MAT 121 Test 2 – Spring, 2017 Name KEY

100 Points Covers Chapter 2 NO GRAPHING CALCULATORS!!!

Do your work and circle (or square-box) final answers on the separate paper, provided.

Only write on one side of each sheet. No points for what's on the back (either for or against!).

All that you should put on this cover sheet is your name.

1. (5 kindness points) I can really see how you made an effort to be clear, use one column per page, one side of writing per page, and plenty of room for my silly comments. Thanks for writing big enough to read, and really dark, which helped me see your work so easily. You helped me serve your classmates, better, too, because your paper was quick and easy to grade. I thank you on their behalf.

2. Consider the relation $R=\{(1,3),(2,7),(3,4),(4,7)\}$.

   a. (5 pts) Is R a function? If not, why not?

   b. (5 pts) What is the domain of R ?

   c. (5 pts) What is the range of R ?

   d. (5 pts) If you answered "No" to part 'a.,' then the appropriate response, here, is "'1-to-1' only applies to functions, so this question is poorly posed." Assuming R is a function (You answered "Yes" to part 'a.,'), is R one-to-one? If not, explain why not.

3. Let $f(x)=\sqrt{x+8}$ and $g(x)=x^2-4x-5$.

   a. (5 pts) What is the domain of f ?

   b. (5 pts) What is the domain of g ?

   c. (5 pts) Write the function $\frac{f}{g}$. Do not simplify.

   d. (5 pts) What is the domain of $\frac{f}{g}$?

   e. (5 pts) Write the function $f \circ g$. Do not simplify.

   f. (5 pts) What is the domain of $f \circ g$? (Highest level of synthesis.)

4. (5 pts) Simplify the difference quotient for $f(x)=3x^2-5x-12$.

5. (5 pts) Write the difference quotient for $f(x)=\sqrt{x}$, and explain its connection to the slope of a secant line between x and $x+h$ on a graph of f.

6. Let $g(x)=7(-2x+10)^3-11$.

   a. (10 pts) Sketch the graph of $g(x)$, by transforming the basic function $f(x)=x^3$. I want to see 3 points labeled in the graph of f – preferably $(-1,-1),(0,0)$, and $(1,1)$ – and track where those points are moved to after every step, as demonstrated by the instructor. This will take 5 graphs, counting the first graph of $f(x)=x^3$ as the first. (I've been numbering them 0 thru 4, for some reason, this semester.)

   b. (5 pts) State the domain and range of $g(x)$, based on your final graph.

   c. (5 pts) Find the x- and y-intercepts of $g(x)$, and label them, clearly, on the graph.

7. (5 pts) Prove that $p(x)=5x-7$ is one-to-one.

8. (5 pts) The force of gravity, F, varies jointly with the masses, m_1 and m_2 of the two planets, and inversely with the square of the distance, r, between them. (To be precise, r is the distance between their respective centers of mass.). Using the proportionality constant, G, which stands for Newton's *gravitational constant*, write an equation relating force to the masses of the two planets and the distance between them.

9. (5 pts) Explain why $x=|2y-4|$ does *not* define y as a function of x.

Bonus Section (5 pts each) Are you smarter than the average bear?

B1. Write down your answer to #4, again, and pass to the limit as h approaches zero, and show me some calculus.

B2. Simplify the difference quotient for the function $f(x)=\sqrt{x}$ **(See #5.)**. Then pass to the limit, as h approaches zero, and demonstrate an early aptitude for Calculus.

B3. Add the line to your picture in #5, that represents the tangent to f at the point $(x, f(x))$.

B4. Complete the square to re-write the function $h(x)=-4x^2-5x+3$ in the form $a(x-h)^2+k$. What is the vertex?

B5. Sketch the graph of the piecewise-defined function $h(x)=\begin{cases}\sqrt{x+8} & \text{if } x<1\\ x^2-4x-5 & \text{if } x\geq 1\end{cases}$.

