MAT 121-G81-G82
30 Points

Take-Home Test 3 – Fall, 2017
Covers Chapter 3

Name KEY
NO GRAPHING CALCULATORS!!!

Remember to use separate paper for everything except your name. Leave a margin in the top left corner. Spread out your work. Use only one column for your work. Submit problems in order.

Online Students: 10% extra if you can get hardcopy to me (EDBH 134K) by Friday, October 20th. I will grade it and give you feedback before the test; otherwise, just bring it with you and hand it in when you take the test. The sit-down test is available Tuesday-Thursday, 10/24-10/26.

Face-to-Face Students: Deadline for 10% bonus is Friday, 10/20. Sit-down test is Wednesday, 10/25.

BEGIN TEST:

We will be working with $f(x)=2x^5+x^4-6x^3+16x^2+8x-48$ for most of this test. We'll say everything about this polynomial that's worth saying.

1. (2 pts) Describe the end behavior of f with a simple graphic.

2. (2 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeros.

3. (2 pts) Use the Rational Zeros Theorem to determine the *possible* rational zeros (roots) of f.

4. (2 pts) Using a graphing utility (only smart to do so on a take-home) and the information, above, to find all the *real* roots of f. This will involve using synthetic division to split off one factor at a time, and, at each step, working with the remaining, very depressed polynomial.

5. (2 pts) From your work, above, factor f over the real numbers. This will involve an irreducible quadratic factor that your grapher has no way of helping you to see, without the synthetic divisions in #4, bringing you closer and closer, step by step, to the irreducible quadratic.

6. (2 pts) Give a rough sketch of f from all of the above information. This is an *art* whose essence is really only found in my videos. If you're too tied to your grapher's output, you'll not capture the real essence of what's going on, or the key features I'm always looking for.

7. (2 pts) Now we've covered everything *real* about f. Let's use that work to find *all* the roots of f and *split* f *into linear factors*. 5 roots are *guaranteed by the Fundamental Theorem of Algebra,* and we have found the 3 real ones. The other 2 are nonreal, hiding inside the irreducible quadratic polynomial, that's the last, very very depressed piece, remaining from your answer to #5. Now do your quadratic equation thing to *find* the 2 nonreal roots, and, *finally*, apply the Factor Theorem to *all* the above work, and represent f as a product of linear factors, $f(x)=a(x-r_1)^{m_1}(x-r_2)^{m_2}\cdots(x-r_w)^{m_w}$. Don't forget the leading coefficient, a.

This wrings (almost) every useful drop of the Theorems on Polynomials out of f, so now on to Rational Functions, which are *quotients* of polynomials!

8. (5 pts) Sketch the graph of $R(x)=\dfrac{x^2+3x-10}{2x^2-13x+15}$, showing all intercepts, asymptotes, and capturing the *essential features* including the shape of the graph. If you're a slave to your grapher, and oblivious to the features I'm looking for, it'll jump off the page at me (and be bad).

9. (2 pts) Sketch the graph of $Q(x)=\dfrac{x^3+6x^2-x-30}{2x^3-7x^2-24x+45}$. All the work you did for #8 applies to this one, *except* for the *hole* in the graph of Q, which I expect you to find and clearly label in your graph.

10. (5 pts) Sketch the graph of $T(x)=\dfrac{x^3-7x^2-9x+63}{x^2-6x+5}$, showing all intercepts and asymptotes.

Now for a pair of questions many struggle with on the sit-down test, but which are actually *very simple* if you can synthesize your skills and *apply* them to these sorts of questions. Often the downfall of people on the sit-down, but designed to be easy points for people who are putting things together.

For HELP on these problems, you want to look at Test Prep Videos, in particular the Test-Prep Videos for the SIT-DOWN Test 3, because the old Take-Home 3/Writing Project #3 didn't have these type-questions.

11. (2 pts) What is the domain of $W(x)=\sqrt{(x-5)(x+1)(x-2)^2(x+7)}$?

