

Leave a margin at the top. Write DARK. A couple borderline papers, last time, that won't get credit if they're as faint, this time.

1. (20 pts) Starting with $f(x) = 3^x$, sketch the graph of $g(x) = -2 \cdot 3^{2x-14} + 3$ in 5 steps (counting $f(x) = 3^x$ as the first step). Use $x = -1, x = 0$, and $x = 1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$.
2. (10 pts) Find the *exact* x - and y -intercepts for $g(x)$ from #1. That means no decimal approximations.
 - a. x -intercept: $A =$
 - b. y -intercept: $B =$

Label your final graph for #1 with the intercepts labeled with A and B .

3. (5 pts) Find the inverse, $g^{-1}(x)$, for $g(x)$ in #1. The moves are very similar to what you did in #2a.
4. Let $f(x) = \sqrt{x+11}$ and $g(x) = x^2 + 2x - 35$.
 - a. (5 pts) What is the domain of f ?
 - b. (5 pts) What is the domain of g ?
 - c. (5 pts) Determine $\left(\frac{f}{g}\right)(x)$. (Sometimes just called $\frac{f}{g}$ in the text.). Do not simplify.
 - d. (5 pts) What is the domain of $\left(\frac{f}{g}\right)(x)$? Leave your answer in simplified radical form.
 - e. (5 pts) Determine $(f \circ g)(x)$ (Again, sometimes just called $f \circ g$). Simplify.
 - f. (5 pts) What is the domain of $f \circ g$?

5. (5 pts) What is the domain of $\sqrt{\frac{(x-3)^2(x+5)}{(x-4)(x+8)^2}}$?

6. (5 pts) What is the domain of $\log_7\left(\frac{(x-3)^2(x+5)}{(x-4)(x+8)^2}\right)$?

7. (10 pts) Solve $\ln(x-4) + \ln(x+3) = \ln(8)$. Give an exact solution, then round to 3 decimal places.

8. Suppose the half-life of C-14 is 6000 years. (It isn't, quite, but just suppose...).
- (10 pts) Derive the exponential decay model, $A(t) = A_0 e^{kt}$. The trick is to use the half-life to find the relative decay rate, k .
 - (5 pts) How old is a sample of charcoal from a prehistoric fire pit, if 30% of the C-14 has decayed (i.e., 70% is left.)? Round to the nearest year in your final answer. If it makes it easier for you, use an initial mass of 100 g of and a final mass of 70 g. It's the same thing.

Bonus Answer up to three (3) 5-pointers. That's a total of 15 bonus points possible.

B 1 (5 pts) Solve the absolute value inequality: $|-5x + 8| - 11 > -2$

B 2 (5 pts) Re-write $f(x) = 2x^2 - 3x + 1$ in the form $a(x - h)^2 + k$.