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① —‖—

② $R = \{ (1,3), (2,7), (3,4), (4,7) \}$

ⓐ R is a func. $\boxed{\text{Yes}}$

ⓑ $\mathcal{D}(R) = \{ 1, 2, 3, 4 \}$

ⓒ $\mathcal{R}(R) = \{ 3, 7, 4 \}$

ⓓ R is not a function, b/c (2,7) & (4,7) both contain the same y-coordinate, y=7, so it flunks the horizontal line test for 1-to-1'ness. $\boxed{\text{No.}}$

③ $f(x) = \sqrt{x+8}$ & $g(x) = x^2 - 4x - 5$

ⓐ $\mathcal{D}(f) = \{ x \mid x \geq -8 \} = [-8, \infty)$

(Need $x + 8 \geq 0 \implies x \geq -8$)

ⓑ $\mathcal{D}(g) = \boxed{(-\infty, \infty)}$, b/c g is a polynomial

$= \mathbb{R}$

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③ (c) $\boxed{\dfrac{f}{g} = \dfrac{\sqrt{x+8}}{x^2-4x-5}}$

(d) $\mathcal{D}(f/g) = \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \text{ and } g(x) \neq 0\}$

NO RESTRICTION

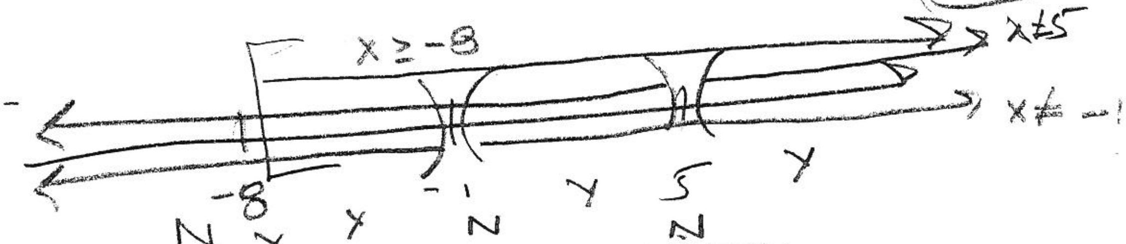
$= \{x \mid x \geq -8 \text{ and } x \in \mathbb{R} \text{ and } x^2-4x-5 \neq 0\}$

Scratch: Need $x^2-4x-5 \neq 0$

$\Rightarrow (x-5)(x+1) \neq 0$

$\Rightarrow x \neq 5 \text{ and } x \neq -1$

AND

$=$ 

$= \boxed{[-8,-1) \cup (-1,5) \cup (5,\infty)}$

STOP!

(e) $\boxed{f \circ g = \sqrt{x^2-4x-5+8}}$

$= \sqrt{x^2-4x+3}$

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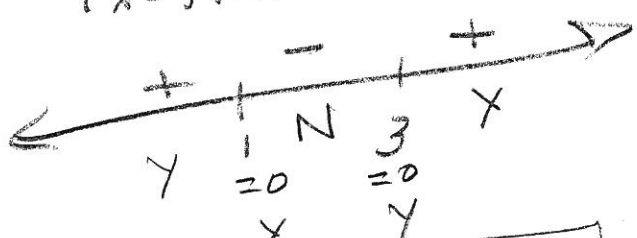
3 f $\mathcal{D}(f \circ g) = \{ x \mid \underbrace{x \in \mathcal{D}(g)}_{\substack{(-\infty, \infty) \\ \text{NO RESTRICT.}}} \text{ and } \underbrace{g(x) \in \mathcal{D}(f)}_{\text{Need } g(x) \geq -8} \}$

Scratch: $g(x) \geq -8$

$x^2 - 4x - 5 \geq -8$

$x^2 - 4x + 3 \geq 0$ $\longrightarrow$

$(x-3)(x-1) \geq 0$



$= \boxed{(-\infty, 1] \cup [3, \infty)}$

4 $f(x) = 3x^2 - 5x - 12 \Longrightarrow \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{3(x+h)^2 - 5(x+h) - 12 - (3x^2 - 5x - 12)}{h}$

