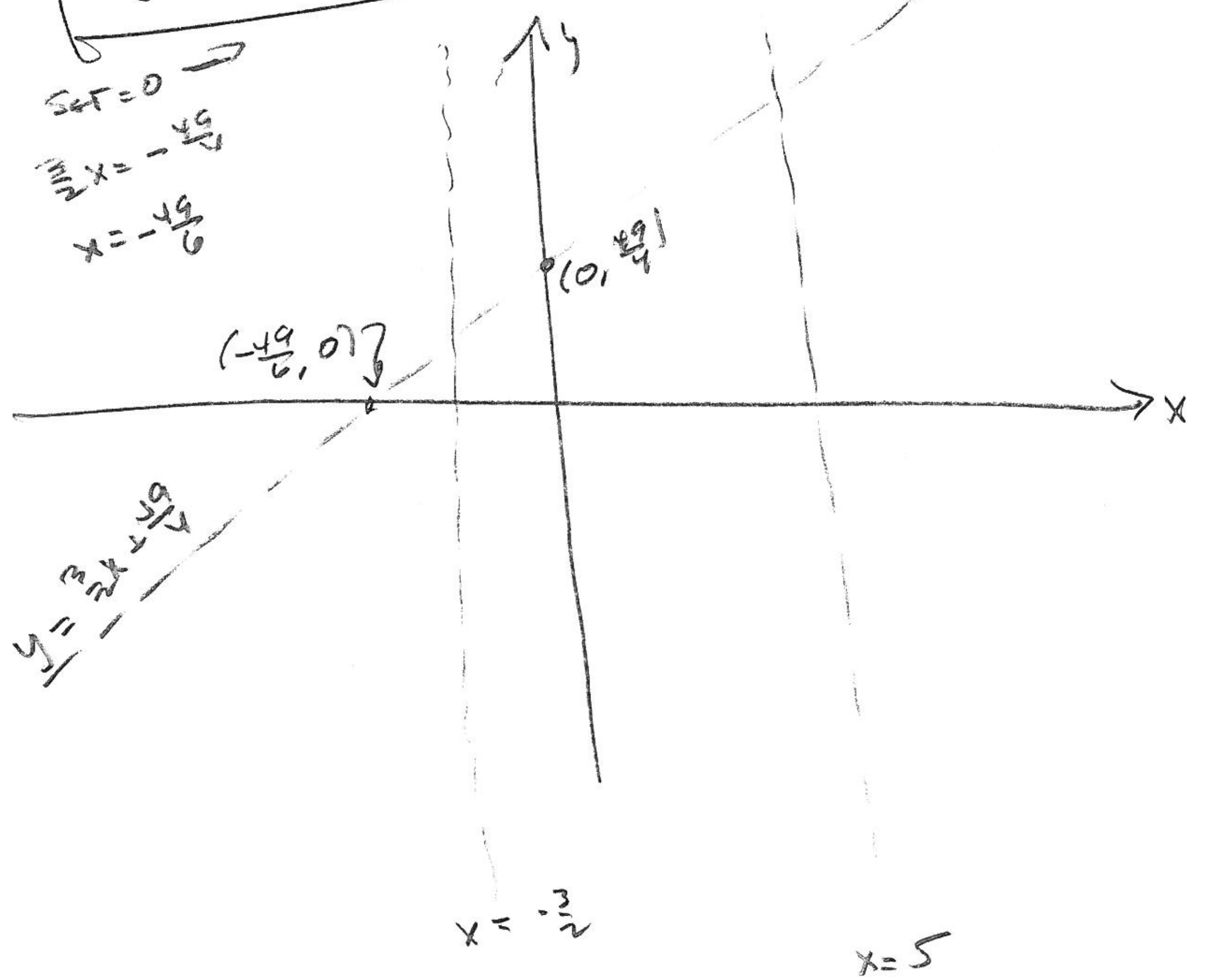
$$\begin{array}{r} \frac{3}{2}x + \frac{49}{4} \\ 2x^2-7x-15 \overline{)\ 3x^3+14x^2-15x-50} \\ -\left(3x^3 - \frac{21}{2}x^2 - \frac{45}{2}x\right) \\ \hline \frac{49}{2}x^2 \end{array}$$

$\boxed{y = \frac{3}{2}x + \frac{49}{4}}$

Set $= 0$ →

$\frac{3}{2}x = -\frac{49}{4}$

$x = -\frac{49}{6}$



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9 $f(x) = \dfrac{3x^2 - x - 10}{2x^2 - 7x - 15} = \dfrac{(3x+5)(x-2)}{(2x+3)(x-5)}$

