MAT 121 | Test 2 – Spring, 2017 | Name KEY
100 Points | Covers Chapter 2 | NO GRAPHING CALCULATORS!!!

Do your work and circle (or square-box) final answers on the separate paper, provided.

Only write on one side of each sheet. No points for what's on the back (either for or against!).

All that you should put on this cover sheet is your name.

1. (5 kindness points) I can really see how you made an effort to be clear, use one column per page, one side of writing per page, and plenty of room for my silly comments. Thanks for writing big enough to read, and really dark, which helped me see your work so easily. You helped me serve your classmates, better, too, because your paper was quick and easy to grade. I thank you on their behalf.

2. Consider the relation $R=\{(1,3),(2,4),(3,5),(4,3)\}$.

   a. (5 pts) Is R a function? If not, why not?

   b. (5 pts) What is the domain of R ?

   c. (5 pts) What is the range of R ?

   d. (5 pts) What stops R from being 1-to-1?

3. Let $f(x)=\dfrac{1}{x-4}$ and $g(x)=\sqrt{x+5}$.

   a. (5 pts) What is the domain of f ?

   b. (5 pts) What is the domain of g ?

   c. (5 pts) Write the function $\dfrac{f}{g}$. Do not simplify.

   d. (5 pts) What is the domain of $\dfrac{f}{g}$?

   e. (5 pts) Write the function $f \circ g$. Do not simplify.

   f. (5 pts) What is the domain of $f \circ g$? (Highest level of synthesis.)

4. (5 pts) Let $f(x)=x^2-4x+6$. Provide a simple graph f by finding its vertex. You don't need to go step-by-step like you did in your Writing Project. Then write the difference quotient for f and show what it represents, with a secant line from x to $x+h$ on the graph of f.

5. (5 pts) Simplify the difference quotient for $f(x)=x^2-4x+6$.

6. Let $g(x)=\dfrac{-4}{(3x-6)^2}-5$. THIS is your Writing-Project-#2 question.

   a. (20 pts) Sketch the graph of $g(x)$, by transforming the basic function $f(x)=\dfrac{1}{x^2}$. I want to see two (2) points labeled in the graph of f, namely $(-1,1)$, and $(1,1)$, and track where those points are moved to after every step, as demonstrated in video, until you've transformed f into g. This will take 5 graphs, counting the first graph of $f(x)=\dfrac{1}{x^2}$ as the first. (I number them 0 thru 4, for no good reason.)

   b. (5 pts) State the domain and range of $g(x)$, based on your final graph. Use interval notation in your answer.

   c. (5 pts) Find any x- or y-intercept(s) of $g(x)$, and label them, clearly, on the graph.

7. (5 pts) Prove that $f(x)=\dfrac{x-1}{x+3}$ is one-to-one.

8. (5 pts) The force of gravity, F, varies jointly with the masses m_1 and m_2 of the two bodies, and inversely with the square of the distance r between their centers of mass. The constant of proportionality, here, is the universal gravitational constant G. Write the equation relating F to m_1, m_2, and r.

Bonus Section (5 pts each) Are you smarter than the average bear? Find out, by answering up to 3 of the following for up to 15 points.

B1. Write down your answer to #5, again, and pass to the limit as h approaches zero, and show me some calculus.

B2. Simplify the difference quotient for the function $f(x)=\sqrt{3x}$. Then pass to the limit, as h approaches zero, and demonstrate an early aptitude for Calculus.

B3. Add the line to your picture in **#5**, that represents the tangent to f at the point $(x, f(x))$.

B4. Complete the square to re-write the function $h(x)=5x^2-2x-13$ in the form $a(x-h)^2+k$. What is the vertex?

B5. Sketch the graph of the piecewise-defined function $h(x)=\begin{cases}\sqrt{x+8} & \text{if } x<3 \\ x^2-5x+4 & \text{if } x\geq 3\end{cases}$.

B6. Solve the absolute-value inequality $|3-2x|>25$. Express the solution iu interval notation.

