..121 FINAL FALL '16

1 (a) $x^2-5x-14 = (x-7)(x+2) \overset{\text{SET}}{=} 0$

$\rightarrow \boxed{x \in \{-2, 7\}}$

10pts

(b) $x^2-5x-14 = 0$

$x^2-5x = 14$

$x^2-5x+\left(\frac{5}{2}\right)^2 = 14+\frac{25}{4}$

$\left(x-\frac{5}{2}\right)^2 = \frac{56}{4}+\frac{25}{4} = \frac{81}{4}$

$x-\frac{5}{2} = \pm\sqrt{\frac{81}{4}} = \pm\frac{9}{2}$

$x = \frac{5}{2} \pm \frac{9}{2} = \frac{5 \pm 9}{2}$ $\frac{14}{2} = 7$; $\frac{-4}{2} = -2$

$\boxed{x \in \{-2, 7\}}$

10pts

(c) $a = 1$, $b = -5$, $c = -14$

10pts

$b^2-4ac = (-5)^2-4(1)(-14)$

$= 25+56 = 81$

$\sqrt{81} = 9$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{5 \pm 9}{2}$ $\frac{14}{2} = 7$; $\frac{-4}{2} = -2$

$\boxed{x \in \{-2, 7\}}$

0

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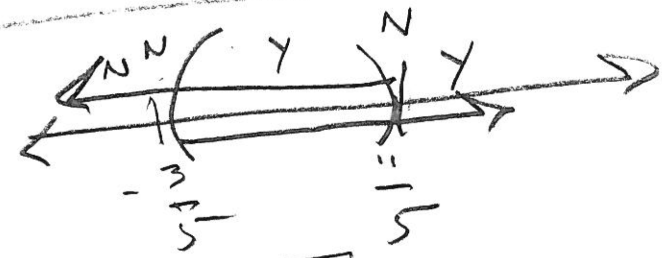
2) a) $|-5x+4| < 7$

$-5x+4 < 7$ and $-5x+4 > -7$

$-5x < 3$ $\qquad$ $-5x > -11 \implies x \in \ldots$

$\boxed{\left\{x \mid x > \frac{-3}{5} \text{ and } x < \frac{11}{5}\right\}}$ $\qquad$ 10 pts



AND

$= \boxed{\left(-\frac{3}{5}, \frac{11}{5}\right)}$

b) $|-5x+4| \geq 7$

$-5x+4 \geq 7$ OR $-5x+4 \leq -7 \implies$

$-5x \geq 3$ $\qquad$ $-5x \leq -11$

$x \in \boxed{\left\{x \mid x \leq \frac{-3}{5} \text{ OR } x \geq \frac{11}{5}\right\}}$ $\qquad$ 10 pts

OR

$= \boxed{\left(-\infty, -\frac{3}{5}\right] \cup \left[\frac{11}{5}, \infty\right)}$

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3 $f(x) = \sqrt{x+5}$, $g(x) = x^2-4x-5$

a Need $x+5 \geq 0$

$\Rightarrow \mathcal{D} = \boxed{\{x \mid x \geq -5\} = [-5, \infty)}$ 10pts

b g is a polynomial $\mathcal{D} = \boxed{\{x \mid x \in \mathbb{R}\} = (-\infty, \infty)}$ 10pts

c $(f \circ g)(x) = \sqrt{x^2-4x-5+5}$

$\boxed{(f \circ g)(x) = \sqrt{x^2-4x}}$ 10pts

d $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$\mathcal{D}(g) = (-\infty, \infty)$, so just worried about

$g(x) \in \mathcal{D}(f)$ part $\Rightarrow$

$g(x) \geq -5$

$x^2-4x-5 \geq -5$

$x^2-4x \geq 0$

$x(x-4) \geq 0$

$+ \quad - \quad +$

Y 0 N 4 Y

Y Y

$\Rightarrow \boxed{\mathcal{D}(f \circ g) = (-\infty, 0] \cup [4, \infty) = \{x \mid x \leq 0 \text{ OR } x \geq 4\}}$ 10 pts

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Should be a '2.' Does not affect the work in any real way, because the square behaves the same as the quartic with regard to sign changes.