$-\frac{5}{3}, 2, -\frac{3}{2}, 5$

$\mathcal{D}: \mathbb{R} \setminus \{-\frac{3}{2}, 5\}$

$+ \quad - \quad + \quad - \quad +$

$-\frac{5}{3} \quad -\frac{3}{2} \quad 2 \quad 5$

V.A.: $x = -\frac{3}{2}, x = 5$

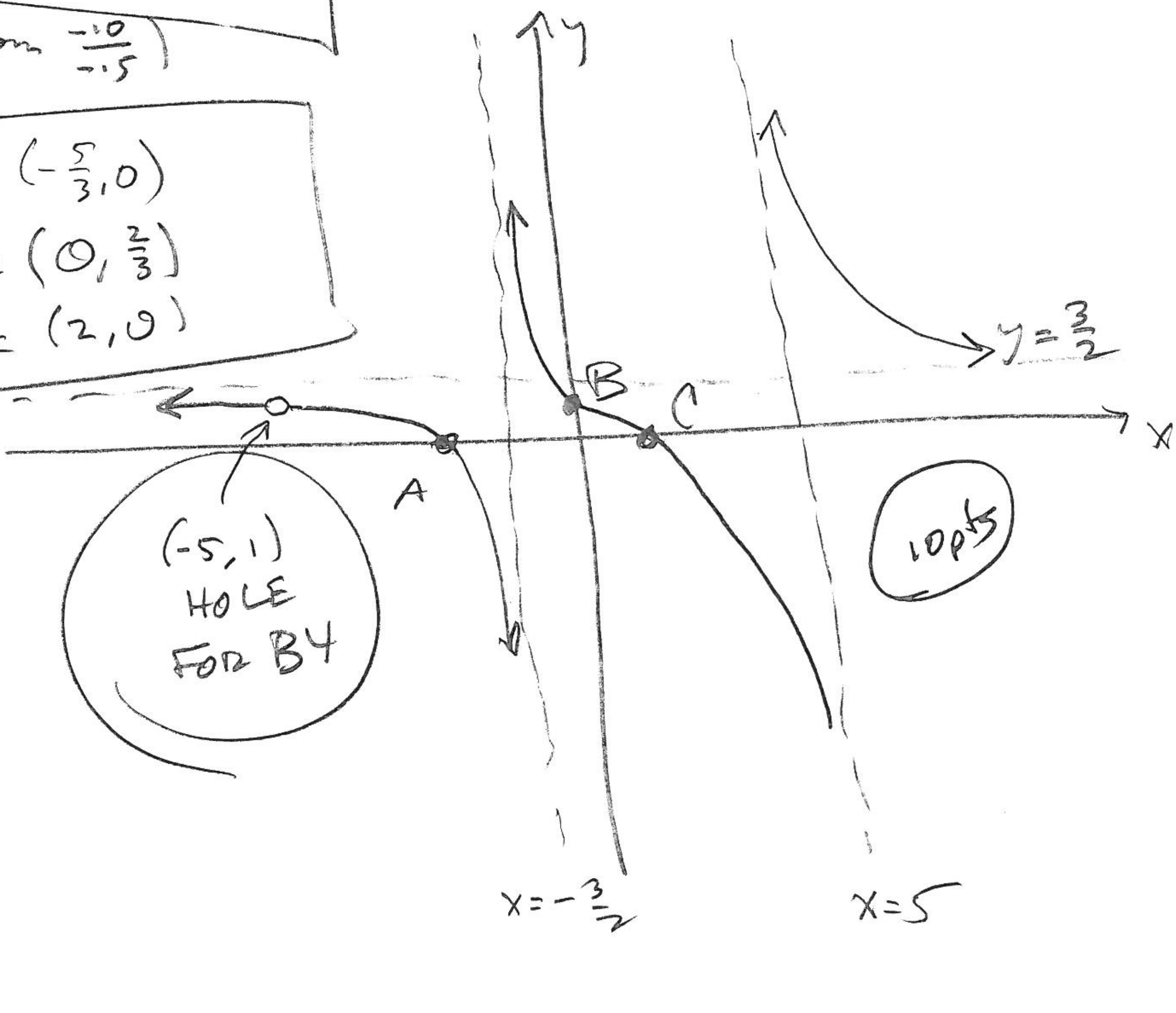
HA: $y = \frac{3}{2}$

x-int: $(-\frac{5}{3}, 0)$
$(2, 0)$

y-int: $(0, \frac{2}{3})$

(from $\frac{-10}{-15}$)

