

Do your work and circle final answers on separate paper, provided. Remember, to write big and bold, but mostly bold, or your teacher can't read it and give you any points for it. Leave a margin for the staple!

1. ~~10~~⁴ people are going to be chosen out of ~~20~~⁴ people to sit on stage at an awards ceremony.
- (10 pts) How many ways can the ~~10~~⁴ people be chosen?
 - (10 pts) How many ways could you choose the ~~10~~⁴ people and arrange their individual seating on stage?

2. (10 pts) Determine a , r , and n for the finite geometric sequence $5, 2, \frac{4}{5}, \frac{8}{25}, \dots, \frac{128}{15625}$.

3. (10 pts) Use a , r , and n to determine the *exact* value of the sum $\sum_{k=1}^n a \cdot r^{k-1} = a \left(\frac{1-r^n}{1-r} \right)$. For full credit, a fraction in lowest terms is required.

4. (10 pts) Find the sum of the infinite geometric series $\sum_{k=1}^{\infty} 3 \cdot \left(\frac{2}{7} \right)^{k-1}$.

Finance Formulas: Future Value of Savings: $A = P \left(1 + \frac{r}{m} \right)^{mt} = P(1+i)^n$

Future Value of Annuity:
$$FV = R \left(\frac{\left(1 + \frac{r}{m} \right)^{mt} - 1}{\left(\frac{r}{m} \right)} \right) = R \left(\frac{(1+i)^n - 1}{i} \right)$$

5. (10 pts) **(Present Value of Savings)** What is the *present* value of an \$8,000 payment in 9 years if 4.5% annual percentage rate, compounded weekly, is being charged? (Recall, there are 52 weeks in a year.)
6. (10 pts) **(Present Value of Annuity)** If your monthly house payments are \$900, and the loan is for 30 years, at 4.5% annual percentage rate, compounded monthly, how much did you borrow in the first place?
7. Let $g(x) = -2 \cdot \log_4(5x - 15) + 7$.
- (20 pts) Start with the graph of $f(x) = \log_4(x)$, and proceed to the graph of $g(x)$ with 4 more graphs, demonstrating each of the 4 basic transformations. So, 5 graphs, in all, counting the graph of $f(x)$ as the first. Label the 3 points, $\left(\frac{1}{4}, -1 \right)$, $(1, 0)$, and $(4, 1)$ in the graph of $f(x)$, and track where they end up after each transformation.
 - (10 pts) Find the x - and y -intercepts of the graph of $g(x)$, above. Report them as ordered pairs, using the labels A and B . Label the x - and y -intercepts A and B , respectively, in your final graph, above.

Bonus Answer up to 30 points' worth of the following:

- B 1** (10 pts) Use the Binomial Theorem (or Pascal's Triangle) to expand $(2x - 3y)^4$. No points awarded for doing this by brute force.
- B 2** (10 pts) Let $f(x) = 7 \cdot 5^{3x-8} - 6$. Find $f^{-1}(x)$.
- B 3** Suppose the half-life of C-14 is 5800 years. (It isn't, quite, but just suppose...).
- (5 pts) Derive the exponential decay model, $A(t) = A_0 e^{kt}$. The trick is to use the half-life to find the relative decay rate, k . Symbolic (Non-calculator) answers accepted (preferred).
 - (5 pts) How old is a sample of charcoal from the prehistoric fire pit, if 65% of the C-14 has decayed? Round to the nearest year in your final answer.
- B 4** (10 pts) Re-write $f(x) = 3x^2 - 2x + 1$ in the form $a(x - h)^2 + k$.
- B 5** (10 pts) John can finish a job in 7 hours that it takes Bill 13 hours to finish. Suppose Bill shows up and starts working 2 hours before John shows up, and then they work together until the job is done. How many hours does each of the two end up working?
- B 6** (10 pts) Solve the absolute value inequality $|-5x + 4| \geq 7$. Give your answers in set-builder *and* interval notation.
- B 7** What is the domain of...
- (5 pts) $f(x) = \sqrt{(x+6)(x-1)^2(x-6)}$?
 - (5 pts) $g(x) = \log_5((x+6)(x-1)^2(x-6))$?

