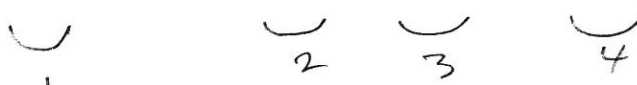


WP#3 Spring 2027

$$f(x) = 4x^5 - 8x^4 - 3x^3 + 39x^2 - 34x + 8$$

①  $4x^5$   2pts

②  $4x^5 - 8x^4 - 3x^3 + 39x^2 - 34x + 8$   
  
 4, 2, or 0 positive roots 2pts

$$f(-x) = -4x^5 - 8x^4 + 3x^3 + 39x^2 + 34x + 8$$

1 negative root

③  $\frac{p}{q} : \frac{8}{-4} : \pm 1, \pm 2, \pm 4, \pm 8,$  2pts

$\pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{4}{2}, \pm \frac{8}{2},$

$\pm \frac{1}{4}, \pm \frac{2}{4}, \pm \frac{4}{4}, \pm \frac{8}{4},$

$\pm \frac{1}{8}, \pm \frac{2}{8}, \pm \frac{4}{8}, \pm \frac{8}{8},$

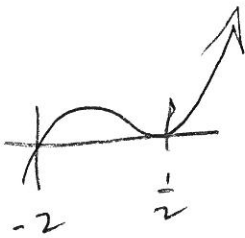
$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}$  No!

$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}, \pm \frac{1}{4}$

121

WP3

(4)



2pts

$$\begin{array}{r}
 -2 \mid 4 \quad -8 \quad -3 \quad 39 \quad -34 \quad 8 \\
 \quad \quad -8 \quad 32 \quad -58 \quad 38 \quad -8 \\
 \hline
 \frac{1}{2} \mid 4 \quad -16 \quad 29 \quad -19 \quad 4 \quad 0 \\
 \quad \quad 2 \quad -7 \quad 11 \quad -4 \\
 \hline
 \frac{1}{2} \mid 4 \quad -14 \quad 22 \quad -8 \quad 0 \\
 \quad \quad 2 \quad -6 \quad 8 \\
 \hline
 4 \quad -12 \quad 16 \quad 0 \\
 \hline
 \end{array}$$

$\hookrightarrow 4x^2 - 12x + 16$

Real Roots

$x = -2$

$x = \frac{1}{2} \quad (m=2)$

$$\begin{aligned}
 b^2 - 4ac &= (-12)^2 - 4(4)(16) \\
 &= 144 - 256 = -112
 \end{aligned}$$

No more real solutions

So  $f(x) = (x+2)(x-\frac{1}{2})^2(4x^2-12x+6)$

2pts

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12 \pm \sqrt{-112}}{2(4)} = \frac{12 \pm 4i\sqrt{7}}{8}$$

$$= \frac{3 \pm i\sqrt{7}}{2} = x$$

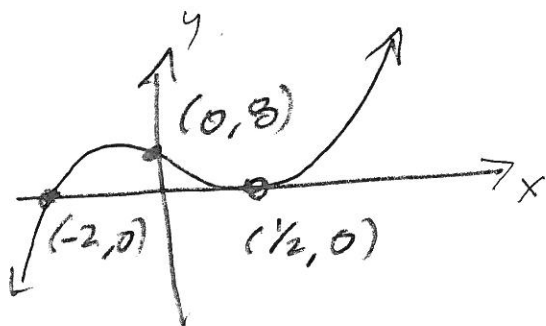
2pts

$$\begin{array}{l}
 2 \sqrt{112} \\
 2 \sqrt{56} \\
 2 \sqrt{28} \\
 2 \sqrt{14} \\
 7
 \end{array}$$

$\sqrt{112} = 4\sqrt{7}$

$$f(x) = 4(x+2)\left(x-\frac{1}{2}\right)^2\left(x-\left(\frac{3+i\sqrt{7}}{2}\right)\right)\left(x-\left(\frac{3-i\sqrt{7}}{2}\right)\right)$$