$A = (-\frac{5}{3}, 0)$
$B = (0, \frac{2}{3})$
$C = (2, 0)$



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10 pts each

B1 $\boxed{(x-(3-\sqrt{2}))^2(x-(3+\sqrt{2}))^2(x-(5+2i))(x-(5-2i))(x-7)^2}$

B2 2 5pts $|3x+8|+9<7$

$|3x+8|<-2$ Nevah! $\boxed{\varnothing}$

b $|3x+11|+7>10$

$|3x+11|>3$

$3x+11>3$ OR $3x+11<-3$

$3x>-8$ $\quad$ $3x<-14$

$\left\{x \mid x>-\frac{8}{3} \text{ OR } x<-\frac{14}{3}\right\}$

$= \boxed{\left(-\infty,-\frac{14}{3}\right)\cup\left(-\frac{8}{3},\infty\right)}$

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B3 $R(x) = \dfrac{3x^3+14x^2-15x-50}{2x^2-7x-15} = \dfrac{(x-2)(x+5)(3x+5)}{(2x+3)(x-5)}$

$$\begin{array}{r|rrrr} 2 & 3 & 14 & -15 & -50 \\ & & 6 & 40 & 50 \\ \hline & 3 & 20 & 25 & 0 \end{array}$$

x --

$3 \,|\, 75$

$5 \,|\, 25$

5

$3x^2+20x+25$

$= 3x^2+15x+5x+25$

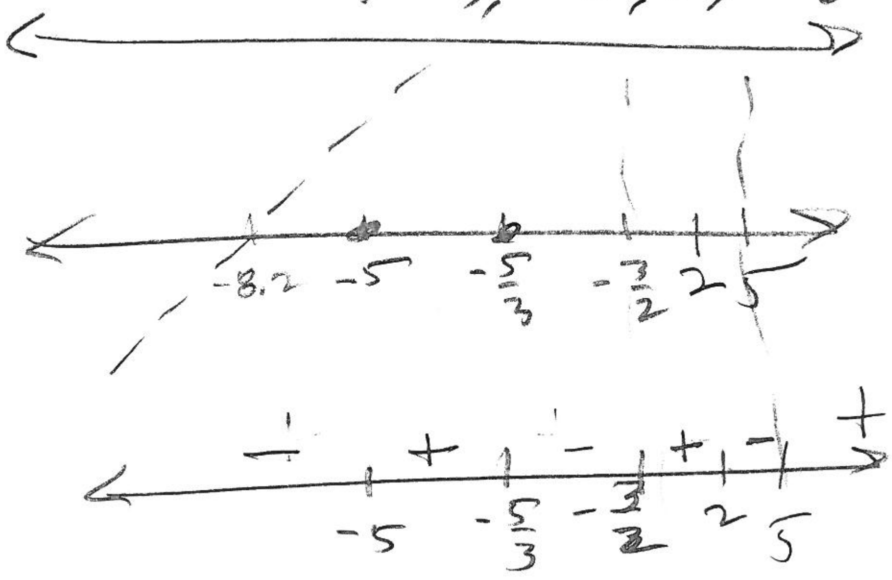
$= 3x(x+5)+5(x+5)$

$= (x+5)(3x+5)$

3,

x-int: $(2,0), (-5,0), (-\frac{5}{3},0)$

$2, -5, -\frac{5}{3}, -\frac{3}{2}, 5, -\frac{48}{6} \approx -8.2$



B3

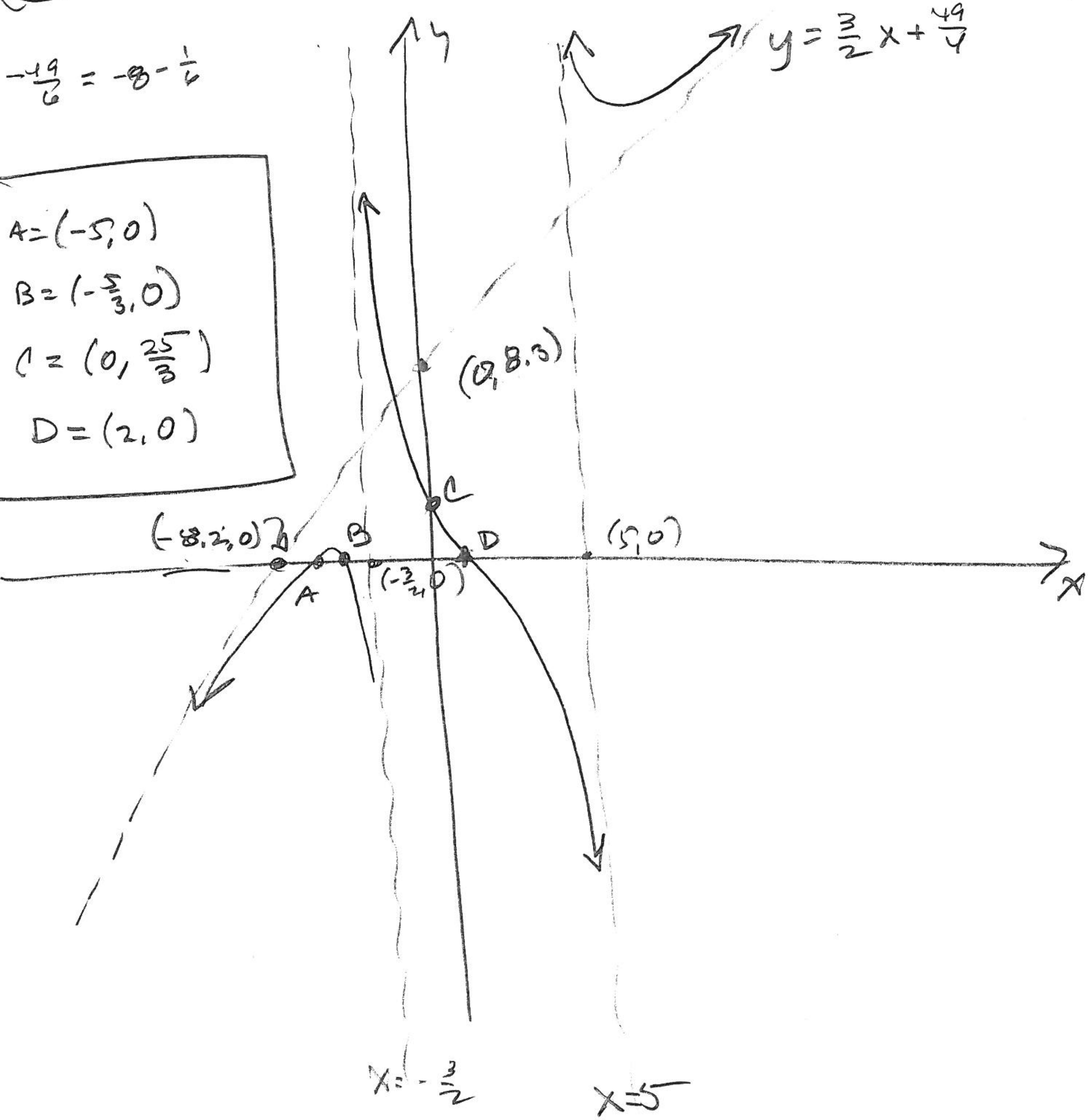
$-\frac{49}{6} = -8 - \frac{1}{6}$

$A = (-5, 0)$

$B = (-\frac{5}{3}, 0)$