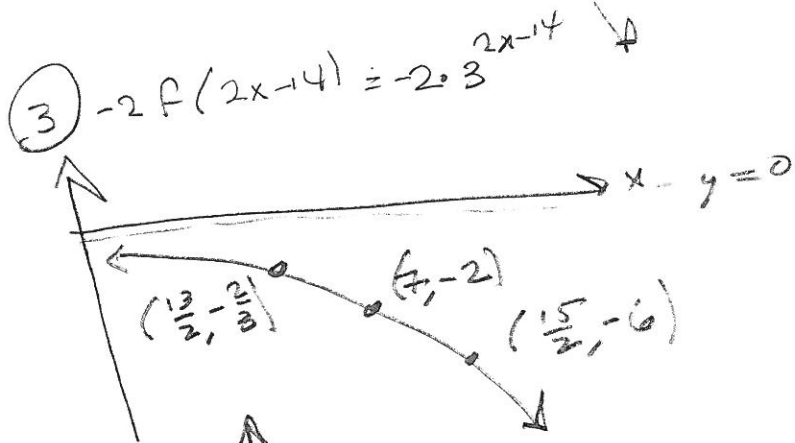
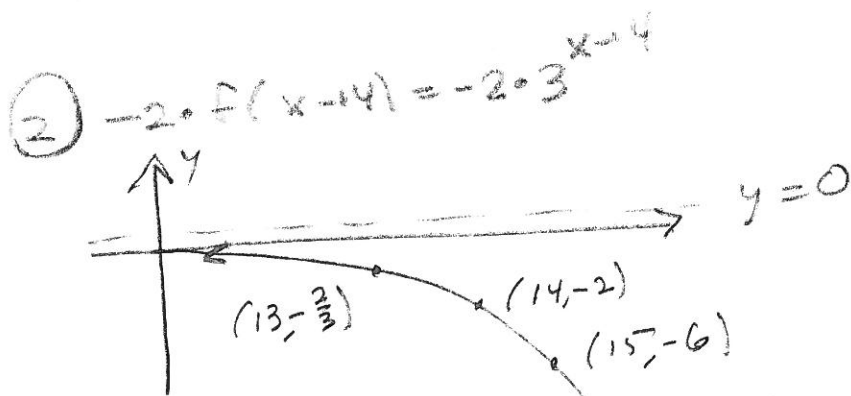
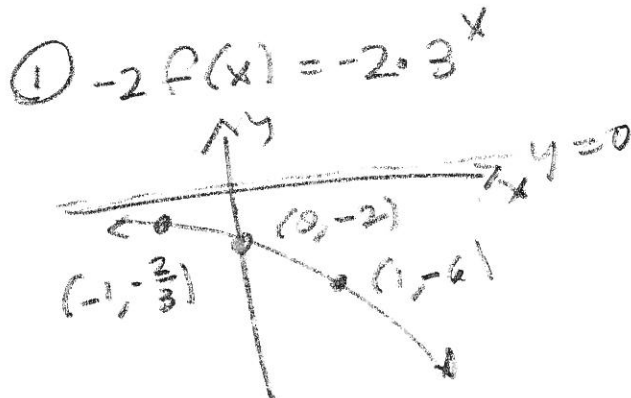
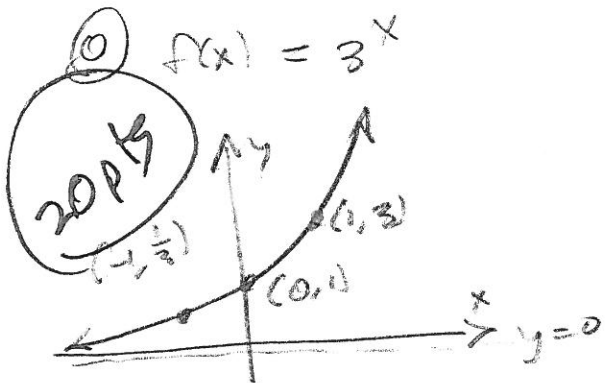
B 3 (5 pts) Solve the exponential equation $4 \cdot 7^x = 11 \cdot 2^x$.

B 4 What is the future value of \$5,000 in 9 years, if interest is 7%, compounded daily? (Use 360 days in a banker's year.).

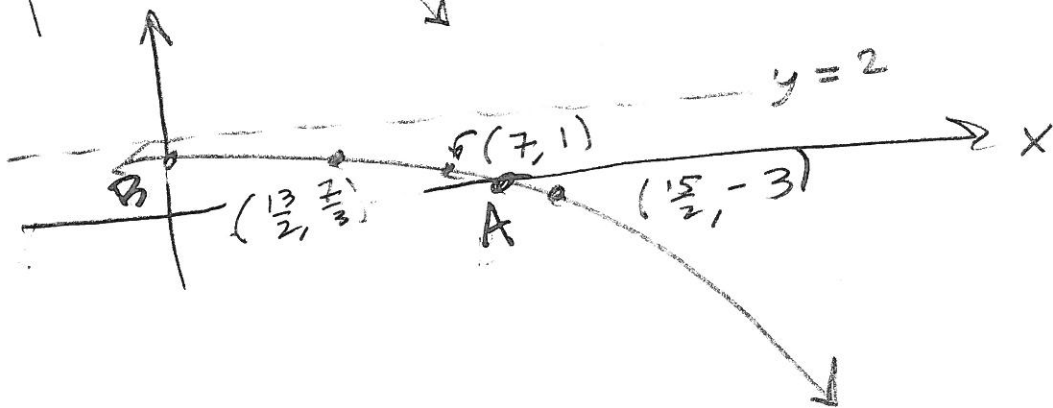
B 5 What is the present value of \$5,000 in 9 years, if interest is 7%, compounded daily?

B 6 Sketch the graph of $g(x) = -2 \cdot \log_3(2x - 14) + 3$. (Same exact moves as #1. Different basic function.).