$= \dfrac{3(x^2 + 2xh + h^2) - 5x - 5h - 12 - 3x^2 + 5x + 12}{h}$

$= \dfrac{3x^2 + 6xh + 3h^2 - 5h - 3x^2}{h} = \dfrac{6xh + 3h^2 - 5h}{h}$

$= \dfrac{h(6x + 3h - 5)}{h} = \boxed{6x + 3h - 5}$

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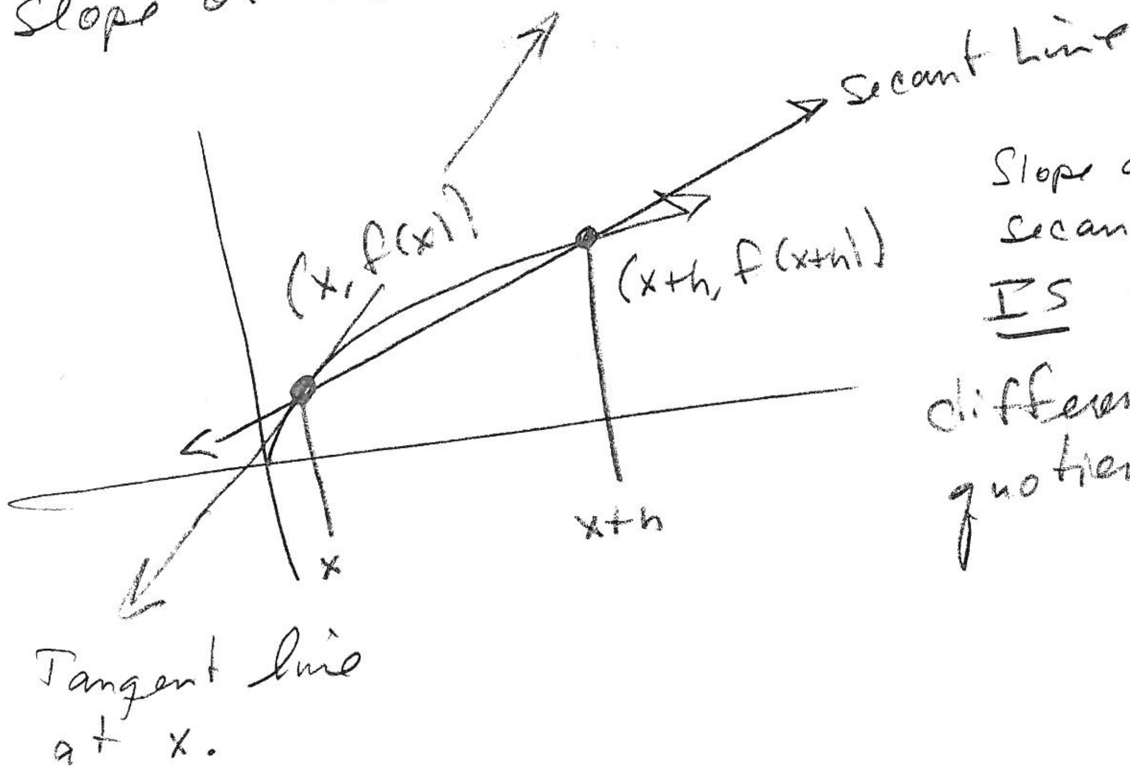
(5) $f(x) = \sqrt{x} \implies \dfrac{f(x+h) - f(x)}{h} = \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$

= Slope of the line thru $(x, \sqrt{x})$ & $(x+h, \sqrt{x+h})$



Slope of secant line <u>IS</u> the difference quotient!