12. (2 pts) What is the domain of $\mathrm{K}(x)=\sqrt{\dfrac{(x-2)^2(x+1)}{(x-5)(x+7)}}$?

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(1) $f(x) = 2x^5 + x^4 - 6x^3 + 16x^2 + 8x - 48$

$2x^5$

(2) 3 or 1 positive

$f(-x) = -2x^5 + x^4 + 6x^3 + 16x^2 - 8x - 48$

2 or 0 negative

(3) $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16,$

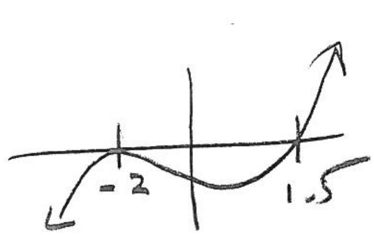
$\pm 24, \pm 48$

$\pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \pm \frac{3}{2}, \cancel{\pm \frac{4}{2}}, \cancel{\pm \frac{6}{2}}, \cancel{\pm \frac{8}{2}},$

$\pm \cancel{\frac{12}{2}}, \pm \cancel{\frac{16}{2}}, \cancel{\pm \frac{24}{2}}, \cancel{\pm \frac{48}{2}}$

12 of 'em.

(4)



$$
\begin{array}{r|rrrrrr}
-2 & 2 & 1 & -6 & 16 & 8 & -48 \\
 & & -4 & 6 & 0 & -32 & 48 \\
\hline
-2 & 2 & -3 & 0 & 16 & -24 & 0 \\
 & & -4 & 14 & -28 & 24 & \\
\hline
\frac{3}{2} & 2 & -7 & 14 & -12 & 0 & \\
 & & 3 & -6 & 12 & & \\
\hline
 & 2 & -4 & 8 & 0 & &
\end{array}
$$

$a = 2, b = -4, c = 8$

$b^2 - 4ac = 16 - 4(2)(8) < 0$

$\Rightarrow$ No real roots.

$$\boxed{\begin{array}{l} x = -2, m = 2 \\ x = \frac{3}{2}, m = 1 \end{array}}$$

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(5) $f(x) = 2(x+2)^2\left(x-\frac{3}{2}\right)(2x^2-4x+8)$

$b^2-4ac = 16-64 = -48$

$\sqrt{-48} = ?$

$\sqrt{\ } = 4i\sqrt{3}$

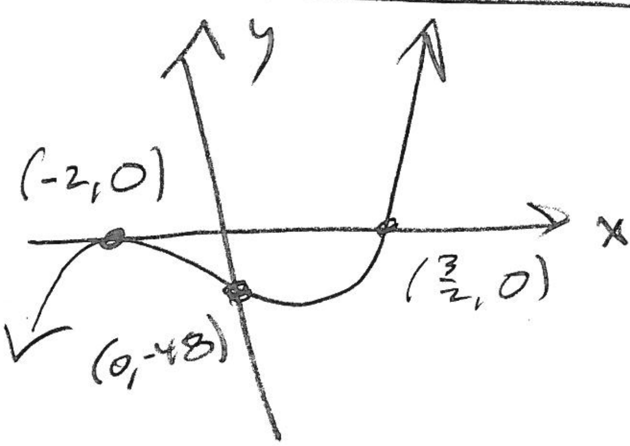
$2\,|\,48$
$2\,|\,24$
$2\,|\,12$
$2\,|\,6$
3

(7) $x = \dfrac{4 \pm 4i\sqrt{3}}{2(2)}$

$= \dfrac{4(1 \pm i\sqrt{3})}{4} = \boxed{1 \pm i\sqrt{3}}$

$\boxed{f(x) = 2(x+2)^2\left(x-\frac{3}{2}\right)\left(x-(1+i\sqrt{3})\right)\left(x-(1-i\sqrt{3})\right)}$

(6)



(8)

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(8) $R(x) = \dfrac{x^2+3x-10}{2x^2-13x+15} = \dfrac{(x+5)(x-2)}{(2x-3)(x-5)}$