① $g(x) = -2 \cdot 3^{2x-14} + 3$



$$-\frac{2}{3} + 3 = \frac{-2+9}{3} = \frac{7}{3}$$



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E4

(2) (a) $-2 \cdot 3^{2x-14} + 3 = 0$

$$-2 \cdot 3^{2x-14} = -3$$

$$3^{2x-14} = \frac{3}{2}$$

$$\log_3(m) = \log_3\left(\frac{3}{2}\right)$$

$$2x-14 = \log_3\left(\frac{3}{2}\right) = C$$

$$2x = C + 14$$

$$x = \frac{C+14}{2}$$

$$A = \left(\frac{\log_3\left(\frac{3}{2}\right) + 14}{2}, 0 \right)$$

$$\text{OR } \left(\frac{\ln\left(\frac{3}{2}\right) + 14}{\ln(3)}, 0 \right)$$

$$\text{OR } \left(\frac{\ln 3 - \ln 2 + 7}{2 \ln 3}, 0 \right)$$

$$\text{OR } (7.184535123, 0)$$

SPK

SPK (b)

$$g(0) = -2 \cdot 3^{-14} + 3$$

$$\Rightarrow B = (0, -2 \cdot 3^{-14} + 3)$$

$$\approx (0, 2.999999582)$$

3
SPK

$$-2 \cdot 3^{2y-14} + 3 = x$$

$$3^{2y-14} = \frac{x-3}{-2}$$

$$2y-14 = \log_3\left(\frac{x-3}{-2}\right) = D$$

$$2y = D + 14$$

$$y = \frac{D+14}{2} =$$

$$\frac{\log_3\left(\frac{x-3}{-2}\right) + 14}{2} = g^{-1}(x)$$

(4) $f(x) = \sqrt{x+11}$, $g(x) = x^2 + 2x - 35$

(a) $D(f) = \{x \mid x+11 \geq 0\} = [-11, \infty)$ SPS

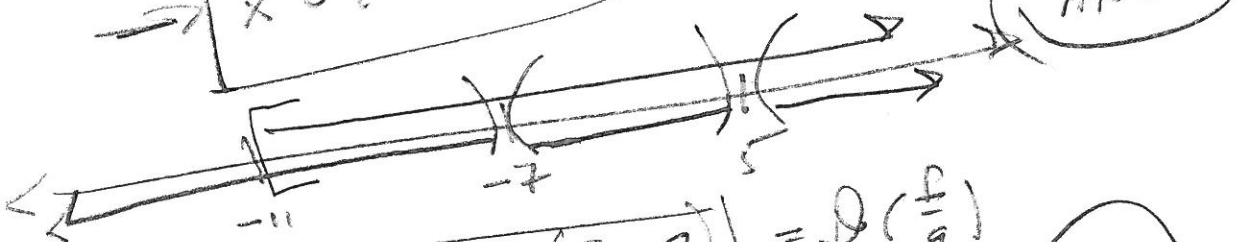
(b) $D(g) = \mathbb{R} = (-\infty, \infty)$ SPS

(c) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+11}}{x^2 + 2x - 35}$ SPS