(6) Graph of  $f(x)$



2 pts

(8)  $R(x) = \frac{x^2 - 9}{x^2 - 6x + 5} = \frac{(x-3)(x+3)}{(x-5)(x-1)}$

5 pts

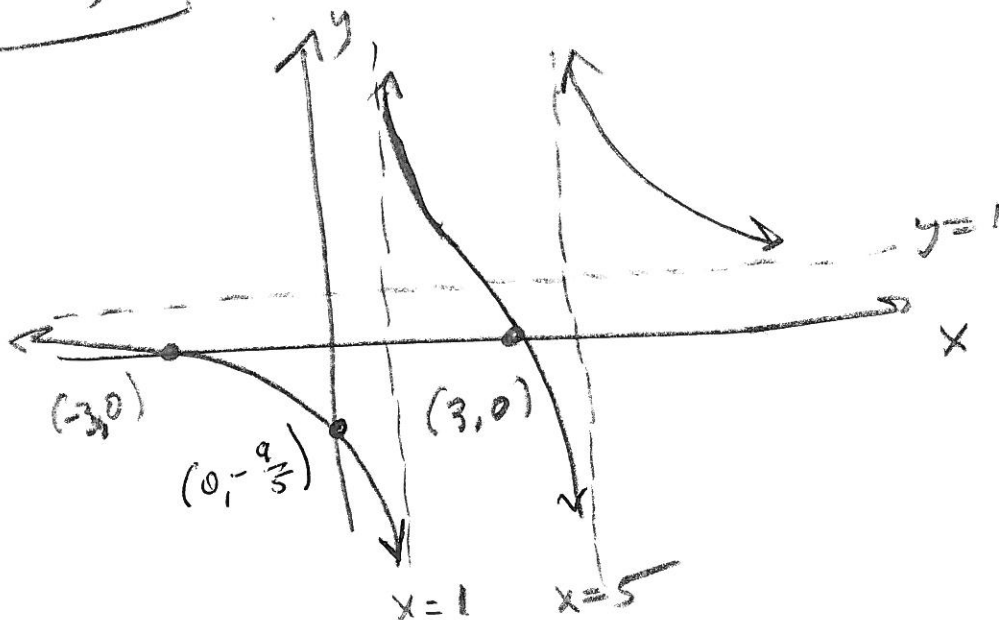
$D: \mathbb{R} \setminus \{1, 5\}$

V.A.:  $x=1, x=5$

x-int:  $(3, 0), (-3, 0)$  from  $(x-3)(x+3)=0$

y-int:  $R(0) = \frac{-9}{5} \rightarrow (0, -\frac{9}{5})$

H.A.:  $\frac{x^2}{x^2} = 1 = y$



9)  $Q(x) = \frac{x^3 - 7x^2 - 9x + 63}{x^3 - 13x^2 + 47x - 35} = \frac{(x-3)(x+3)(x-7)}{(x-1)(x-5)(x-7)}$

$$\begin{array}{r|rrrr} 3 & 1 & -7 & -9 & 63 \\ & & 3 & -12 & -63 \\ \hline -7 & 1 & -4 & -21 & 0 \\ & & -3 & 21 & \\ \hline & 1 & -7 & 0 & \end{array}$$

