121 E3 TAKE-HOME / Writing Project #3

(1) $4x^5$ in control

(2) Descartes:

$$f(x) = 4x^5 - 12x^4 - x^3 + 44x^2 - 57x + 22$$

1 2 3 4

4, 2, or 0 positive roots

$$f(-x) = -4x^5 - 12x^4 + x^3 + 44x^2 + 57x + 22$$

1

<u>Exactly</u> one negative root.

(Look for it, first!

(3) $\frac{p}{q} : \frac{22}{4} \longrightarrow$ $\pm 1, \pm 2, \pm 11, \pm 22,$

$\pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \pm \frac{11}{2}, \cancel{\pm \frac{22}{2}},$

$\pm \frac{1}{4}, \cancel{\pm \frac{2}{4}}, \pm \frac{11}{4}, \cancel{\pm \frac{22}{4}}$

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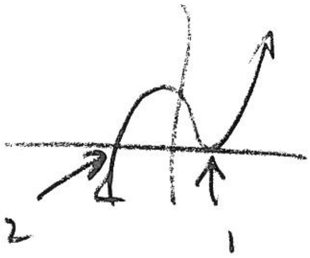
MP3

(3) Grapher shows



So, using #2, go after the one negative root, first:

$$
\begin{array}{r|rrrrrr}
-2 & 4 & -12 & -1 & 44 & -57 & 22 \\
 & & -8 & 40 & -78 & +68 & -22 \\
\hline
1 & 4 & -20 & 39 & -34 & 11 & 0 \\
 & & 4 & -16 & 23 & -11 & \\
\hline
1 & 4 & -16 & 23 & -11 & 0 & \\
 & & 4 & -12 & 11 & & \\
\hline
 & 4 & -12 & 11 & 0 & &
\end{array}
$$

Check if $4x^2 - 12x + 11$ has real zeros:

$a = 4,\ b = -12,\ c = 11 \longrightarrow$

$b^2 - 4ac = (-12)^2 - 4(4)(11) = 144 - 176 = -32$

$\sqrt{-32} = i\sqrt{32} = 4i\sqrt{2}$

$2\,|\,32$
$2\,|\,16$
$2\,|\,8$
$2\,|\,4$
2

No real zeros, pluss I started on #6 already.

ANS

(4) $\boxed{x = 1,\ m = 2;\ x = -2,\ m = 1}$

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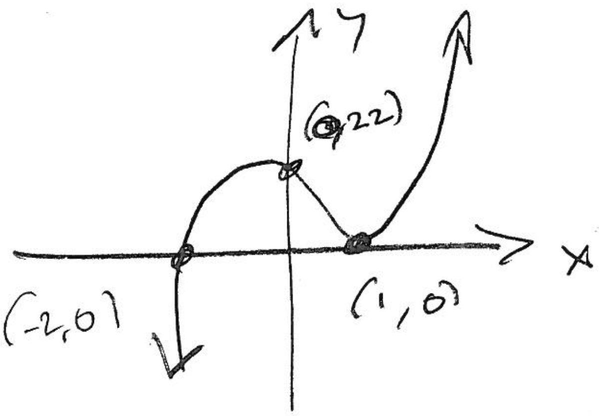
(5) $\boxed{f(x) = (x-1)^2(x+2)(4x^2-12x+11)}$

(6) $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{12 \pm 4i\sqrt{2}}{2(4)} = \boxed{\dfrac{3 \pm i\sqrt{2}}{2}}$

So $\boxed{f(x) = 4(x-1)^2(x+2)\left(x - \dfrac{3+i\sqrt{2}}{2}\right)\left(x - \dfrac{3-i\sqrt{2}}{2}\right)}$

(7)



(8)

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(8) $R(x) = \dfrac{2x^2 - x - 3}{x^2 + 2x - 15} = \dfrac{(2x-3)(x+1)}{(x+5)(x-3)}$