Fall, 2017

$$\textcircled{1} \textcircled{a} \textcircled{10 \text{ pts}} \quad C(20, 4) = \frac{20!}{16! 4!} = \frac{20 \cdot 19 \cdot 18 \cdot 17}{4 \cdot 3 \cdot 2}$$

$$= 5 \cdot 19 \cdot 3 \cdot 17 = \boxed{4845} \text{ ways}$$

$$\textcircled{b} \textcircled{10 \text{ pts}} \quad P(20, 4) = 4! C(20, 4) = 24 (4845)$$

$$= \boxed{116,280} \text{ ways}$$

Check:

$$\frac{20!}{(20-4)!} = \frac{20!}{16!} = 20 \cdot 19 \cdot 18 \cdot 17 \quad \checkmark$$

$$\textcircled{2} \textcircled{10 \text{ pts}}$$

$$5, 2, \frac{4}{5}, \frac{8}{25}, \dots, \frac{128}{15625}$$

$$\frac{2}{5}, \frac{\frac{4}{5}}{2}, \frac{\frac{8}{25}}{\frac{4}{5}} = \boxed{\frac{2}{5} = r} \quad \checkmark$$

$$\boxed{a=5}$$

$$ar^{n-1} = \frac{128}{15625}$$

$$5 \left(\frac{2}{5}\right)^{n-1} = \frac{128}{15625}$$

$$\left(\frac{2}{5}\right)^{n-1} = \frac{128}{78125}$$

$$\left(\frac{2}{5}\right)^{n-1} = \frac{2^7}{5^7} = \left(\frac{2}{5}\right)^7$$

$$\boxed{\begin{array}{l} n-1=7 \\ n=8 \end{array}}$$

| | | | | | |
|---|---|-----|---|---|-------|
| 1 | 2 | 128 | 1 | 5 | 78125 |
| 2 | 2 | 64 | 2 | 5 | 15625 |
| 3 | 2 | 32 | 3 | 5 | 3125 |
| 4 | 2 | 16 | 4 | 5 | 625 |
| 5 | 2 | 8 | 5 | 5 | 125 |
| 6 | 2 | 4 | 6 | 5 | 25 |
| | | | 7 | 5 | |

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(3) #2 $\rightarrow a \left(\frac{1-r^n}{1-r} \right) = 5 \left(\frac{1 - \left(\frac{2}{5}\right)^8}{1 - \frac{2}{5}} \right)$

10 pts

$= 5 \left(\frac{1 - \frac{256}{390625}}{\frac{3}{5}} \right) = \frac{25}{3} \left(1 - \frac{256}{390625} \right)$

$\left(\frac{3}{5}\right)$ Invert. Multiply

$= \frac{25}{3} \left(\frac{390625 - 256}{390625} \right) = \frac{25}{3} \left(\frac{390369}{390625} \right)$

$= \frac{25}{3} \left(\frac{7 \cdot 7 \cdot 29 \cdot 641}{390625} \right)$

$= \frac{7 \cdot 29 \cdot 641}{15625}$

$= \frac{130123}{15625}$

$\frac{1}{5}$
~~5~~
25
 $\frac{390625}{78125}$
78125
15625

≈ 8.327872

3 | 390369
7 130123
29 | 18589

641 Prime.

(4) 10 pts

$\sum_{k=1}^{\infty} 3 \cdot \left(\frac{2}{7}\right)^{k-1} = 3 \left(\frac{1}{1 - \frac{2}{7}} \right)$

$= 3 \left(\frac{1}{\frac{5}{7}} \right) = (3) \left(\frac{7}{5} \right) = \frac{21}{5}$

(5)

10 pts

$P = 8000 \left(1 + \frac{0.045}{52} \right)^{-52(9)}$

$\approx 8000 (1.000865385)^{-52(9)}$

$\approx 8000 (.667093633)$

$\approx \$5336.75$

6 (10pts)

Present Value:

