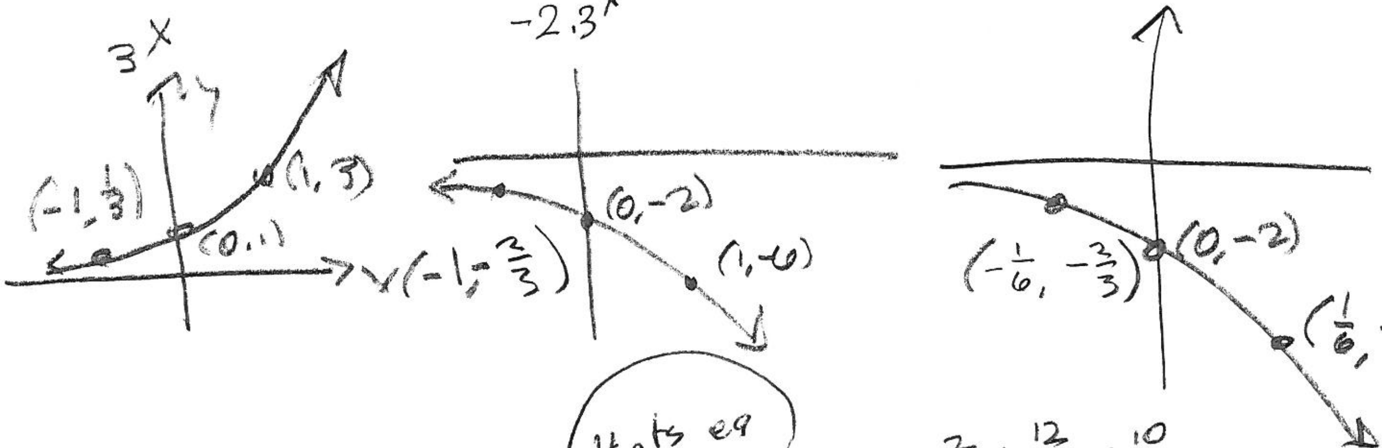
121 E4 FALL '16

(1) $g(x) = -2 \cdot 3^{6x+18} + 4$

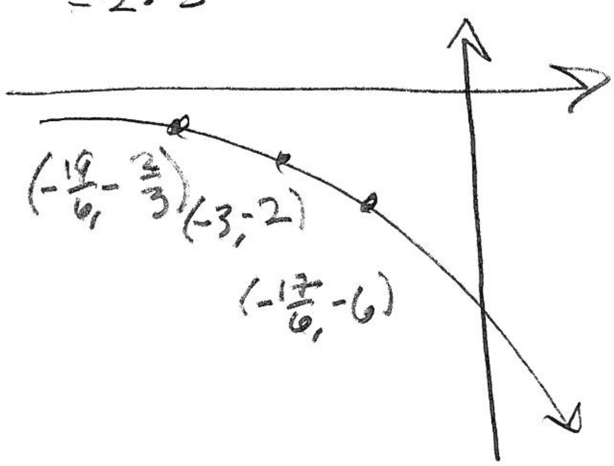
3^x $\qquad$ -2.3^x $\qquad$ $-2 \cdot 3^{6x}$