$= \frac{(x-3)(x+3)}{(x-1)(x-5)} \quad (x \neq 7)$

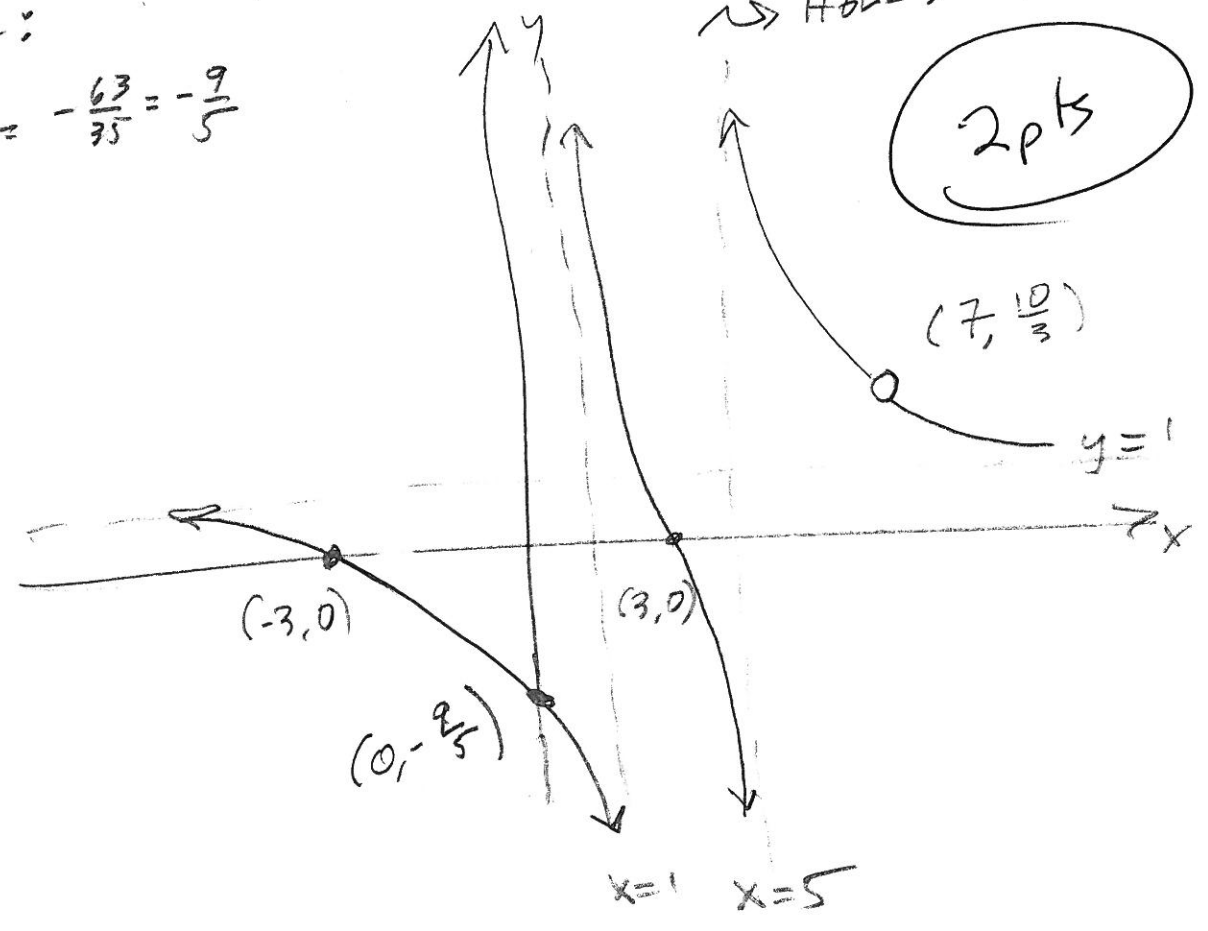
$= Q^*(x)$   
 $Q^*(7) = \frac{(4)(10)}{(6)(2)} = \frac{40}{12}$