Set $A = FV$ & solve for P :

$$A = P(1+i)^n \stackrel{\text{SET}}{=} R \left(\frac{(1+i)^n - 1}{i} \right)$$

$$\Rightarrow P = R \left(\frac{1 - (1+i)^{-n}}{i} \right)$$

$$= 900 \left(\frac{1 - \left(1 + \frac{.045}{12}\right)^{-(30)(12)}}{\frac{.045}{12}} \right)$$

$$\approx 900 \left(\frac{1 - (.259895654)}{\frac{.045}{12}} \right) \quad \text{Invert. Multiply}$$

$$= 900 (1 - .259895654) \left(\frac{12}{.045} \right)$$

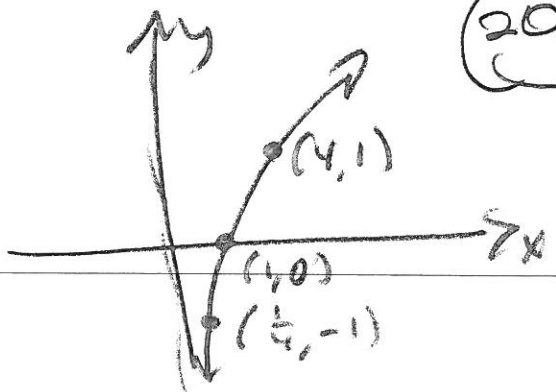
$$= 900 (.740104346) \left(\frac{12}{.045} \right)$$

$$\approx 900 (197.361159) \approx \boxed{\$177,625.04}$$

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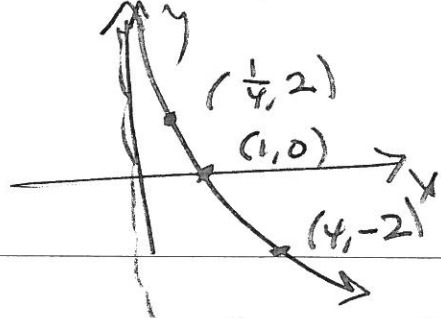
(7) $g(x) = -2 \log_4(5x-15) + 7$ ↙ m. copy!

(1) $f(x) = \log_4(x)$



20pts

(1) $-2 \log_4(x)$



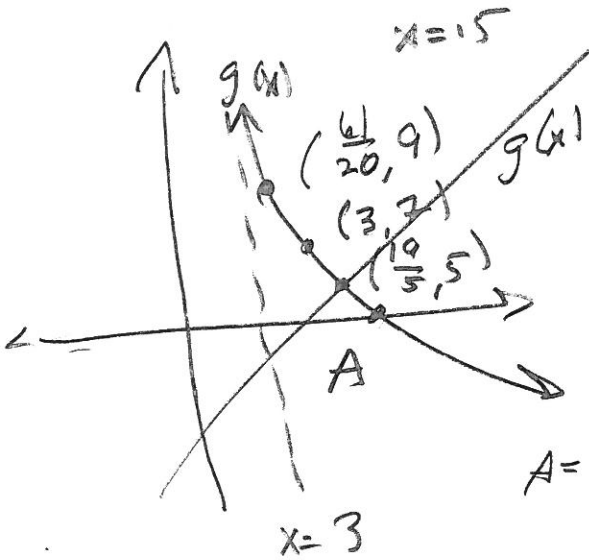
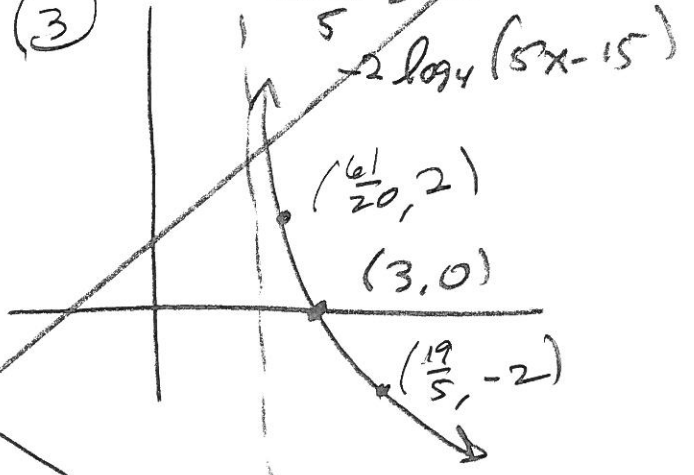
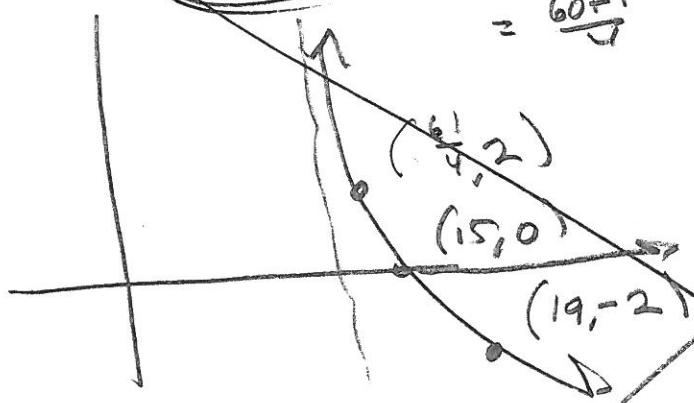
(2) $-2 \log_4(x-15)$