(6) Start fresh

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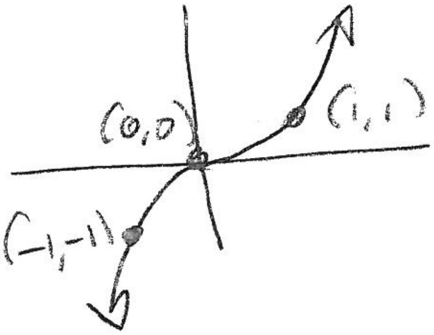
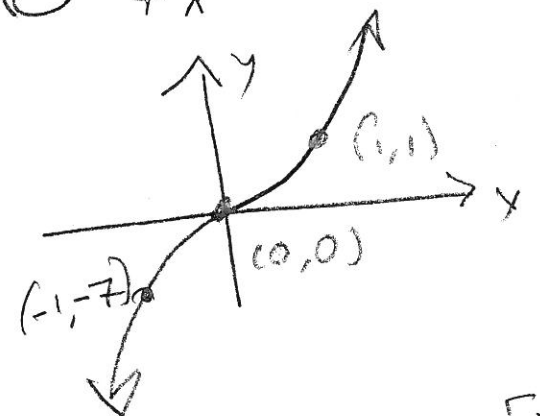
6 10 R^{B} $g(x) = 7(-2x+10)^3 - 11$

M1 FAILED!

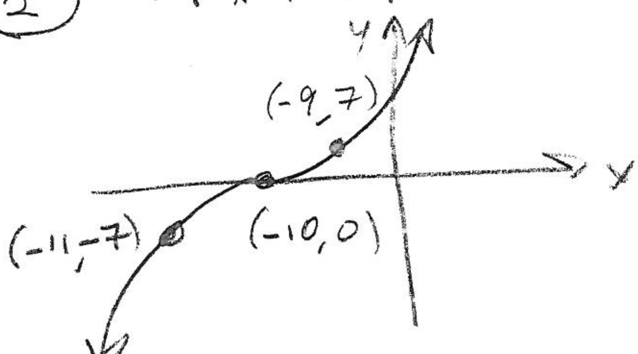
0 $f(x) = x^3$



1 $7x^3$



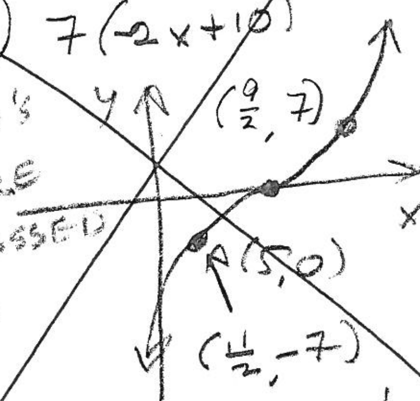
2 $7(x+10)^3$



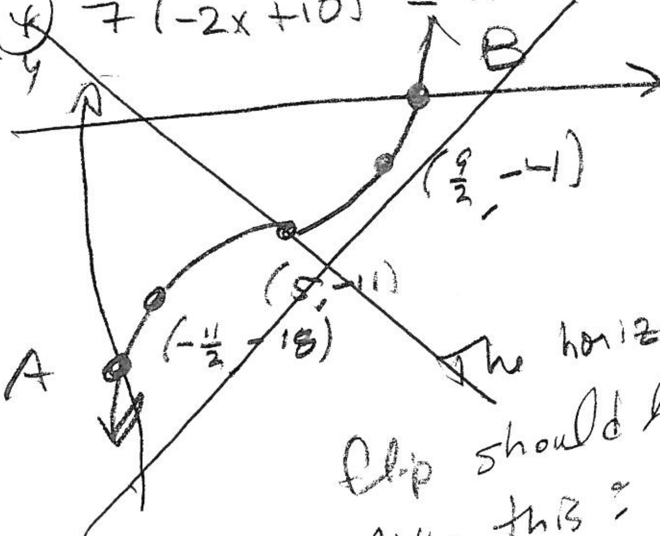
HORIZONTAL FLIP FAIL

3 $7(-2x+10)^3$

HERE'S WHERE I MESSED UP



4 $7(-2x+10)^3 - 11$



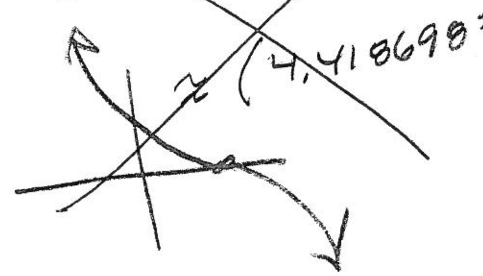
Doesn't match!