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1. Kindness

2. $R = \{(1,3), (2,4), (3,5), (4,3)\} \Longrightarrow$

a) R is a function $\boxed{\text{Yes}}$

b) $\boxed{\mathcal{D}(R) = \{2, 3, 4, 1\}}$

c) $\boxed{\mathcal{R}(R) = \{4, 3, 5\}}$

d) $\boxed{f(1) = f(4) = 3 \text{ prevents it from being 1-to-1.}}$

3. $f(x) = \dfrac{1}{x-4}$, $g(x) = \sqrt{x+5}$

$x \neq 4$:

a) $\boxed{\mathcal{D}(f) = \mathbb{R} \setminus \{4\}}$ $= (-\infty, 4) \cup (4, \infty)$

$x \geq -5$

b) $\boxed{\mathcal{D}(g) = [-5, \infty)}$

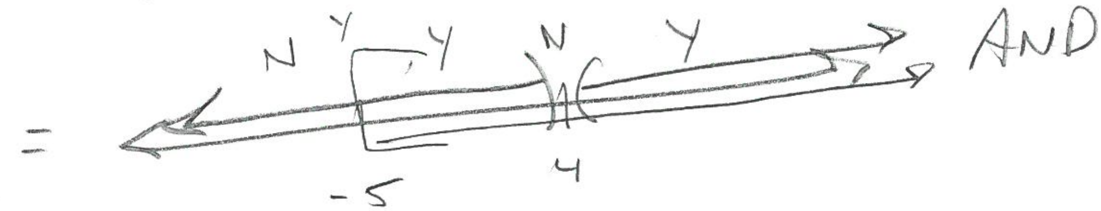
c) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \boxed{\dfrac{\frac{1}{x-4}}{\sqrt{x+5}}}$

121 TEST 2 $\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid \underbrace{x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g)}\text{ and } g(x) \neq 0\}$

(d) $\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid \boxed{x \in \mathcal{D}(f) \cap \mathcal{D}(g)} \text{ and } g(x) \neq 0\}$

$= \{x \mid x \neq 4 \text{ and } x \geq -5 \text{ and } x+5 \neq 0\}$

$= \{x \mid x \neq 4 \text{ and } x > -5\}$

$=$ 

$\boxed{= [-5,4) \cup (4,\infty)}$

(e) $(f \circ g)(x) = f(g(x)) = \boxed{\dfrac{1}{\sqrt{x+5} - 4}}$

(f) $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \geq -5 \text{ and } \sqrt{x+5} \neq 4\}$

(Scratch: $\sqrt{x+5} \neq 4$
$x+5 \neq 16$
$x \neq 11$)

$\boxed{= \{x \mid x \geq -5 \text{ and } x \neq 11\} = [-5,11) \cup (11,\infty) = \mathcal{D}(f \circ g)}$

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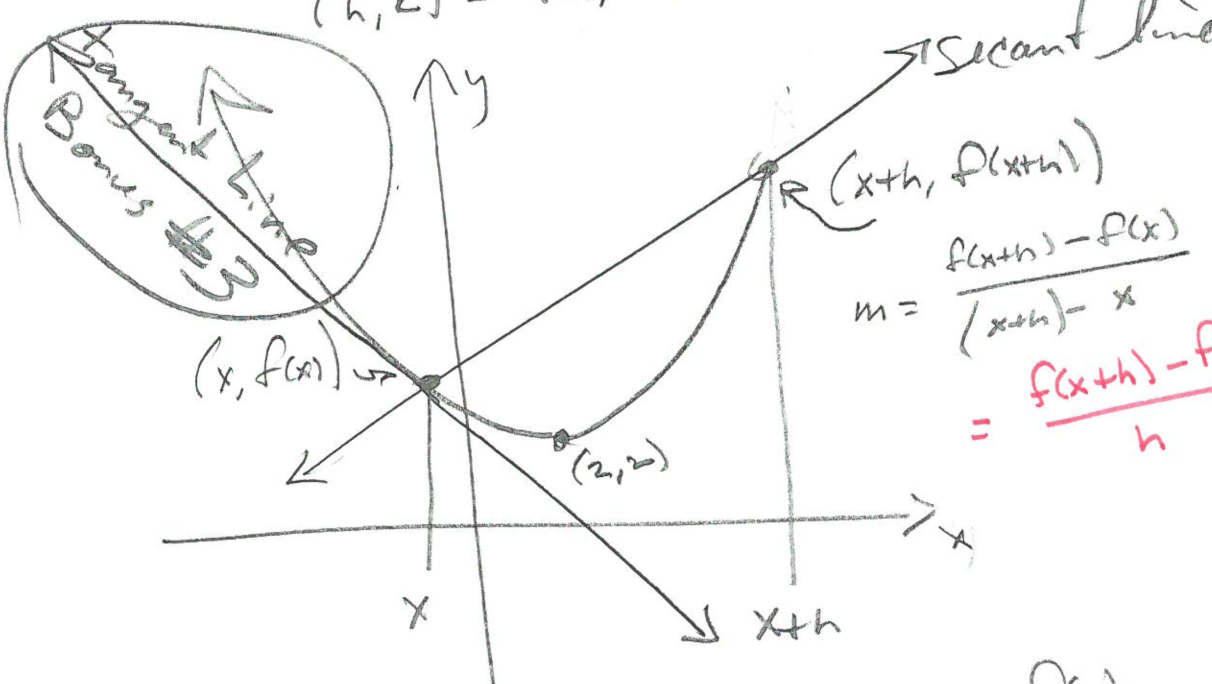
(4) $f(x) = x^2 - 4x + 6 = x^2 - 4x + 4 - 4 + 6$