Should be $-2 \log_4(5x+15) + 7$

$15 + \frac{1}{4} = \frac{60+1}{4}$

(3)

$\frac{61}{5} = \frac{61}{5}$



(b) 10pts No y-intercept

x-int: $-2 \log_4$

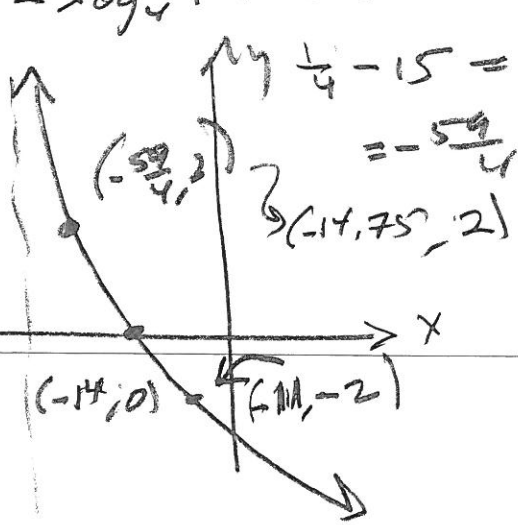
Start fresh page,

idiot

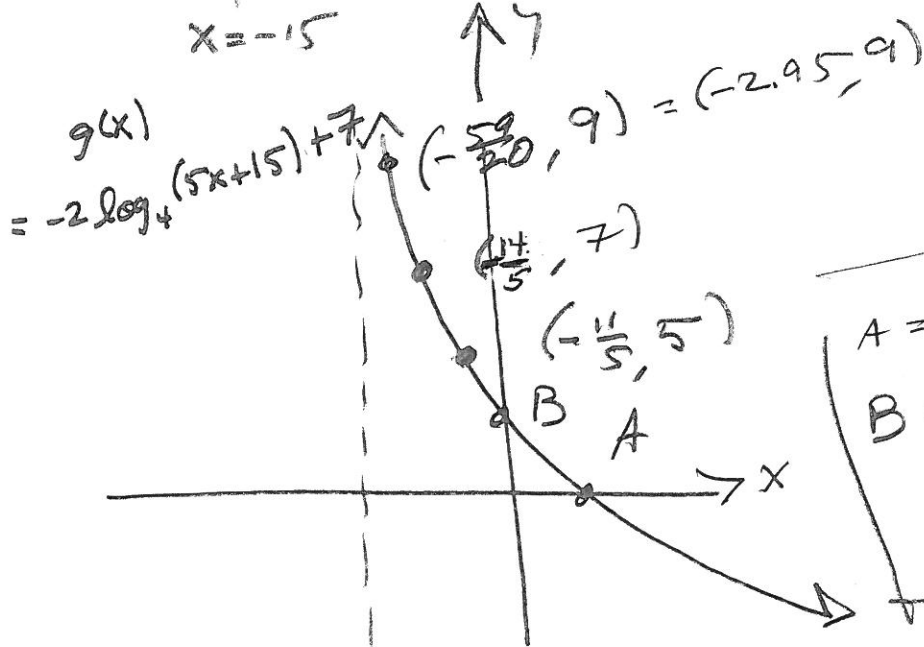
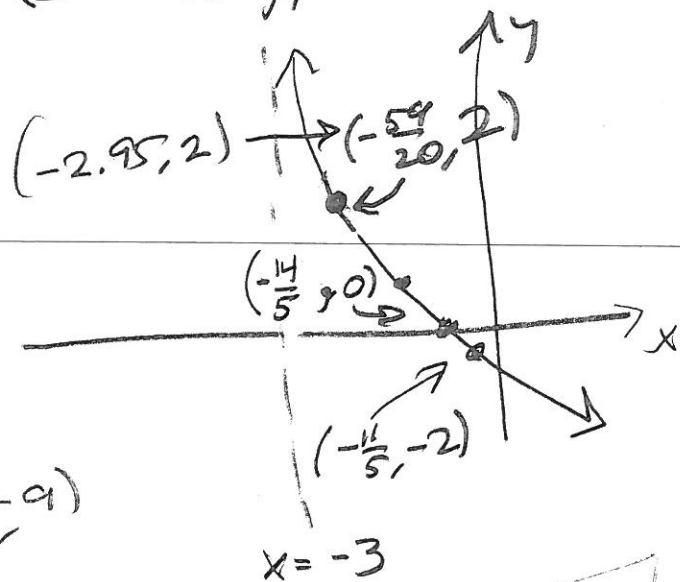
$A = (\frac{143}{5}, 0)$ by part b, next pg.