$= \frac{10}{3}$

→ HOLE;  $(7, \frac{10}{3})$

2 pts

y-int:  
 $Q(0) = -\frac{63}{35} = -\frac{9}{5}$



$$(10) \quad T(x) = \frac{x^3 - 7x^2 - 9x + 63}{x^2 - 6x + 5}$$

$$= \frac{(x-3)(x+3)(x-7)}{(x-1)(x-5)}$$

$$D: \mathbb{R} \setminus \{1, 5\}$$

$$\text{V.A.: } x=1, x=5$$

$$\text{x-Int: } (-3, 0), (3, 0), (7, 0)$$

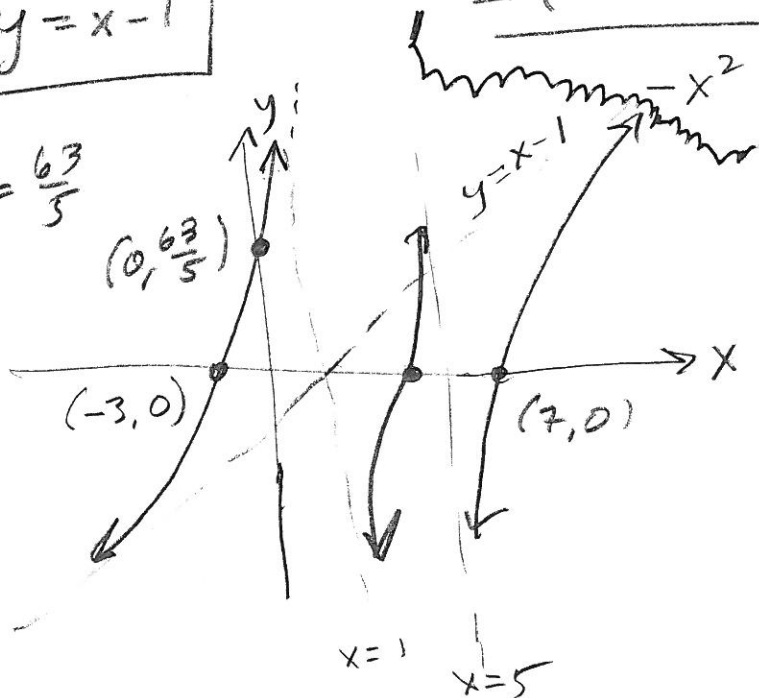
$$\text{H.A.: NONE}$$

$$\text{O.A.:}$$

$$x^2 - 6x + 5 \begin{array}{r} x-1 \\ x^3 - 7x^2 - 9x + 63 \\ - (x^3 - 6x^2 + 5x) \end{array}$$

$$y = x - 1$$

$$T(0) = \frac{63}{5}$$

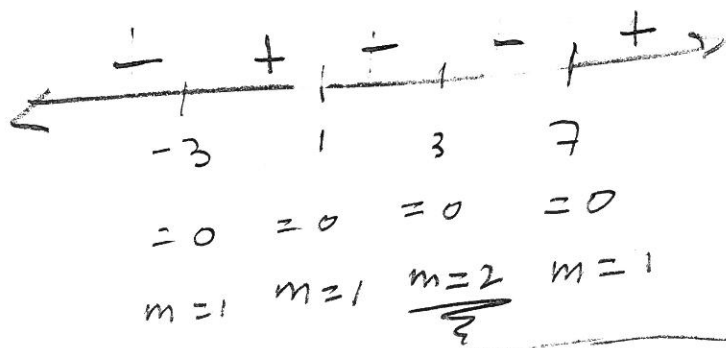


5 pl's

121 WP #2

(11)  $W(x) = \sqrt{(x+3)(x-1)(x-3)^2(x-7)}$

Need:  $(x+3)(x-1)(x-3)^2(x-7) \geq 0$



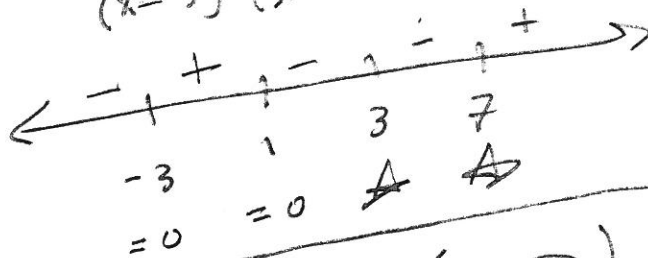
$D(W) = [-3, 1] \cup \{3\} \cup [7, \infty)$

2pts

(12)

$K(x) = \sqrt{\frac{(x+3)(x-1)}{(x-3)^2(x-7)}}$

Need:  $\frac{(x+3)(x-1)}{(x-3)^2(x-7)} \geq 0$



$D(K) = [-3, 1] \cup (7, \infty)$

2pts