$= (x-2)^2 + 2$

$(h, k) = (2, 2)$



$m = \frac{f(x+h) - f(x)}{(x+h) - x}$

$= \frac{f(x+h) - f(x)}{h}$

Difference Quotient $= \frac{f(x+h) - f(x)}{h}$

$=$ slope of secant line $= \frac{(x+h)^2 - 4(x+h) + 6 - (x^2 - 4x + 6)}{h}$

(5) $= \frac{x^2 + 2xh + h^2 - 4x - 4h + 6 - x^2 + 4x - 6}{h}$

$= \frac{2xh + h^2 - 4h}{h} = \frac{h(2x + h - 4)}{h} = \boxed{2x + h - 4}$

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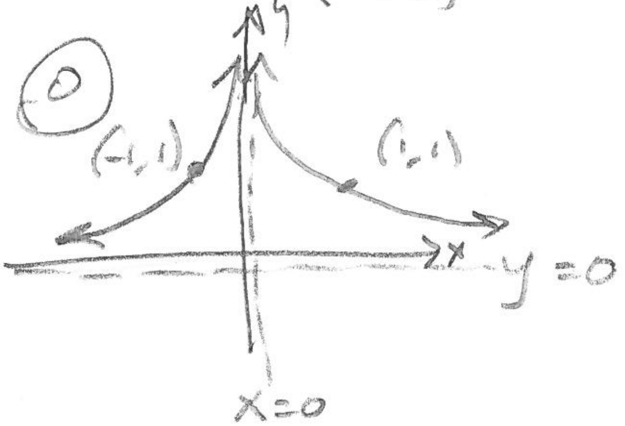
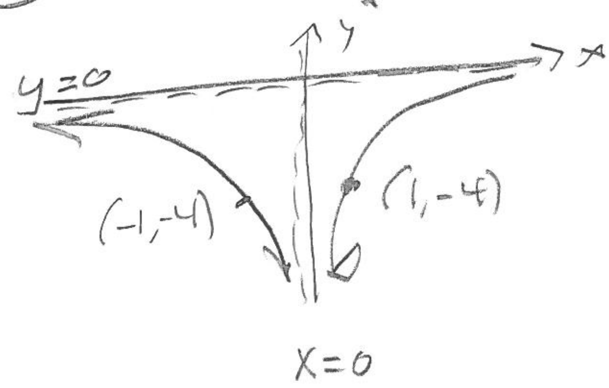
(6) $g(x) = \dfrac{-4}{(3x-6)^2} - 5$

(0)