(7a) cont'd

(2) $-2 \log_4 (x+15)$



(3) $-2 \log_4 (5x+15)$



$A = (\frac{113}{5}, 0) = (22.6, 0)$
 $B = (0, -2 \log_4 (15) + 7)$
 $\approx (0, 3.0931)$

$x = -3$

A =

B =

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⑦ ⑥ ~~Along intercept~~

x-intercept =

$$-2 \log_4(5x+15) + 7 = 0$$

$$-2 \log_4(5x+15) = -7$$

$$\log_4(5x+15) = \frac{7}{2}$$

$$5x+15 = 4^{\frac{7}{2}} = 2^7 \quad \left(4^{\frac{7}{2}} = (2^2)^{\frac{7}{2}} = \left((2^2)^{\frac{1}{2}} \right)^7 \right)$$

$$5x = 2^7 - 15 = 113$$

$$x = \frac{113}{5}$$

$$\rightarrow A = \left(\frac{113}{5}, 0 \right) = (22.6, 0)$$

E1 y-int: $g(0) = -2 \log_4(15) + 7$

$$\rightarrow B = (0, -2 \log_4(15) + 7)$$

$$\approx (0, 3.093109404) \approx B$$

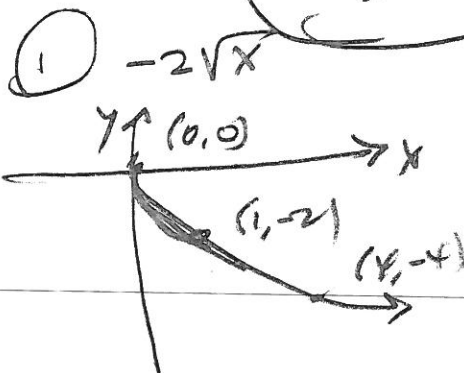
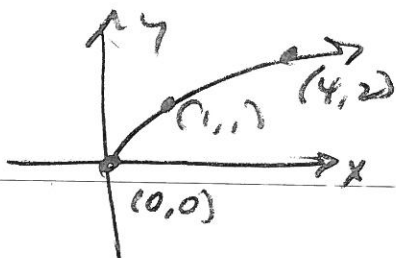
$$\approx (0, 3.0931) \approx B$$

7a) Half-credit version.

