## Comprehensive

NO GRAPHING CALCULATORS!!!

Do your work and circle final answers on separate paper, provided. Remember, to write big and bold, but mostly bold, or your teacher can't read it and give you any points for it.

- 1. Solve the equation  $x^2 5x 14 = 0$  in three different ways:
  - a. (10 pts) Factoring
  - b. (10 pts) Completing the Square
  - c. (10 pts) Quadratic Formula
- 2. Solve the absolute value inequalities. Give your answers in set-build and interval notation.
  - a.  $(10 \text{ pts}) \left| -5x + 4 \right| < 7$
  - b.  $(10 \text{ pts}) |-5x+4| \ge 7$
- 3. Let  $f(x) = \sqrt{x+5}$  and  $g(x) = x^2 4x 5$ 
  - a. (10 pts) What is the domain of f? Give your answer in set-builder and interval notation.
  - b. (10 pts) What is the domain of g? Give your answer in set-builder and interval notation.
  - c. (10 pts) Determine  $(f \circ g)(x)$  (Just  $f \circ g$ , for short). Simplify your answer.
  - d. (10. pts) What is the domain of  $f \circ g$ ?
- 4. (10 pts) Solve  $(x+6)(x-1)^4(x-6) > 0$ . Give your answer in interval notation.
- 5. (10 pts) What is the domain of  $f(x) = \sqrt{(x+6)(x-1)^2(x-6)}$ ?
- 6. (10 pts) What is the domain of  $g(x) = \sqrt{\frac{(x+6)(x-6)}{(x-1)^2}}$ ?
- 7. (10 pts) Use synthetic division to find P(2), for  $P(x) = 2x^5 5x^4 2x^2 + 6x 13$
- 8. (10 pts) Determine a, r, and n for the finite geometric sequence  $3, 2, \frac{4}{3}, \dots, \frac{256}{2187}$ . Use a, r, and n to

determine the *exact* value of the sum  $\sum_{k=1}^{n} a \cdot r^{k-1} = a \left( \frac{1-r^n}{1-r} \right)$ . For full credit, a fraction in lowest terms is required.

9. (10 pts) Find the sum of the infinite geometric series  $\sum_{k=1}^{\infty} 5 \cdot \left(\frac{2}{3}\right)^{k-1}$ 

Finance Formulas: 
$$A = P\left(1 + \frac{r}{m}\right)^{mt} = P\left(1 + i\right)^n$$
 and  $FV = R\left(\frac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{\left(\frac{r}{m}\right)}\right) = R\left(\frac{\left(1 + i\right)^n - 1}{i}\right)$ 

- 10. (10 pts) What is the future value, in 7 years, of \$5,000 deposited at 5% annual percentage rate, compounded weekly? (Recall, there are 52 weeks in a year.)
- 11. (10 pts) If your monthly loan payments are \$5,000, and the loan is for 5 years, how much did you borrow in the first place, if you're paying 9.5% annual percentage rate, compounded monthly?
- 12. Let  $g(x) = 5 \cdot \log_6(3x + 12) 11$ .
  - a. (20 pts) Start with the graph of  $f(x) = \log_6(x)$ , and proceed to the graph of g(x) with 4 more graphs, demonstrating each of the 4 basic transformations. So, 5 graphs, in all, counting the graph of f(x) as the first. Label the 3 points,  $\left(\frac{1}{6}, -1\right)$ , (1,0), and (6,1) in the graph of f(x), and track where they end up after each transformation.
  - b. (10 pts) Find the x- and y-intercepts of the graph of g(x), above. Report them as ordered pairs, using the labels A and B. Label the x- and y-intercepts A and B, respectively, in your final graph, above.

## **Bonus**

- **B 1** (5 pts) Let  $f(x) = 3^{5x+2} + 7$ . Find  $f^{-1}(x)$ .
- **B 2** Suppose the half-life of C-14 is 6000 years. (It isn't, quite, but just suppose...).
  - a) (5 pts) Derive the exponential decay model,  $A(t) = A_0 e^{kt}$ . The trick is to use the half-life to find the relative decay rate, k. Symbolic (Non-calculator) answers accepted (preferred).
  - b) (5 pts) How old is a sample of charcoal from the prehistoric fire pit, if 80% of the C-14 has decayed? Round to the nearest year in your final answer.
- **B 3** (5 pts) Sketch the graph of  $r(x) = -3\sqrt{2x+8} + 27$ . I don't need to see all the steps. But you better *nail* the shape, orientation, endpoint and x- and y-intercepts.
- **B 4** (5 pts) Re-write  $f(x) = 4x^2 3x + 1$  in the form  $a(x h)^2 + k$ .

**B 5** (5 pts) John can finish a job in 6 hours that it takes Bill 11 hours to finish. Suppose Bill shows up and starts working 2 hours before John shows up, and then they work together until the job is done. How many hours does each of the two end up working?