(d) $D\left(\frac{f}{g}\right) = D(f) \cap D(g) \cap \{x \mid g(x) \neq 0\}$ SPS

$g(x) = x^2 + 2x - 35$
 $= (x+7)(x-5) \stackrel{\text{set } 0}{=}$

$\rightarrow x \in \{-7, 5\}$

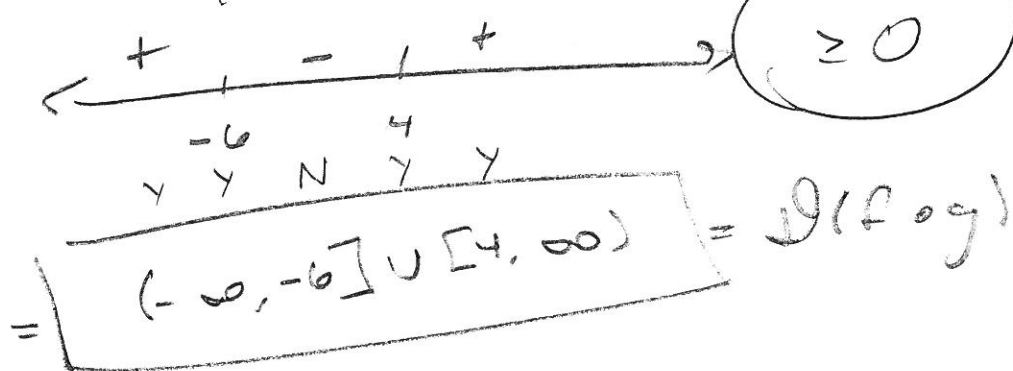


$[-11, -7) \cup (-7, 5) \cup (5, \infty) = D\left(\frac{f}{g}\right)$ SPS

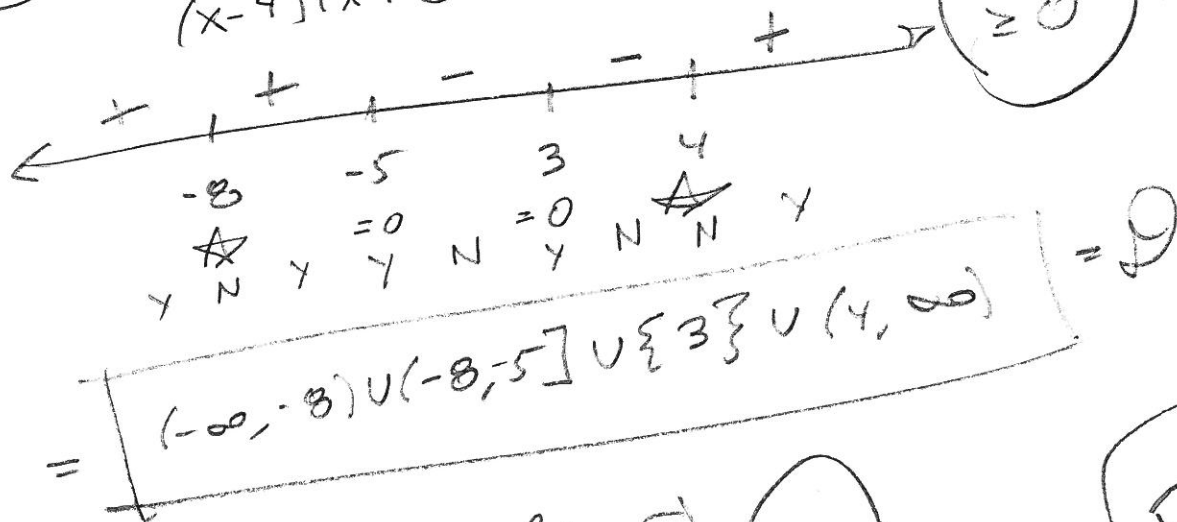
(e) $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)+11}$
 $= \sqrt{x^2 + 2x - 35 + 11} = \sqrt{x^2 + 2x - 24} = f \circ g$ SPS

(f)

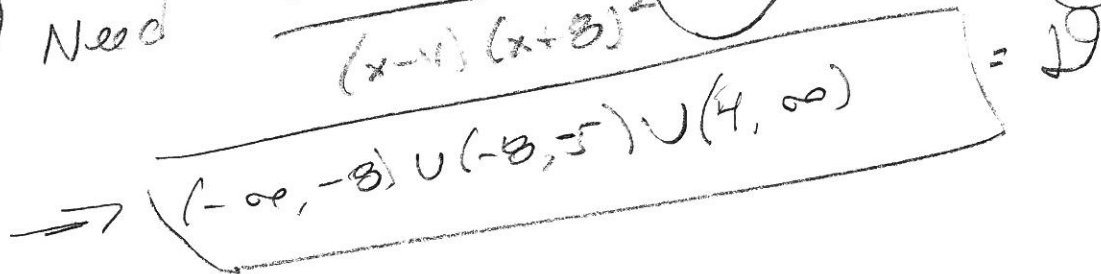
(4) Need $x^2 + 2x - 24 \geq 0$
 $(x+6)(x-4) \geq 0$



(5) $\frac{(x-3)^2(x+5)}{(x-4)(x+8)^2} \geq 0$



(6) Need $\frac{(x-3)^2(x+5)}{(x-4)(x+8)^2} > 0$



(7)

$$\ln(x-4) + \ln(x+3) = \ln(8)$$

$$\ln(x-4)(x+3) = \ln(8)$$

$$x^2 - x - 12 = 8$$

$$x^2 - x - 20 = 0$$

$$(x-5)(x+4) = 0$$

$$x \in \{-4, 5\} \rightarrow \boxed{x=5}$$

$$\ln(1) + \ln(8) = 0 + \ln(8) \checkmark$$

10 Pts

(8) $\frac{1}{2}$ -life is 6,000 yrs

(a)