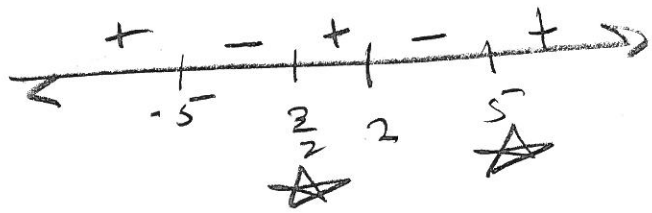
$\mathcal{D} = \mathbb{R} \setminus \{\frac{3}{2}, 5\}$

5 pts

$\boxed{\text{V.A.: } x = \frac{3}{2},\ x = 5}$

$R(x) = 0 \longrightarrow x \in \{-5, 2\}$

$\boxed{\text{x-int: } (-5,0),\ (2,0)}$

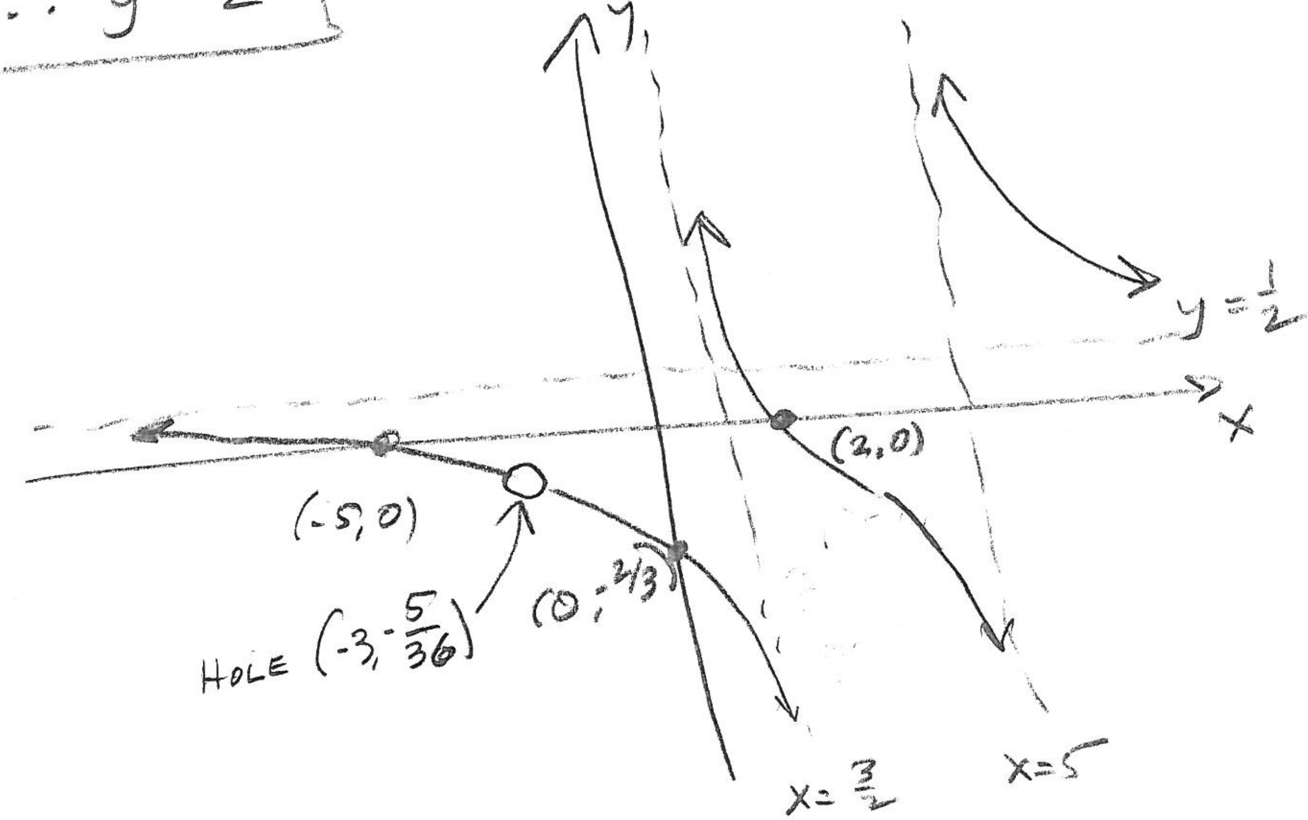


$R(0) = -\frac{10}{15} = -\frac{2}{3}$

$\boxed{\text{y-int: } (0, -\frac{2}{3})}$

$R(x) = \dfrac{x^2 + \cdots}{2x^2 + \cdots} \xrightarrow{|x| \to \infty} \frac{1}{2}$

$\boxed{\text{H.A.: } y = \frac{1}{2}}$



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(9) $Q(x) = \dfrac{x^3+6x^2-x-30}{2x^3-7x^2-24x+45} = R(x)\left(\dfrac{x-c}{x-c}\right)$

Find R(x) inside:

$$\begin{array}{r|rrrr} 2 & 1 & 6 & -1 & -30 \\ & & 2 & +16 & 30 \\ \hline -5 & 1 & 8 & 15 & 0 \\ & & -5 & -15 & \\ \hline & 1 & 3 & & \end{array}$$