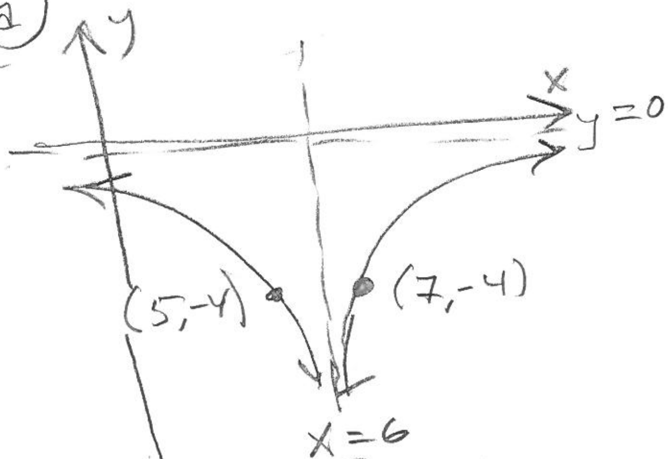
$f(x) = \dfrac{1}{x^2}$

(1) $-4f(x) = \dfrac{-4}{x^2}$

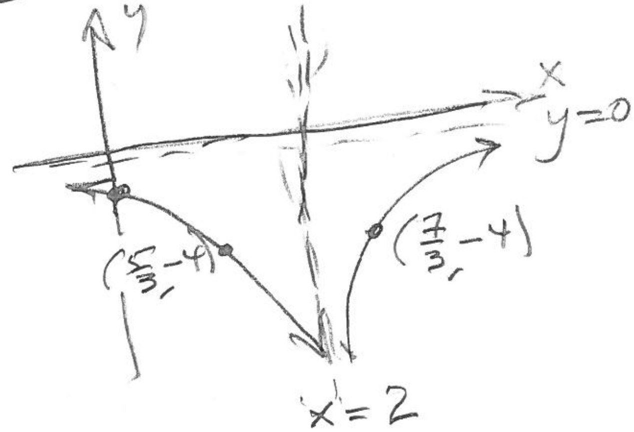


(2) $-4f(x-6) = \dfrac{-4}{(x-6)^2}$

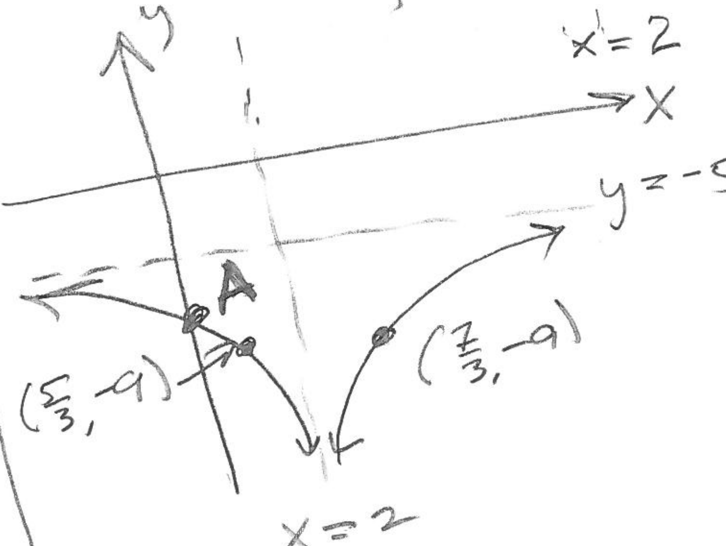
(a)



(3) $-4f(3x-6) = \dfrac{-4}{(3x-6)^2}$



(4) $g(x)$



(b) $D = \mathbb{R} \setminus \{2\}$
$R = (-\infty, -5)$

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(c) No x-int, but $g(0) = \frac{-4}{(-6)^2} - 5 = \frac{-4}{36} - 5$

$= -\frac{1}{9} - \frac{45}{9} = -\frac{46}{9} \rightsquigarrow \boxed{A = (0, -\frac{46}{9})}$

(7) $f(x) = \frac{x-1}{x+3}$ is 1-to-1.

Pf Suppose $f(x_1) = f(x_2)$. Then

$\frac{x_1 - 1}{x_1 \; 3} = \frac{x_2 - 1}{x_2 - 3} \implies$

$(x_2 - 3)(x_1 - 1) = (x_2 - 1)(x_1 - 3) \implies$

$x_2x_1 - x_2 - 3x_1 + 3 = x_2x_1 - 3x_2 - x_1 + 3 \implies$

$-x_2 - 3x_1 = -3x_2 - x_1 \implies$

$-2x_1 = -2x_2 \implies$

$x_1 = x_2$ ∎

(8) $\boxed{F = \frac{G m_1 m_2}{r_2}}$

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B1 $2x+h-4 \xrightarrow{h \to 0} \boxed{2x-4 = f'(x)}$