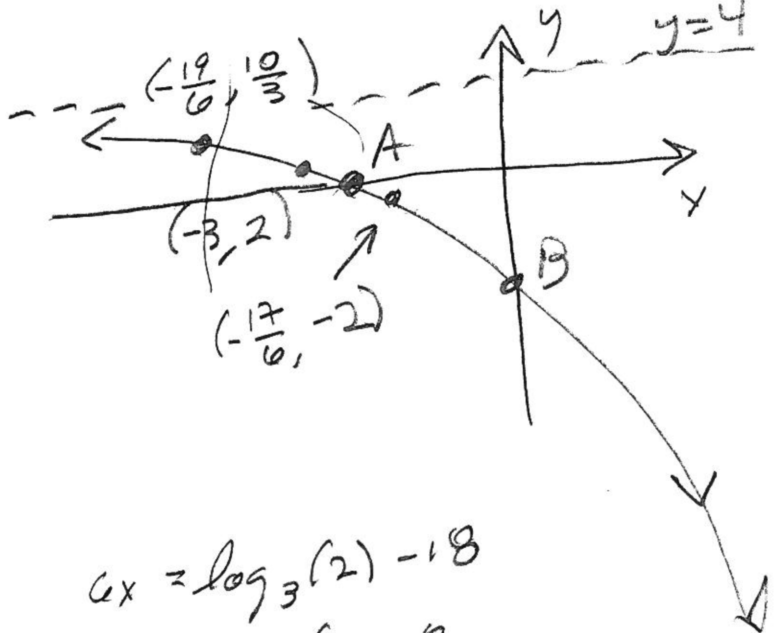


4pts ea

$-\frac{2}{3} + \frac{12}{3} = \frac{10}{3}$

$-2 \cdot 3^{6(x+3)}$





(2) A : $g(x) = 0$

$-2 \cdot 3^{6x+18} + 4 = 0$

$-2 \cdot 3^{6x+18} = -4$

$3^{6x+18} = \frac{4}{2} = 2$

$6x + 18 = \log_3(2)$

$6x = \log_3(2) - 18$

$x = \frac{\log_3(2) - 18}{6}$

-2.894845041

$\boxed{A = \left(\frac{\log_3(2) - 18}{6}, 0 \right)}$ 5pts

B : $g(0) = -2 \cdot 3^{18} + 4$

$\boxed{B = (0, -2 \cdot 3^{18} + 4)}$ 5pts

-774840974

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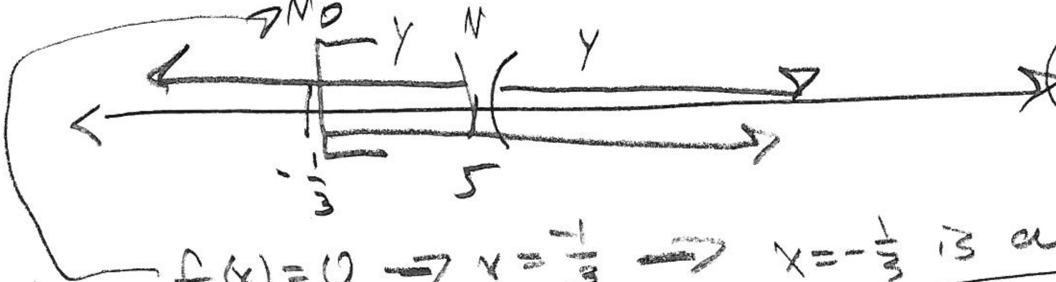
(3) $f(x) = \sqrt{3x+1}$, $g(x) = \dfrac{5}{x-5}$

(a) $\boxed{\mathcal{D}(f) = [-\tfrac{1}{3}, \infty)}$ 5pts

(b) $\boxed{\mathcal{D}(g) = \mathbb{R} \setminus \{5\}}$ 5pts

(c) $\boxed{\left(\dfrac{g}{f}\right)(x) = \dfrac{\frac{5}{x-5}}{\sqrt{3x+1}}}$ 5pts

(d) $\mathcal{D}\left(\frac{g}{f}\right) = \{x \mid x \in \mathcal{D}(f) \cap \mathcal{D}(g) \text{ and } f(x) \neq 0\}$



AND

$f(x) = 0 \Rightarrow x = -\frac{1}{3} \Rightarrow x = -\frac{1}{3}$ is also bad.