(4) $(x+6)(x-1)^4(x-6) > 0$

Y + | N | N − | N | N − | N | Y +

-6 $=0$ | 1 $=0$ | 6 $=0$

>0

10 pts

$\rightarrow \boxed{x \in (-\infty, -6) \cup (6, \infty)}$

(5) $f(x) = \sqrt{(x+6)(x-1)^4(x-6)}$

$\rightarrow \mathcal{D}(f)$ needs $(x+6)(x-1)^4(x-6) \geq 0$

Y + | Y | N − | Y | N − | Y | Y +

-6 $=0$ | 1 $=0$ | 6 $=0$

10 pts

$\rightarrow \boxed{\mathcal{D}(f) = (-\infty, -6] \cup \{1\} \cup [6, \infty)}$

(6) $f(x) = \sqrt{\dfrac{(x+6)(x-6)}{(x-1)^4}}$ $\rightarrow$

$\mathcal{D}(f)$ needs $\dfrac{(x+6)(x-6)}{(x-1)^4} \geq 0$

Y + | Y | N − | N | N − | Y | Y +

-6 $=0$ | 1 ☆ | 6 $=0$

10 pts

$\rightarrow \boxed{\mathcal{D}(f) = (-\infty, -6] \cup [6, \infty)}$

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(7) $P(x) = 2x^5 - 5x^4 - 2x^2 + 6x - 13$

We find $P(2)$:

$$\begin{array}{r|rrrrrr} 2 & 2 & -5 & 0 & -2 & 6 & -13 \\ & & 4 & -2 & -4 & -12 & -12 \\ \hline & 2 & -1 & -2 & -6 & -6 & \boxed{-25 = P(2)} \end{array}$$

10 pts

(8) $3, 2, \frac{4}{3}, \ldots, \frac{256}{2187}$

$\boxed{\begin{array}{c} a = 3, \\ r = \frac{2}{3} \\ n = 9 \end{array}}$

r: $\frac{2}{3} \stackrel{?}{=} \frac{\frac{4}{3}}{2} = \frac{2}{3}$ ✓

n: $\frac{256}{2187} = 3 \cdot \left(\frac{2}{3}\right)^{n-1}$

$= \frac{2^8}{3^7} = 3 \cdot \frac{2^8}{3^8}$

$= 3 \cdot \left(\frac{2}{3}\right)^8 \Longrightarrow$

$\Longrightarrow n - 1 = 8$

$\boxed{n = 9}$

$$\begin{array}{r|l} 2 & 256 \\ 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array} \qquad \begin{array}{r|l} 3 & 2187 \\ 3 & 729 \\ 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

10 pts

So, $\sum_{k=1}^{9} 3 \cdot \left(\frac{2}{3}\right)^{k-1} = 3\left(\frac{1 - \left(\frac{2}{3}\right)^9}{1 - \frac{2}{3}}\right)$

$= 3\left(\frac{1 - \frac{512}{19683}}{\frac{1}{3}}\right) = 3\left(\frac{\frac{19683 - 512}{19683}}{\frac{1}{3}}\right)$

$= 9\left(\frac{19171}{19683}\right) = \boxed{\frac{19171}{2187}} \boxed{\approx 8.765889346}$

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(9) $\sum_{k=1}^{\infty} 5\cdot\left(\frac{2}{3}\right)^{k-1}$ $a=5,\ r=\frac{2}{3},\ n \longrightarrow \infty$ 10pts

$$S = a\left(\frac{1}{1-r}\right) = 5\left(\frac{1}{\frac{1}{3}}\right) = 5(3) = \boxed{15 = S}$$

(10) $A = P(1+i)^n = 5000\left(1+\frac{.05}{52}\right)^{(52)(7)}$ $m=52$

$\approx \boxed{\$7094.14}$ $i = \frac{.05}{12}$ $t=7$

$n = (52)(7)$ 10pts

(11) $A = FV$

$$P(1+i)^n = R\left(\frac{(1+i)^n-1}{i}\right)$$

Divide by $(1+i)^n$ to isolate the P. 10pts

$$P = R\left(\frac{1-(1+i)^{-n}}{i}\right)$$

$i = \frac{.095}{12}$

$\approx \boxed{\$238{,}074.14}$ $n = (12)(5)$

$R = 5{,}000$

(12) We graph...