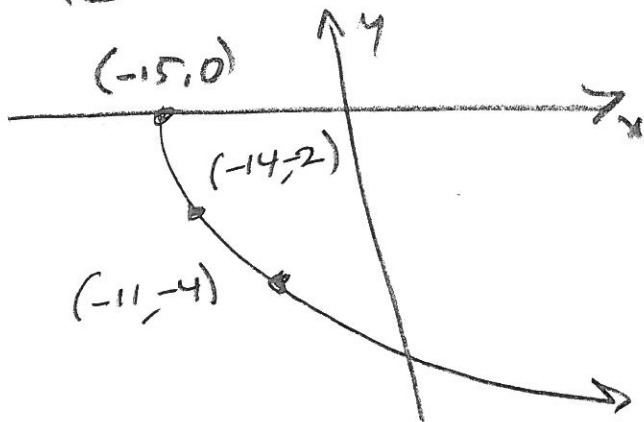
$$g(x) = -2\sqrt{5x+15} + 7$$

$\frac{20}{2}$ pts

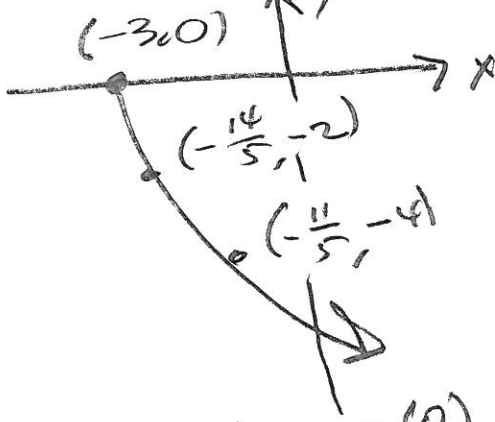
1) $f(x) = \sqrt{x}$



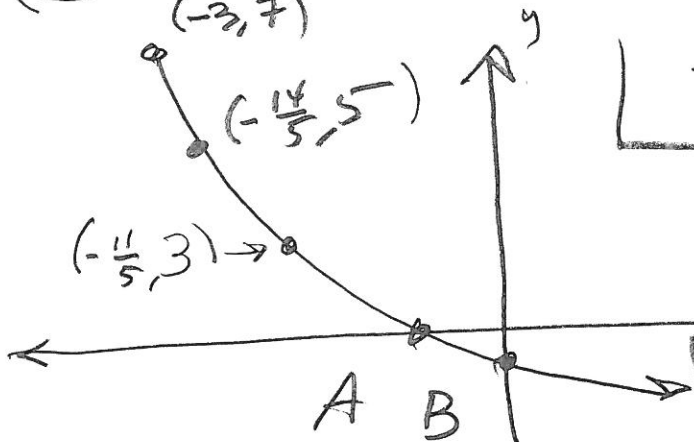
2) $-2\sqrt{x+15}$



3) $-2\sqrt{5x+15}$



4) $-2\sqrt{5x+15} + 7$



7b) y -intercept $g(0)$
 $= -2\sqrt{15} + 7 \approx -7.45967$

$B = (0, -2\sqrt{15} + 7)$
 $\approx (0, -7.459666924)$
 $\approx (0, -7.460)$

$A = (-\frac{11}{20}, 0) = (-.55, 0)$

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7b entid 10pb

$$g(x) = -2\sqrt{5x+15} + 7 \stackrel{\text{SET}}{=} 0$$

$$-2\sqrt{5x+15} = -7$$

$$\sqrt{5x+15} = \frac{7}{2}$$

$$5x+15 = \frac{49}{4}$$

$$5x = \frac{49-60}{4}$$

$$x = \frac{-11}{20} = -.55$$

$$A = (-.55, 0)$$

$$= \left(-\frac{11}{20}, 0\right)$$

(B1) 10pts

$$(2x-3y)^4$$

$$\begin{array}{cccccc} & & & & & 1 \\ & & & & & 1 & 1 \\ & & & & 1 & 2 & 1 \\ & & & 1 & 3 & 3 & 1 \\ & & 1 & 4 & 6 & 4 & 1 \\ & 1 & & & & & \end{array}$$