$$A_0 e^{6000k} = \frac{1}{2} A_0$$

$$e^{6000k} = \frac{1}{2}$$

$$6000k = \ln\left(\frac{1}{2}\right) = -\ln(2)$$

$$k = -\frac{\ln(2)}{6000}$$

$$A(t) = A_0 e^{-\frac{\ln 2}{6000} t}$$

$$\approx A_0 e^{-0.00011552453 t}$$

10 Pts

(b)

$$A_0 e^{kt} = .7 A_0$$

$$e^{kt} = .7$$

$$kt = \ln(.7)$$

$$t = \frac{\ln(.7)}{k}$$

$$t = \frac{\ln(.7)}{-\frac{\ln(2)}{6000}}$$

$$= \frac{6000 \ln(.7)}{-\ln(2)}$$

$$\approx 3087.439037$$

$$\approx 3087 \text{ yrs old}$$

5 Pts

$$B1 \quad |-5x+8| - 11 > -2$$

$$|-5x+8| > 9$$

$$-5x+8 > 9 \quad \text{OR} \quad -5x+8 < -9$$

$$-5x > 1 \quad \text{OR} \quad -5x < -17$$

$$x < -\frac{1}{5} \quad \text{OR} \quad x > \frac{17}{5}$$



$$= \left(-\infty, -\frac{1}{5} \right) \cup \left(\frac{17}{5}, \infty \right)$$

$$\begin{aligned} B2 \quad 2x^2 - 3x + 1 &= 2 \left(x^2 - \frac{3}{2}x \right) + 1 \\ &= 2 \left(x^2 - \frac{3}{2}x + \left(\frac{3}{4} \right)^2 \right) + 1 - 2 \left(\frac{9}{16} \right) \\ &= 2 \left(x - \frac{3}{4} \right)^2 - \frac{1}{8} \end{aligned}$$

$$1 - \frac{9}{8} = \frac{8-9}{8} = -\frac{1}{8}$$

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E4

B3

$$4 \cdot 7^x = 11 \cdot 2^x$$

$$\ln(4 \cdot 7^x) = \ln(11 \cdot 2^x)$$

$$\ln(4) + x \ln(7) = \ln(11) + x \ln(2)$$

$$A + Bx = C + Dx$$

$$Bx - Dx = C - A$$

$$(B - D)x = C - A$$

$$x = \frac{C - A}{B - D}$$

$$\frac{\ln(11) - \ln(4)}{\ln(7) - \ln(2)} = x$$

B4

$$P = 5000, r = .07, m = 360, t = 9$$

$$A = P \left(1 + \frac{r}{m}\right)^{mt} = 5000 \left(1 + \frac{.07}{360}\right)^{(360)(9)}$$

$$\approx 9387.477969 \approx$$

 $\$ 9387.48$

B5

$$P = A \left(1 + \frac{r}{m}\right)^{-mt} = 5000 \left(1 + \frac{.07}{360}\right)^{-360(9)}$$

$$\approx 2663.122096 \approx$$

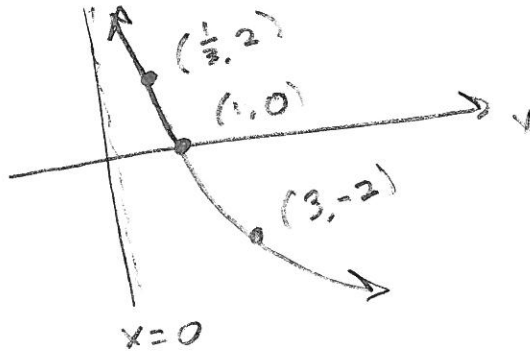
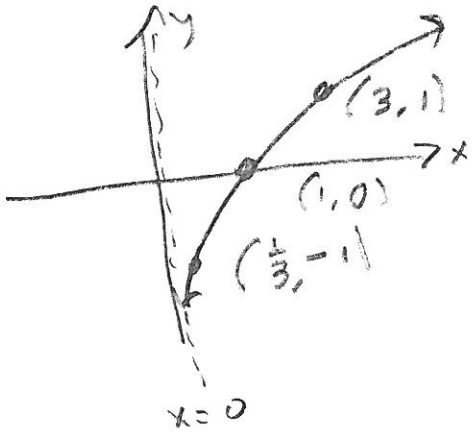
 $\$ 2663.12$

B6

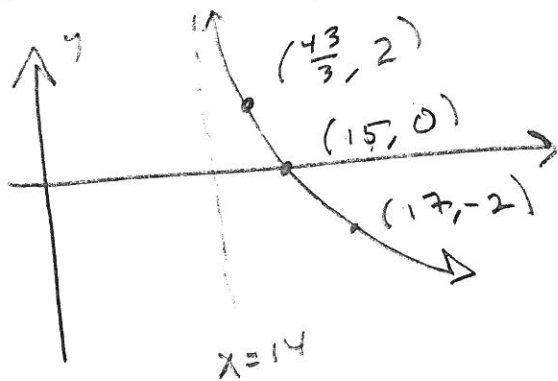
ouch!