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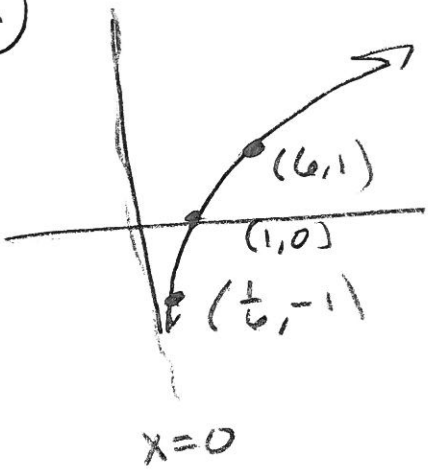
(12) $g(x) = 5\log_6(3x+12) - 11$

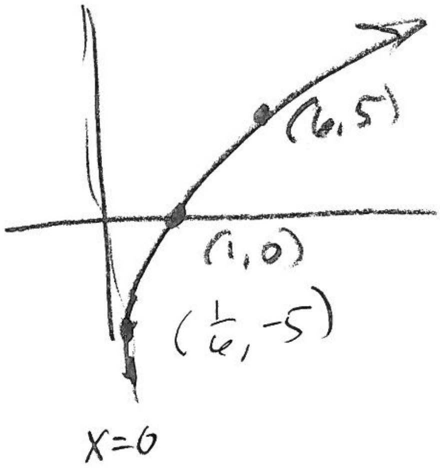
20 pts

$f(x) = \log_6(x)$

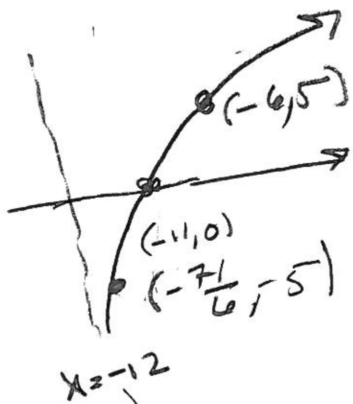
(1)



(2) $5f(x)$

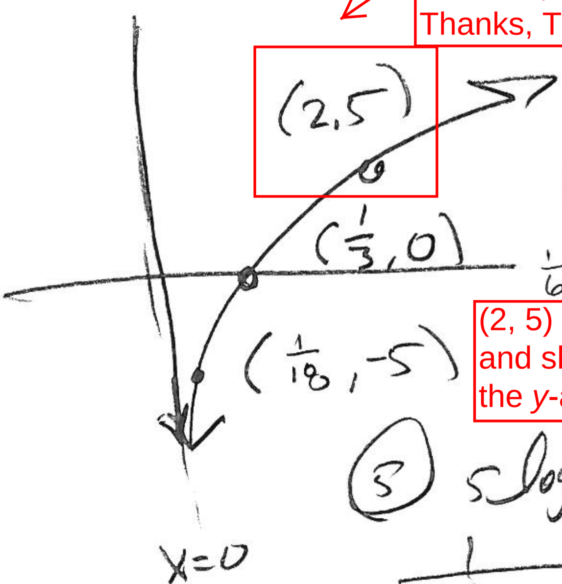


$5\log_6(x+12)$



(3) $5\log_6(3x) = 5f(3x)$

-6/3 = -2, not +2
Thanks, Tanya!