$A = (0, 6989)$

$B = \left(5 - \frac{\sqrt[3]{539}}{14}, 0\right)$

$\approx (4.418698354)$

The horizontal flip should look like this:



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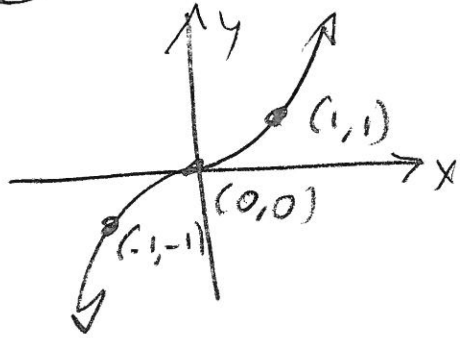
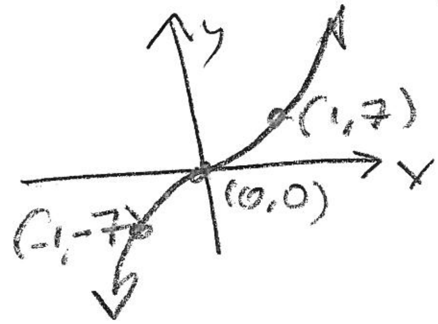
6 a $f(x)=x^3$, $g(x)=7(-2x+10)^3-11$

M1

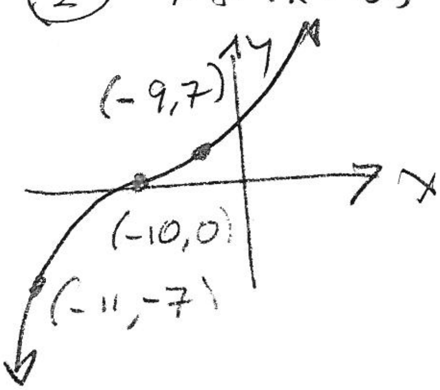
0 $f(x)=x^3$



1 $7f(x)$

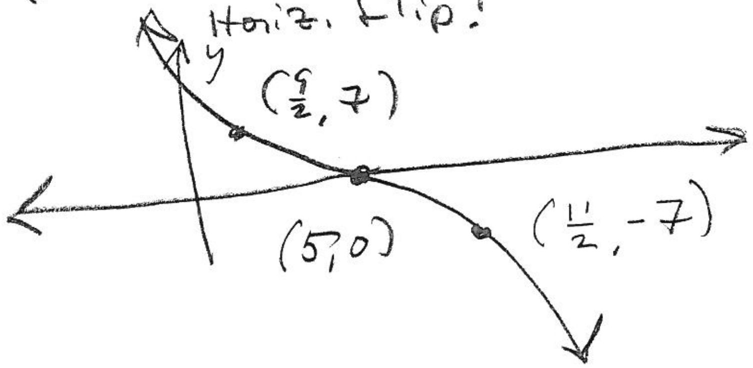


2 $7f(x+10)$

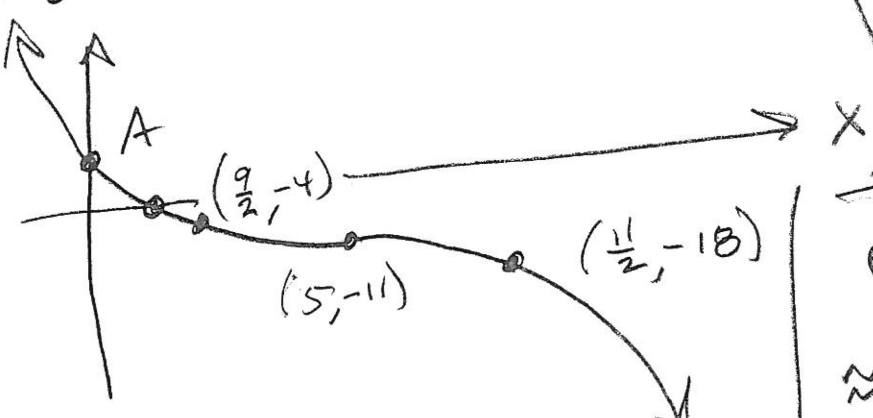


3 $7f(-2x+10)$

Horiz. Flip!