$\to x-3 = x-c$, so $Q(x) = \dfrac{(x+5)(x-2)(x+3)}{(2x-3)(x-5)(x+3)}$

has hole @ $x=-3$:

$R(-3) = \dfrac{(-3+5)(-3-2)}{(2(-3)-3)(-3-5)} = \dfrac{2(-5)}{(-9)(-8)} = \dfrac{-5}{36}$

Hole: $\left(-3, -\frac{5}{36}\right)$

See previous page.

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(10) $T(x)=\dfrac{x^3-7x^2-9x+63}{x^2-6x+5}=\dfrac{(x-7)(x+3)(x-3)}{(x-1)(x-5)}$

5pts

$x^3-7x^2-9x+63 = x^2(x-7)-9(x-7)$

$= (x-7)(x^2-9) = (x-7)(x+3)(x-3)$

$x^2-6x+5 = (x-1)(x-5)$

$$
\begin{array}{r}
x-1 \quad \text{OA.} \\
x^2-6x+5 \,\big)\, x^3-7x^2-9x+63 \\
-(x^3-6x^2+5x) \\
\hline
-x^2-14x
\end{array}
$$

$\mathcal{D} = \mathbb{R} \setminus \{1,5\}$

$\boxed{\text{N.A.: } x=1,\ x=5}$

No H.A.