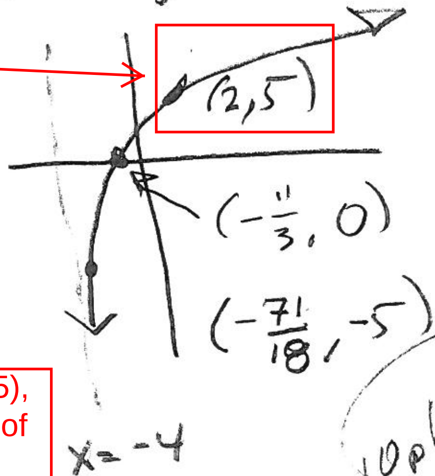


$\frac{1}{3} - 4 = -\frac{11}{3}$

$\frac{1}{6} - 4 = \frac{-23}{6}$

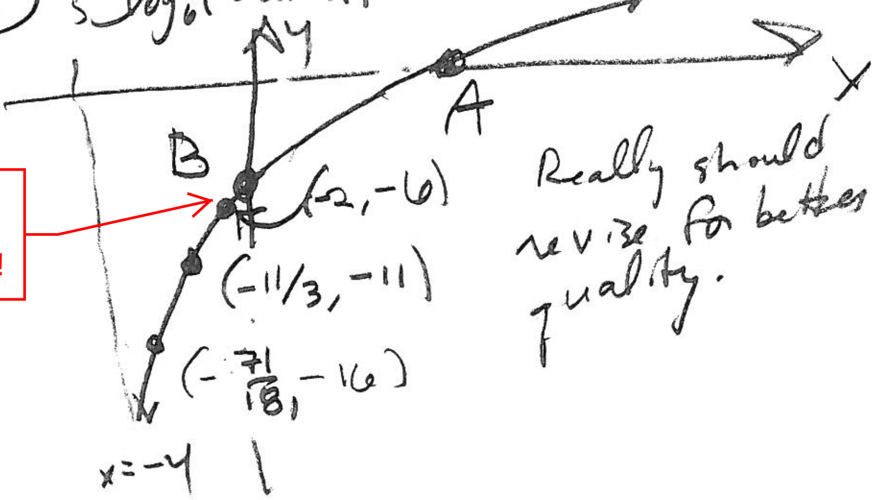
(2, 5) should be (-2, 5), and should be LEFT of the *y*-axis!!!

(4) $5\log_6(3(x+4))$



10 pts

(5) $5\log_6(3(x+4)) - 11$



Really should revise for better quality.

Finally got the values and the position correct, but does not follow from the previous graph!

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12 b

x-int: $g(x)=0$

$5\log_6(3x+12)-11=0$

$5\log_6(3x+12)=11$

$\log_6(3x+12)=\frac{11}{5}$

$3x+12=6^{11/5}$

$3x=6^{11/5}-12$

$x=\frac{1}{3}\cdot 6^{11/5}-4$

$\approx$

$\boxed{A=\left(\frac{1}{3}\cdot 6^{11/5}-4,0\right) \approx (13.17162897,0)}$

y-int: $g(0)=?$

$g(0)=5\log_6(12)-11$

$\boxed{B=(0,5\log_6(12)-11) \approx (0,-4.065736)}$

10pts

B1 $f(x)=3^{5x+2}+7$

$3^{5y+2}+7=x$

$3^{5y+2}=x-7$

$\log_3(\ )=\log_3(\ )$

$5y+2=\log_3(x-7)$

$5y=\log_3(x-7)-2$

$\boxed{y=\frac{\log_3(x-7)-2}{5}}$

$=f^{-1}(x)$

5pts

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B2 $\frac{1}{2}$-life is 6000 yrs $\Longrightarrow$