$$= 1(2x)^4(-3y)^0 + 4(2x)^3(-3y)^1 + 6(2x)^2(-3y)^2 + 4(2x)(-3y)^3 + (-3y)^4$$

$$= 16x^4 + (4)(8)(-3)x^3y + 6(4)(-3)^2x^2y^2 + 4(2)(-3)^3xy^3 + 81y^4$$

$$= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$

(B2) 10pts

$$x = 7 \cdot 5^{3y-8} - 6 = x$$

$$7 \cdot 5^{3y-8} = x+6$$

$$5^{3y-8} = \frac{x+6}{7}$$

$$3y-8 = \log_5\left(\frac{x+6}{7}\right)$$

$$3y = \log_5\left(\frac{x+6}{7}\right) + 8$$

$$y = \frac{1}{3} \left[\log_5\left(\frac{x+6}{7}\right) + 8 \right] = f^{-1}(x)$$

(B3)

$$A_0 e^{5800k} = \frac{1}{2} A_0$$

2

$$e^{5800k} = \frac{1}{2}$$

5 pts

$$5800k = \ln\left(\frac{1}{2}\right) = -\ln(2)$$

$$k = -\frac{\ln(2)}{5800}$$

$$\text{So } A(t) = A_0 e^{kt}$$

(b) 5 pts

65% gone \Rightarrow 35% remains.

$$A_0 e^{kt} = .35 A_0$$

$$e^{kt} = .35$$

$$kt = \ln(.35)$$

$$t = \frac{\ln(.35)}{k} = \frac{5800 \ln(.35)}{-\ln(2)}$$

$$\approx 8784.524402$$

$$\approx 8785 \text{ yrs old}$$

B4 10pts

$$f(x) = 3x^2 - 2x + 1$$

$$= 3\left(x^2 - \frac{2}{3}x\right) + 1$$

$$= 3\left(x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2\right) + 1 - 3\left(\frac{1}{3}\right)^2$$

$$= 3\left(x - \frac{1}{3}\right)^2 + 1 - \frac{1}{3}$$

$$= \left[3\left(x - \frac{1}{3}\right)^2 + \frac{2}{3}\right] = f(x)$$

B5 10pts

Let x = the # of hours John worked.
Then $x+2$ = " " " " Bill worked

of so...

$$\frac{1}{7}x + \frac{1}{13}(x+2) = 1 \quad \text{LCD} = (7)(13) = 91$$

$$\frac{13x + 7(x+2)}{\text{LCD}} = \frac{91}{\text{LCD}} = \frac{\text{LCD}}{\text{LCD}}$$

$$13x + 7x + 14 = 91$$

$$20x = 77$$

$$x = \frac{77}{20} = 3.85$$

$$\Rightarrow x+2 = \frac{77}{20} + 2 = \frac{77+40}{20} = \frac{117}{20} = 5.85 \text{ hrs} = x+2$$

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B6

$$|-5x+4| \geq 7$$

$$-5x+4 \geq 7 \quad \text{OR} \quad -5x+4 \leq -7$$

$$-5x \geq 3$$

$$-5x \leq -11$$

$$\left\{ x \mid x \leq -\frac{3}{5} \quad \text{OR} \quad x \geq \frac{11}{5} \right\}$$

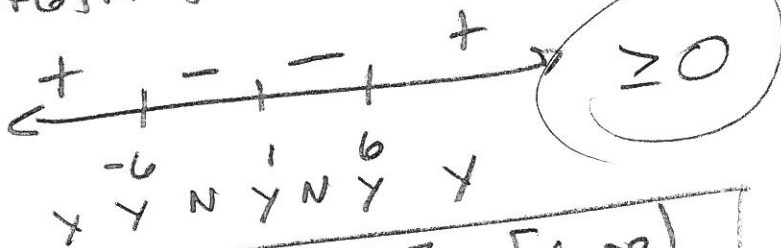


$$= (-\infty, -\frac{3}{5}] \cup [\frac{11}{5}, \infty)$$

B7

$$(x+6)(x-1)^2(x-6) \geq 0$$

2) SPB



$$\mathcal{D} = (-\infty, -6] \cup \{1\} \cup [6, \infty)$$

b)

$$(x+6)(x-1)^2(x-6) > 0$$

$$\mathcal{D} = (-\infty, -6) \cup (6, \infty)$$