B2 $f(x) = \sqrt{3x} \longrightarrow$

$$\frac{f(x+h)-f(x)}{h} = \left(\frac{\sqrt{3(x+h)} - \sqrt{3x}}{h}\right)\left(\frac{\sqrt{3(x+h)} + \sqrt{3x}}{\sqrt{3(x+h)} + \sqrt{3x}}\right)$$

$$= \frac{3(x+h)-3x}{h\left(\sqrt{3(x+h)}+\sqrt{3x}\right)} = \frac{3x+3h-3x}{h\left(\sqrt{3\cdot(x+h)}+\sqrt{3x}\right)}$$

$$= \frac{3h}{h\left(\sqrt{3(x+h)}+\sqrt{3x}\right)} = \frac{3}{\sqrt{3(x+h)}+\sqrt{3x}} \xrightarrow{h \to 0}$$

$$\xrightarrow{h \to 0} \frac{3}{\sqrt{3x}+\sqrt{3x}} = \boxed{\frac{3}{2\sqrt{3x}}}$$

121 $\sqrt{2}$

B3

B4

$$5x^2 - 2x - 13$$

$$= 5\left(x^2 - \tfrac{2}{5}x + \left(\tfrac{1}{5}\right)^2\right) - 13 - 5\left(\tfrac{1}{25}\right)$$

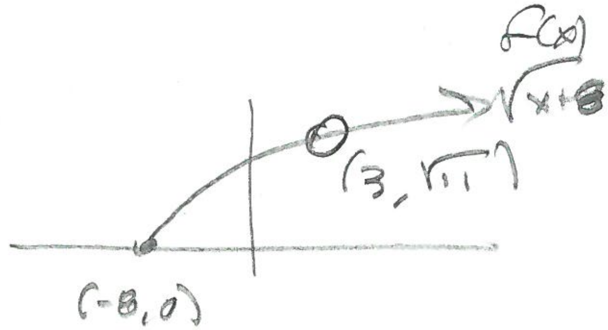
$$\boxed{= 5\left(x - \tfrac{1}{5}\right)^2 - \tfrac{66}{5}}$$

$(h,k) = \left(\frac{1}{5}, -\frac{66}{5}\right)$

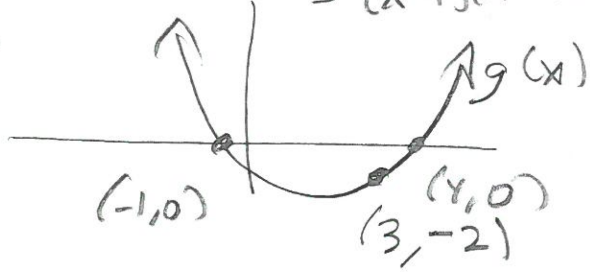
$$-13 - \frac{1}{5}$$

$$= \frac{-65-1}{5} = -\frac{66}{5}$$

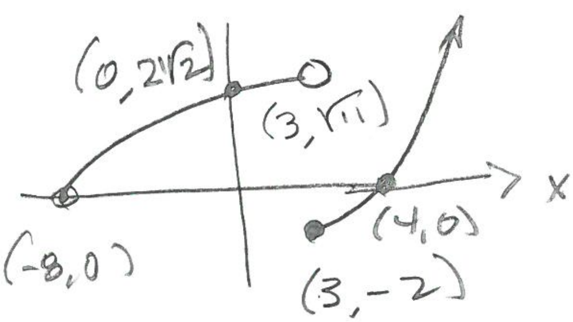
B5



$$x^2 - 5x + 4 = (x-1)(x-4)$$



$$h(x) = \begin{cases} f(x) & x < 3 \\ g(x) & x \geq 3 \end{cases}$$



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B6 $|3-2x| > 25 \longrightarrow$

$3-2x > 25$ OR $3-2x < -25$

$-2x > 22$ $\quad$ $-2x < -28$

$x < \frac{22}{-2} = -11$ $\quad$ $x > \frac{-28}{-2} = 14$

$\{x \mid x < -11 \text{ OR } x > 14\} =$

$= (-\infty, -11) \cup (14, \infty)$