a) $A_0 e^{6000k} = \frac{1}{2}A_0$

$e^{6000k} = \frac{1}{2}$

$6000\ k = \ln(1/2) = -\ln 2$

$\boxed{k = -\frac{\ln 2}{6000}} \approx -.000115524530l$

5pts

b) 80% of C-14 decayed $\Longrightarrow$

20% " " remains $\Longrightarrow$

$A_0 e^{kt} = .2 A_0$

$e^{kt} = .2$

$kt = \ln .2$

$t = \frac{\ln .2}{k} \approx 13,931.56857$

$\approx \boxed{13,932 \text{ yrs}}$

5pts

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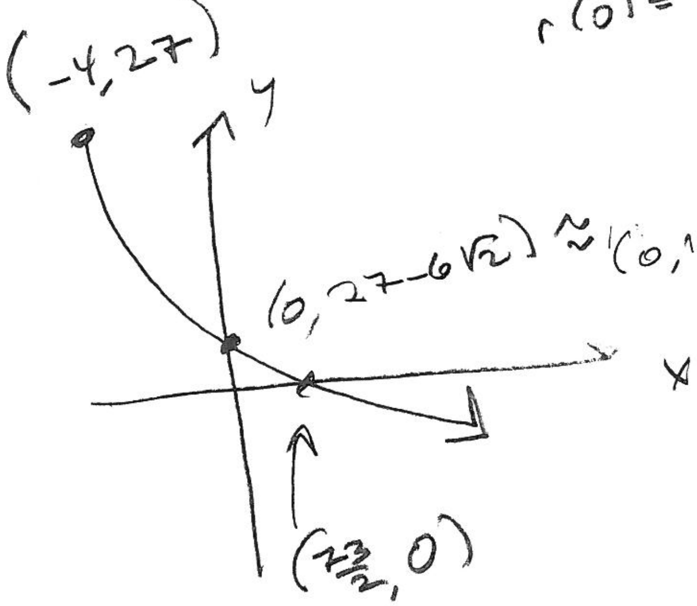
B3 $r(x) = -3\sqrt{2(x+4)} + 27$

$r(0) = -3\sqrt{8} + 27$

$= -3 \cdot 2\sqrt{2} + 27$

$= -6\sqrt{2} + 27 > 0$



$\approx (0, 18.51471863)$

5pts

$r(x) = 0$

$-3\sqrt{2x+8} + 27 = 0$

$-3\sqrt{2x+8} = -27$

$\sqrt{2x+8} = 9$

$2x + 8 = 81$

$2x = 73$

$x = \frac{73}{2}$

5pts

B4 $4x^2 - 3x + 1$

$= 4\left(x^2 - \frac{3}{4}x \quad\right) + 1$

$= 4\left(x^2 - \frac{3}{4}x + \left(\frac{3}{8}\right)^2\right) + 1 - 4\left(\frac{9}{64}\right)$

$= \boxed{4\left(x - \frac{3}{8}\right)^2 + 7/16}$

$1 - 4\left(\frac{9}{64}\right)$

$= 1 - \frac{9}{16} = \frac{16-9}{16} = \frac{7}{16}$

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B5

Let x = # of hours John works.

Then Bill works $x+2$ hours, since John started 2 hrs late so

work by John + work by Bill = 1 Job

$$\frac{1}{6}x + \frac{1}{11}(x+2) = 1$$

5 pts

$$11x + 6(x+2) = 66$$

$$11x + 6x + 12 = 66$$

$$17x = 54$$

$$\boxed{x = \frac{54}{17} \text{ hrs}} \rightarrow 3.1765 \text{ hrs}$$

$$x+2 = \frac{54}{17} + \frac{34}{17} = \boxed{\frac{88}{17} \text{ hrs for Bill}}$$

≈ 5.1765 hrs

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(7) Common mistake is to omit the x^3 placeholder

Enough students missed this one, I did it "right" except for that one mistake, to help award partial credit to those who missed the placeholder.

$$\begin{array}{r|rrrrr} 2 & 2 & -5 & -2 & 6 & -13 \\ & & 4 & -2 & -8 & -4 \\ \hline & 2 & -1 & -4 & -2 & \boxed{-17 = P(2)} \end{array}$$

#8 $n=8$ most common error.

Look: $3\left(\frac{2}{3}\right)^{n-1} = \frac{256}{2187} = \frac{2^8}{3^7} = 3\left(\frac{2^8}{3^8}\right)$

$= 3\left(\frac{2}{3}\right)^8 \longrightarrow$

$n-1=8$

$\longrightarrow n=9$!

So, $3\left(\frac{1-\left(\frac{2}{3}\right)^9}{1-\frac{2}{3}}\right)$, etc.

For $n=8$: $3\left(\frac{1-\frac{256}{6561}}{\frac{1}{3}}\right) = 9\left(\frac{6561-256}{6561}\right) = 9\left(\frac{6305}{6561}\right)$

$= \frac{6305}{729} \approx 8.648834019$