$\boxed{\mathcal{D}\left(\frac{g}{f}\right) = (-\tfrac{1}{3}, 5) \cup (5, \infty)}$

5pts

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(e) $\boxed{f \circ g = \sqrt{3\left(\frac{5}{x-5}\right)+1}}$ 5pts

(f) $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

Scratch: $3\left(\frac{5}{x-5}\right)+1 \geq 0$

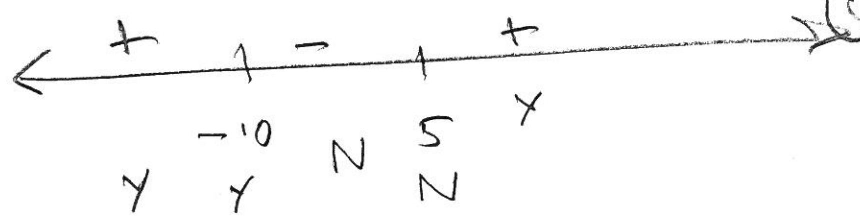
This is B7

$$\frac{15}{x-5} + \frac{x-5}{x-5} \geq 0$$

~~10pts~~

$$\frac{x+10}{x-5} \geq 0$$

≥ 0



$= \{x \mid x \neq 5 \text{ and } x \in (-\infty, -10] \cup (5, \infty)\}$

$= \boxed{(-\infty, -10] \cup (5, \infty)}$ 5pts

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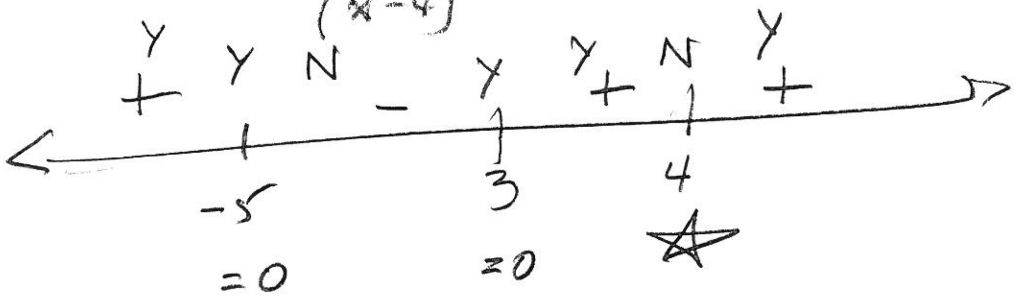
(4) $\mathcal{D}\left(\sqrt{\dfrac{(x-3)(x+5)^3}{(x-4)^2}}\right) = ?$

NEED $\dfrac{(x-3)(x+5)^3}{(x-4)^2} \geq 0$



$\rightarrow \boxed{(-\infty,-5] \cup [3,4) \cup (4,\infty)}$ 5pts

(5) For log, take out all the "=0"s.

$\boxed{(-\infty,-5) \cup (3,4) \cup (4,\infty)}$ 5pts

(6)

10

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EY

(6) $f(x) = 2^{3x-7} + 5$

$2^{3y-7} + 5 = x$

$2^{3y-7} = x - 5$

$3y - 7 = \log_2(x-5)$

$3y = \log_2(x-5) + 7$

$\boxed{y = \frac{\log_2(x-5)+7}{3} = f^{-1}(x)}$

10pts

(7) $\ln(x-3) + \ln(x+2) = \ln(2x+4)$

$\ln((x-3)(x+2)) = \ln(2x+4)$

$x^2 - x - 6 = 2x + 4$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x \in \{\cancel{-2}, 5\}$

$-2 \notin D \rightarrow \boxed{x \in \{5\}}$

10pts

20

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8 $\frac{1}{2}$-life is 5900 yrs