$C = (0, \frac{25}{3})$

$D = (2, 0)$



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B4

$3x^3+14x^2-15x-50=(3x^2-x-10)(x-c)$

$=(3x+5)(x-2)$

$$\begin{array}{r|rrrr} 2 & 3 & 14 & -15 & -50 \\ & & 6 & 40 & 50 \\ \hline & 3 & 20 & 25 & 0 \end{array}$$

$(3x+5)(x+5)$

$$\begin{array}{r|l} 3 & 75 \\ 5 & 25 \\ \hline & 5 \end{array}$$

so, $(3x+5)(x+5)(x-2)$

$$\frac{3x^3+14x^2-15x-50}{2x^3-17x^2+20x+75}=\frac{(3x+5)(x-2)\cancel{(x+5)}}{(2x+3)(x-5)\cancel{(x+5)}}$$

HOLE @ $x=-5$

$G(x)=R(x)$ w/ a hole, so,

$$R(-5)=\frac{(3(-5)+5)(-5-2)}{(2(-5)+3)(-5-5)}=\frac{(-10)(-7)}{(-7)(-10)}=1!$$

See #9

12.1 T3

B5 $f(x)=\dfrac{2}{x-6}$, $\mathcal{D}=\mathbb{R}\setminus\{6\}=(-\infty,6)\cup(6,\infty)=\{x\mid x\neq 6\}$

$g(x)=\sqrt{x+11}$, $\mathcal{D}=[-11,\infty)=\{x\mid x\geq -11\}$

$(f\circ g)(x)=f(g(x))=\dfrac{2}{\sqrt{x+11}-6}$

$\mathcal{D}(f\circ g)=\{x\mid x\in\mathcal{D}(g) \text{ and } g(x)\in\mathcal{D}(f)\}$

$=\{x\mid x\geq -11 \text{ and } \sqrt{x+11}\neq 6\}$

Scratch: $\sqrt{x+11}\neq 6$

$x+11\neq 36$

$x\neq 25$

$=\{x\mid x\geq -11 \text{ and } x\neq 25\}$ AND

$=$

$= \boxed{[-11,25)\cup(25,\infty)=\mathcal{D}(f\circ g)}$