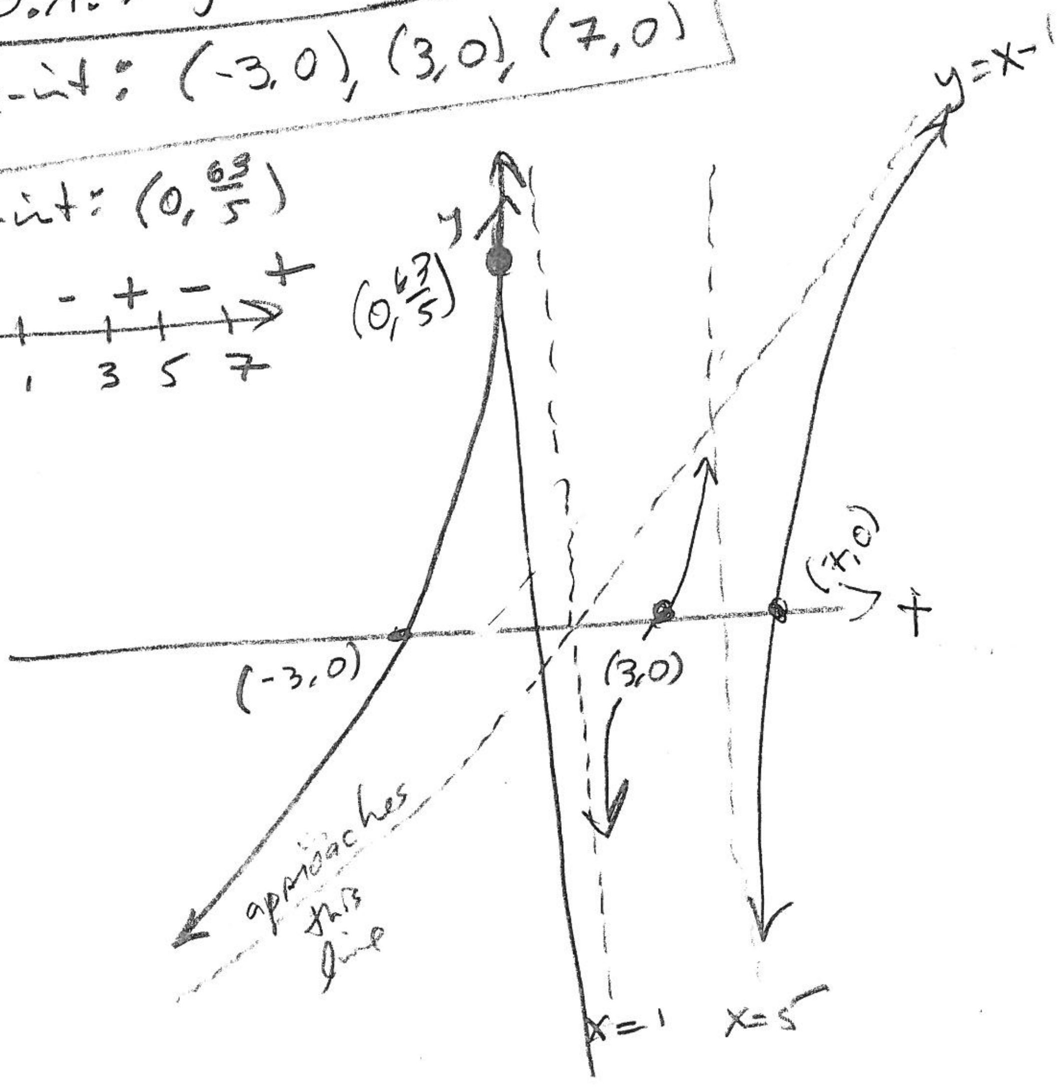
$\boxed{\text{O.A.: } y=x-1}$

$\boxed{\text{x-int: } (-3,0),\ (3,0),\ (7,0)}$

y-int: $(0, \frac{63}{5})$

$+ \quad - \quad + \quad - \quad +$

$-3 \quad 1 \quad 3 \quad 5 \quad 7$



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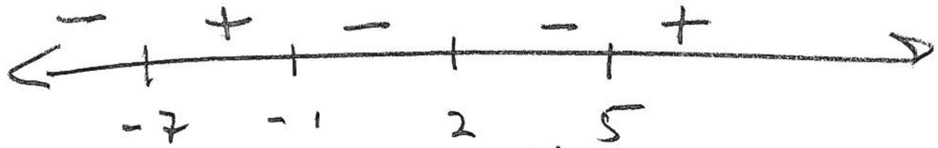
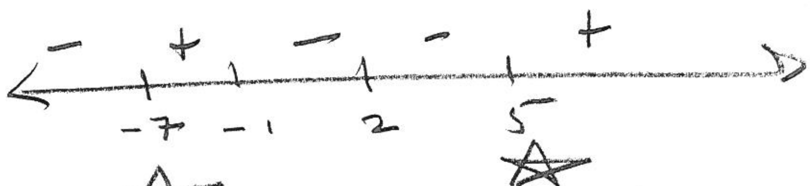
11 Need $(x-5)(x+1)(x-2)^2(x+7) \ge 0$



N Y Y Y N Y N Y Y

$= \boxed{[-7, 1] \cup \{2\} \cup [5, \infty)}$

12 Need $\dfrac{(x-2)^2(x+1)}{(x-5)(x+7)}$



N N Y Y N Y N Y Y

$= \boxed{(-7, -1] \cup \{2\} \cup (5, \infty)}$