(B6) $g(x) = -2 \log_3 (2x-14) + 3$

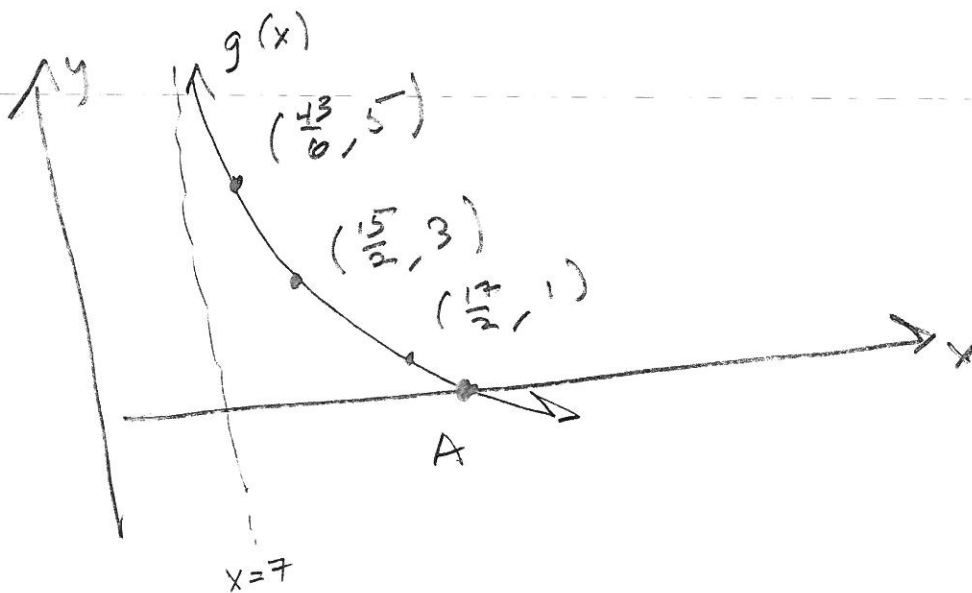
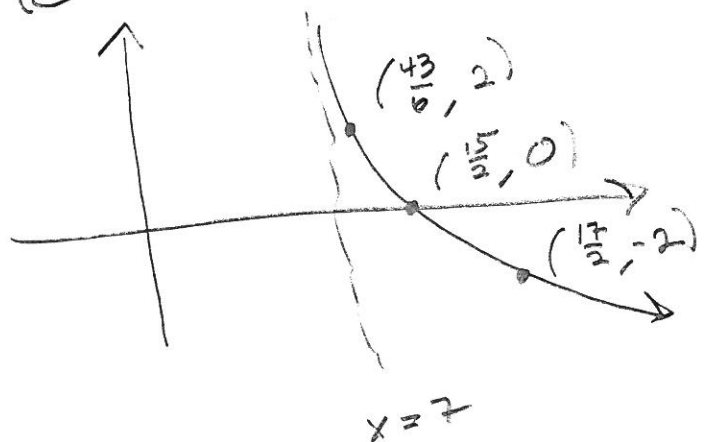
(0) $f(x) = \log_3 (x)$ (1) $-2f(x) = -2 \log_3 (x)$



(2) $-2 \log_3(x-14) = -2f(x-14)$ $\frac{1}{3} + 14 = \frac{1+42}{3}$



(3) $-2 \log_3(2x-14) = -2f(2x-14)$



121

E4

B6

 $x = ?$

$$-2 \log_3 (2x-14) + 3 = 0$$

$$-2 \log_3 (2x-14) = -3$$

$$\log_3 (2x-14) = \frac{3}{2}$$

$$2x-14 = 3^{\frac{3}{2}}$$

$$2x = 3^{\frac{3}{2}} + 14$$

$$x = \frac{3^{\frac{3}{2}} + 14}{2}$$

$$A = \left(\frac{3^{\frac{3}{2}} + 14}{2}, 0 \right)$$