a $P_0 e^{5900k} = \frac{1}{2} P_0$

$e^{5900k} = \frac{1}{2}$

$5900k = \ln(\frac{1}{2})$

$$\boxed{k = \frac{\ln(\frac{1}{2})}{5900} \approx -.000117482573}$$

10pts

b $P_0 e^{kt} = .57 P_0$

$e^{kt} = .57$

$kt = \ln(.57)$

$$t = \frac{\ln(.57)}{k} = \frac{5900 \ln(.57)}{\ln(1/2)}$$

≈ 4784.700436 yrs old

$\approx$ $\boxed{4785 \text{ yrs old}}$

5pts

15

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B1 $|-5x+8|-11>-2$

$|-5x+8|>9$

$-5x+8>9$ OR $-5x+8<-9$

$-5x>1$ $-5x<-17$

$\{x \mid x<-\frac{1}{5}$ OR $x>\frac{17}{5}\}$

$= \boxed{(-\infty,-\frac{1}{5})\cup(\frac{17}{5},\infty)}$ 5pts

B2 $P(x)=5x^2-3x+1$

$\frac{20}{20}-\frac{9}{20}=\frac{11}{20}$

$=5\left(x^2-\frac{3}{5}x \qquad\right)+1$

$=5\left(x^2-\frac{3}{5}x+\left(\frac{3}{10}\right)^2\right)+1-5\left(\frac{9}{100}\right)$

$\boxed{=5\left(x-\frac{3}{10}\right)^2+\frac{11}{20}}$ 5pts

$\boxed{(h,k)=\left(\frac{3}{10},\frac{11}{20}\right)}$

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B3

$$3 \cdot (7.7)^x = 11 \cdot (2.1)^x$$

$$\ln \overset{a}{3} + x \ln \overset{b}{(7.7)} = \ln \overset{c}{(11)} + x \cdot \ln \overset{d}{(2.1)}$$

$$a + x \cdot b = c + x \cdot d$$

$$a + bx = c + dx$$

$$bx - dx = c - a$$

$$x(b-d) = c - a$$

$$x = \frac{c-a}{b-d} = \boxed{\frac{\ln(11) - \ln(3)}{\ln(7.7) - \ln(2.1)} = x}$$

$$= \frac{\ln\left(\frac{11}{3}\right)}{\ln\left(\frac{7.7}{2.1}\right)} = \frac{\ln\left(\frac{11}{3}\right)}{\ln\left(\frac{11}{3}\right)} = 1 \;!$$

B4 x = Time John spends working (in hours)

$$\frac{1}{6}x + \frac{1}{11}(x+2) = 1$$

$$\frac{x}{6} + \frac{x+2}{11} = 1$$

$$\frac{11x + 6x + 12}{66} = \frac{66}{66}$$

$$17x + 12 = 66$$

$$17x = 54$$

John

$$\boxed{x = \frac{54}{17}} \approx 3.176470588$$

Bill

$$x + 2 = \frac{34 + 54}{17}$$

$$= \boxed{\frac{88}{17} = x + 2}$$

$$\approx 5.176470588$$

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B5 $1 + 49 + 343 + \cdots + 5{,}764{,}801$

$a = 1, \ r = 7$

So, 5764801

a

$= ar^{n-1} = ar^{8}$ — 7

$n = 9$

So $a\left(\frac{1-r^n}{1-r}\right) = 1\left(\frac{1-7^9}{1-7}\right)$

$$\boxed{= \frac{40353607 - 1}{6}}$$

1 $7 \,|\, 5764801$

2 $7 \,|\, 823543$

3 $7 \,|\, 117649$

4 $7 \,|\, 16807$

5 $7 \,|\, 2401$

6 $7 \,|\, 343$

7 $7 \,|\, 49$

8 7

$5764801 \times 7 = 40353607$

b $\sum_{k=1}^{\infty} 5\left(\frac{2}{7}\right)^{n-1} = 5\left(\frac{1}{1-\frac{2}{7}}\right) = \frac{5}{\frac{5}{7}} = 5 \cdot \frac{7}{5}$

$\boxed{= 7}$

1

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B7 Dang! See 3f!

B6 $S_n = \sum_{k=1}^{n} ar^{n-1} = a\left(\frac{1-r^n}{1-r}\right)$

$\boxed{\text{Pf}}$ $S_n = a + ar + ar^2 + \cdots + ar^{n-1}$

$- \quad rS_n = ar + ar^2 + ar^3 + \cdots + ar^{n-1} + ar^n$

$S_n - rS_n = (1-r)S_n = a - ar^n = a(1-r^n)$

$\rightarrow \boxed{S_n = \frac{a(1-r^n)}{1-r} \quad \blacksquare}$