MAT 121-G81-G82 Test 3 – Fall, 2014 Name_________________
70 Points Covers Chapter 3 NO GRAPHING CALCULATORS!!!

Remember to use separate paper for everything except your name. Leave a margin in the top left corner. Spread out your work. Use only one column for your work. *Submit problems in order!!!*

1. (10 pts) Form a polynomial of *minimial degree* in *factored form* that has real coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

$x = 2$, multiplicity 3; $x = 6 + 2i$, multiplicity 2; $x = -3$, multiplicity 1.

2. (10 pts) Use synthetic division to find $P(-2)$ if $P(x) = 7x^5 - 2x^4 + 11x^3 + x^2 - 173x - 4$
.
3. (5 pts) Represent the work you just did on the previous problem by writing $P(x)$ in the form $\textit{Dividend} = \textit{Divisor} \bullet \textit{Quotient} + \textit{Remainder}$.

4. Suppose $f(x) = (x-3)^2(x+5)(x-7)(x+1) = x^5 - 7x^4 - 22x^3 + 178x^2 - 123x - 315$. I'm showing you both factored and expanded form to help you answer the following:

   a. Solve the inequality $f(x) \le 0$. Your sign pattern for this one will be helpful in the next two. You just have to interpret what you're seeing.

   b. (10 pts) Provide a rough sketch of *f*, using its zeros, their respective multiplicities and its end behavior. Include *x*- and *y*-intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better quality graph.

   c. (5 pts) What is the domain of $\sqrt{\dfrac{(x-3)^2(x+5)}{(x-7)(x+1)}}$?

5. Let $f(x) = 4x^5 - 9x^3 + 8x^2 - 9x + 6$.
   a. (5 pts) Use Descartes' Rule of Signs to determine the possible number of positive and negative zeros of *f*.

   b. (5 pts) List all possible rational zeros of *f*.
   c. (Bonus 5 pts) Find the smallest possible integer bounds on positive and negative zeros.

6. (10 pts) Find the *real* zeros of $f(x) = 4x^5 - 9x^3 + 8x^2 - 9x + 6$. Then factor *f* over the set of **real numbers**. This should involve an irreducible quadratic factor.

   (If things go haywire, come up with a *plausible*-looking polynomial, in factored form, with the right number of real roots and 2 nonreal roots. The 2 nonreal roots will still be living inside the irreducible quadratic factor, so you'll have to make up a quadratic factor with nonreal zeros).

7. (5 pts) Find the remaining (nonreal) zeros of f and factor f over the set of **complex numbers**. This step requires breaking down the quadratic piece that's irreducible over the real numbers. The fundamental theorem tells us that *nothing* is irreducible over the complex numbers.

   (You can still get full credit for this one, even if things went haywire, in #6, if you solve the depressed equation, correctly, and display your *plausible*-looking follow-up to your *plausible*-looking answer to #6. The more you know about what you're pointing towards, the more points you'll earn.)

   And yes, "*plausible*" must always be italicized.

8. (5 pts) You don't need to graph $R(x)=\dfrac{6x^3-35x^2+64x-35}{2x^2+x-6}$, here, but I do want to see you graph its asymptotes. Hint: This function has no holes.

9. (10 pts) Sketch the graph of $F(x)=\dfrac{5x^2+2x-39}{x^2+2x-8}$. Show all asymptotes and intercepts.

ANSWER ANY TWO (2) OF THE FOLLOWING, FOR UP TO 20 BONUS POINTS!!!

B1 (10 pts) Form a polynomial of *minimal degree* in *factored form* that has ***rational*** coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong.

Zeros: $x=3-\sqrt{5}$, multiplicity 1; $x=2+7i$, multiplicity 2; $x=-5$, multiplicity 17.

B2 Solve both of the following absolute value inequalities.

- **a.** (5 pts) $|3x-7|-2\geq 8$
- **b.** (5 pts) $|3x-7|+8<2$

B3 (10 pts) Sketch the graph of $R(x)=\dfrac{6x^3-35x^2+64x-35}{2x^2+x-6}$. Hints:

- **a.** You already found $R(x)$'s asymptotes.
- **b.** One of $R(x)$'s x-intercepts is $(1,0)$.

B4 (10 pts) Sketch the graph of $G(x)=\dfrac{5x^3-8x^2-43x+78}{x^3-12x+16}$. Hint: $G(x)$ looks exactly like $F(x)$, from #9, except it has a hole.

B5 Let $f(x)=3x^2-5x+10$

- **a.** (5 pts) Solve $f(x)=0$ by completing the square. State its x- and y- intercepts as ordered pairs.
- **b.** (5 pts) Re-write f in the form $f(x)=a(x-h)^2+k$. Give its vertex.