4 $g(x)=7f(-2x+10)-11$



$$\boxed{A=(0,\ 7(10)^3-11) = (0,\ 6989)}$$

$$\boxed{B=\left(5-\frac{\sqrt[3]{539}}{14},\ 0\right) \approx (4.418698354,\ 0)}$$

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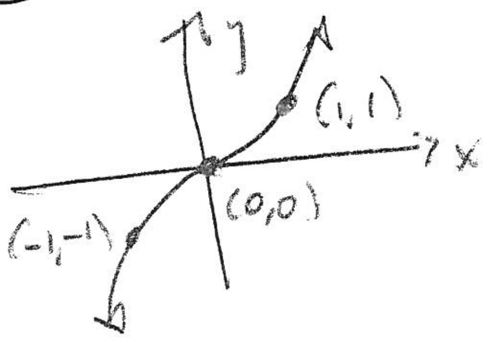
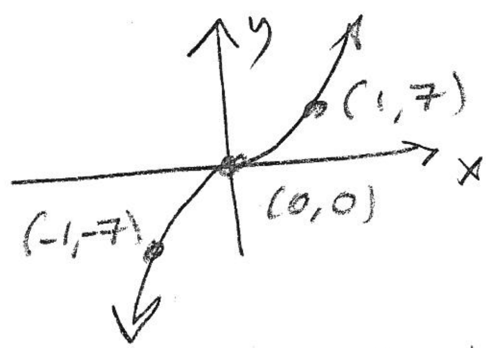
(6 a) $g(x) = 7(-2x+10)^3 - 11$ (M2)

$= 7(-2(x-5))^3 - 11$

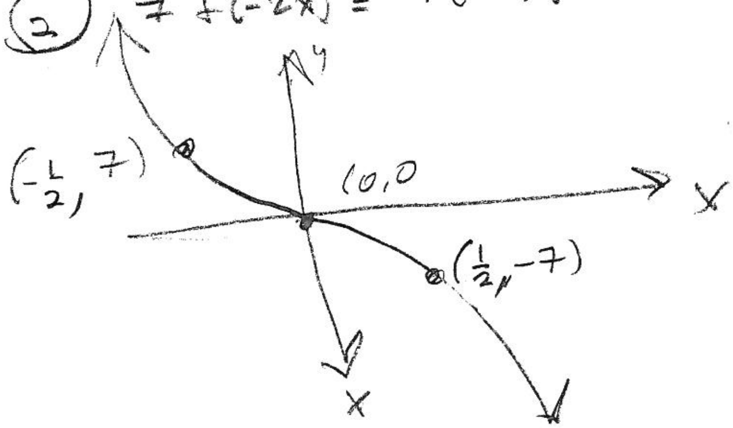
(0) $f(x) = x^3$



(1) $7f(x) = 7x^3$



(2) $7f(-2x) = 7(-2x)^3$



Steps (3) & (4), see previous.
To get to (3) from this pic, it's "delay by 5" or "RIGHT 5."

$-\frac{1}{2} + 5 = -\frac{1}{2} + \frac{10}{2} = \frac{9}{2}$ ✓

$\frac{1}{2} + 5 = \frac{1}{2} + \frac{10}{2} = \frac{11}{2}$ ✓

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6 b $\boxed{\mathcal{D} = \mathbb{R} = (-\infty,\infty)}$

c $g(0) = 7(10)^3 - 11$

$A = (0, 7(10)^3 - 11) = \boxed{(0, 6989) = A}$

Here is where I caught my failure with horizontal reflection.
The # IS +6989, but the PICTURE says it needs to be negative!
That's when I figured out the picture was wrong.

$g(x) = 0 \implies 7(-2x+10)^3 - 11 = 0$

$\implies 7(-2x+10)^3 = 11$

$\implies (-2x+10)^3 = \frac{11}{7}$

$\implies -2x+10 = \sqrt[3]{\frac{11}{7}}$

$\implies -2x = -10 + \frac{\sqrt[3]{539}}{7}$

$\implies x = 5 - \frac{\sqrt[3]{539}}{14}$

$\implies \boxed{B = \left(5 - \frac{\sqrt[3]{539}}{14}, 0\right)}$

$\approx (4.418698354, 0)$

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(7) $p(x) = 5x - 7$ is 1-to-1.

Pf Suppose $5x_1 - 7 = 5x_2 - 7$. Then

$$5x_1 = 5x_2$$

$$\Rightarrow x_1 = x_2$$

$$\Rightarrow p \text{ is 1-to-1}$$

(8) $$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

(9) $x = |2y - 4| = x$

$$\Rightarrow 2y - 4 = \pm x$$

$$\Rightarrow 2y = 4 \pm x$$

$$\Rightarrow y = 2 \pm \tfrac{1}{2}x$$

2 y-values for one x-value,

e.g. $x = 2 \Rightarrow y = 2 \pm 1$ ↗ 3, ↘ 1

$\Rightarrow$ (2,3) & (2,1) are members of the relation $\Rightarrow$ NOT A FUNCTION

TEST 2

BONUS

B1 $6x + 3h - 5 \xrightarrow{h \to 0} \boxed{6x - 5 = f'(x)}$

B2 $$\frac{\sqrt{x+h} - \sqrt{x}}{h} = \left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$

$$= \frac{\left(\sqrt{x+h}\right)^2 - \left(\sqrt{x}\right)^2}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \frac{x + h - x}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$\xrightarrow{h \to 0} \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}} \ !$$

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B3 DONE

B4 $h(x) = -4x^2 - 5x + 3$

$= -4\left(x^2 + \frac{5}{4}x + \left(\frac{5}{8}\right)^2\right) + 3 + 4\left(\frac{25}{64}\right)$

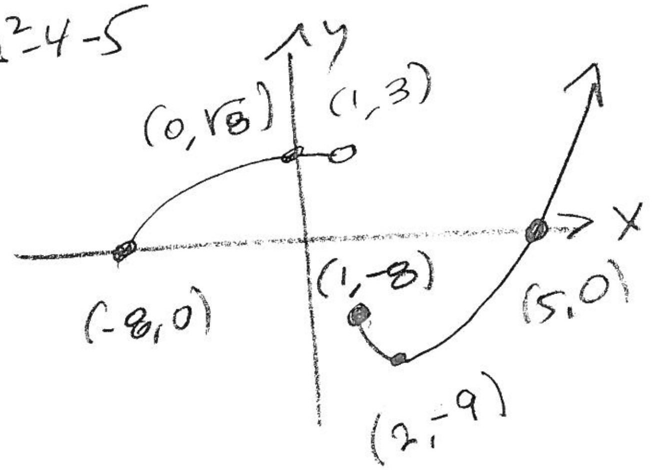
$= \boxed{-4\left(x + \frac{5}{8}\right)^2 + \frac{73}{16}}$ $\quad 3 + \frac{25}{16} = \frac{48 + 25}{16}$

$\rightarrow \boxed{(h, k) = \left(-\frac{5}{8}, \frac{73}{16}\right)}$

B5 $\sqrt{1+8} = \sqrt{9} = 3 \quad x < 1 \Longrightarrow$ o

$1^2 - 4(1) - 5 = -8 \quad x \geq 1 \Longrightarrow$ •

$x^2 - 4x + 2^2 - 4 - 5$



$x^2 - 4x - 5 = 0$

@ $x = -1, 5$

by prev. wk.

$(x-5)(x+1)$