$\mathcal{D} = \mathbb{R} \setminus \{-5, 3\}$ No holes

$\boxed{x = -5,\ x = 3 \quad \text{V.A.}}$

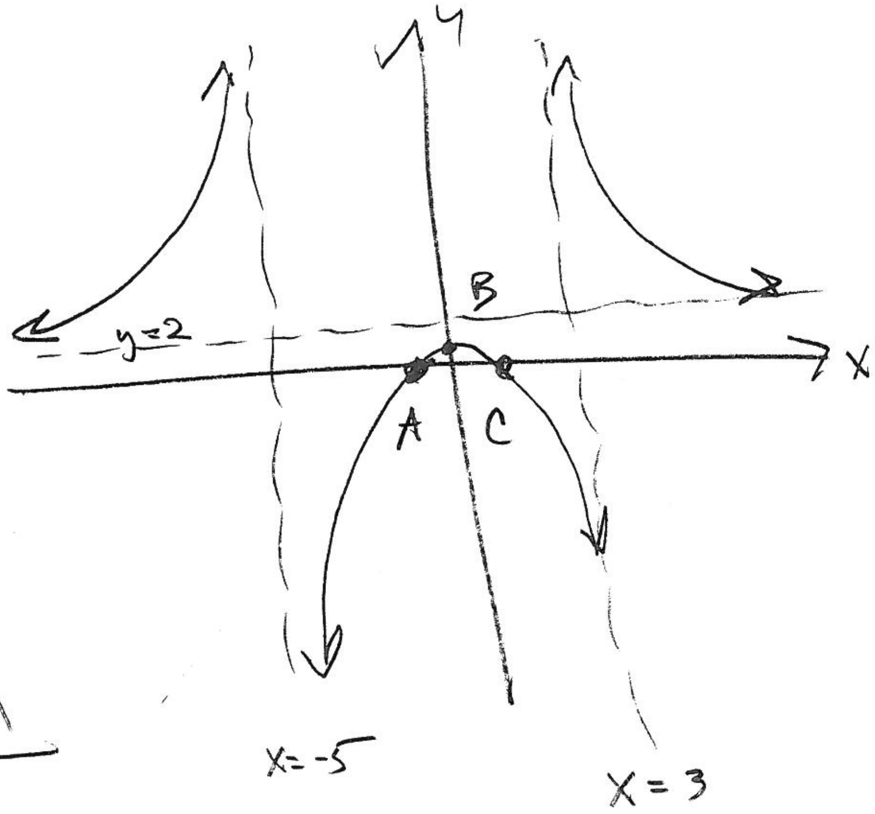
$2x - 3 = 0 \qquad x + 1 = 0$

$x = \frac{3}{2} \qquad x = -1$

$\boxed{\left(\frac{3}{2}, 0\right), \quad (-1, 0) \quad \text{x-int}}$

$R(0) = \dfrac{-3}{-15} = \dfrac{1}{5}$

$\boxed{\text{y-int} \approx \left(0, \tfrac{1}{5}\right)}$



$R(x) \xrightarrow{x \to \infty} \dfrac{2x^2}{x^2} =$

$\boxed{y = 2 \quad \text{H.A.}}$

$A = (-1, 0)$

$B = \left(0, \frac{1}{5}\right)$

$C = \left(\frac{3}{2}, 0\right)$

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(9) $g(x) = \dfrac{2x^3-9x^2+x+12}{x^3-2x^2-23x+60}$ is $R(x)$ with a hole.

We peel away the $R(x)$, inside by breaking down numerator/denominator

We KNOW this

$$g(x) = \frac{(2x-3)(x+1)(x-?)}{(x+5)(x-3)(x-?)}$$

$$\frac{(2(-4)-3)(-4+1)}{(-4+5)(-4-3)} = \frac{(-8-3)(-3)}{(1)(-7)} = \frac{33}{-7}$$

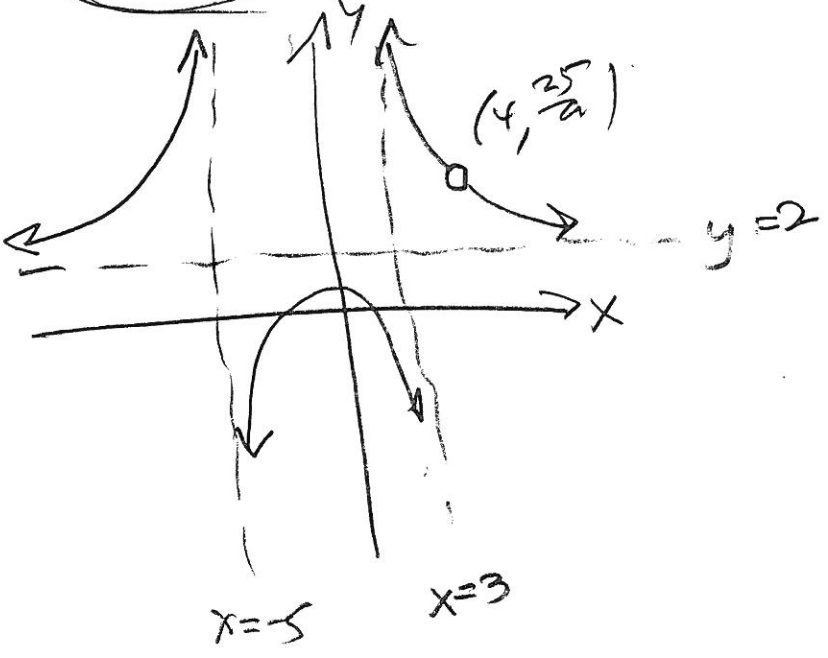
$\boxed{M1}$

$$\begin{array}{r|rrrr}
-5 & 1 & -2 & -23 & 60 \\
 & & -5 & 35 & -60 \\
\hline
3 & 1 & -7 & 12 & \\
 & & 3 & -12 & \\
\hline
 & 1 & -4 & &
\end{array}$$

$\rightarrow$ so $x-? = x-4$!

$$R(4) = \frac{(2(4)-3)(4+1)}{(4+5)(4-3)} = \frac{25}{9}$$

$\left(4, \frac{25}{9}\right)$ = HOLE IN $g(x)$!



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(110) $h(x) = -\dfrac{x^2-6x+3}{x-4} = \dfrac{-x^2+6x-3}{x-4} =$

$D = \mathbb{R} \setminus \{4\}$ Check for hole:

V..

$x^2-6x+3=0$

$x^2-6x = -3$

$x^2-6x+3^2 = -3+9$

$(x-3)^2 = 6$

$x-3 = \pm\sqrt{6}$

$x = 3 \pm \sqrt{6}$ → 5.4495, .5505

No cancellation with x-4.

x-int: $(3-\sqrt{6},0)$

$(3+\sqrt{6},0)$

$\boxed{V.A.: x=4}$

Oblique (slant) asymptote:

$$\begin{array}{r|rrr} 4 & -1 & 6 & -3 \\ & & -4 & 8 \\ \hline & -1 & 2 & 5 \end{array}$$

$\boxed{y=-x+2 \text{ is O.A.}}$

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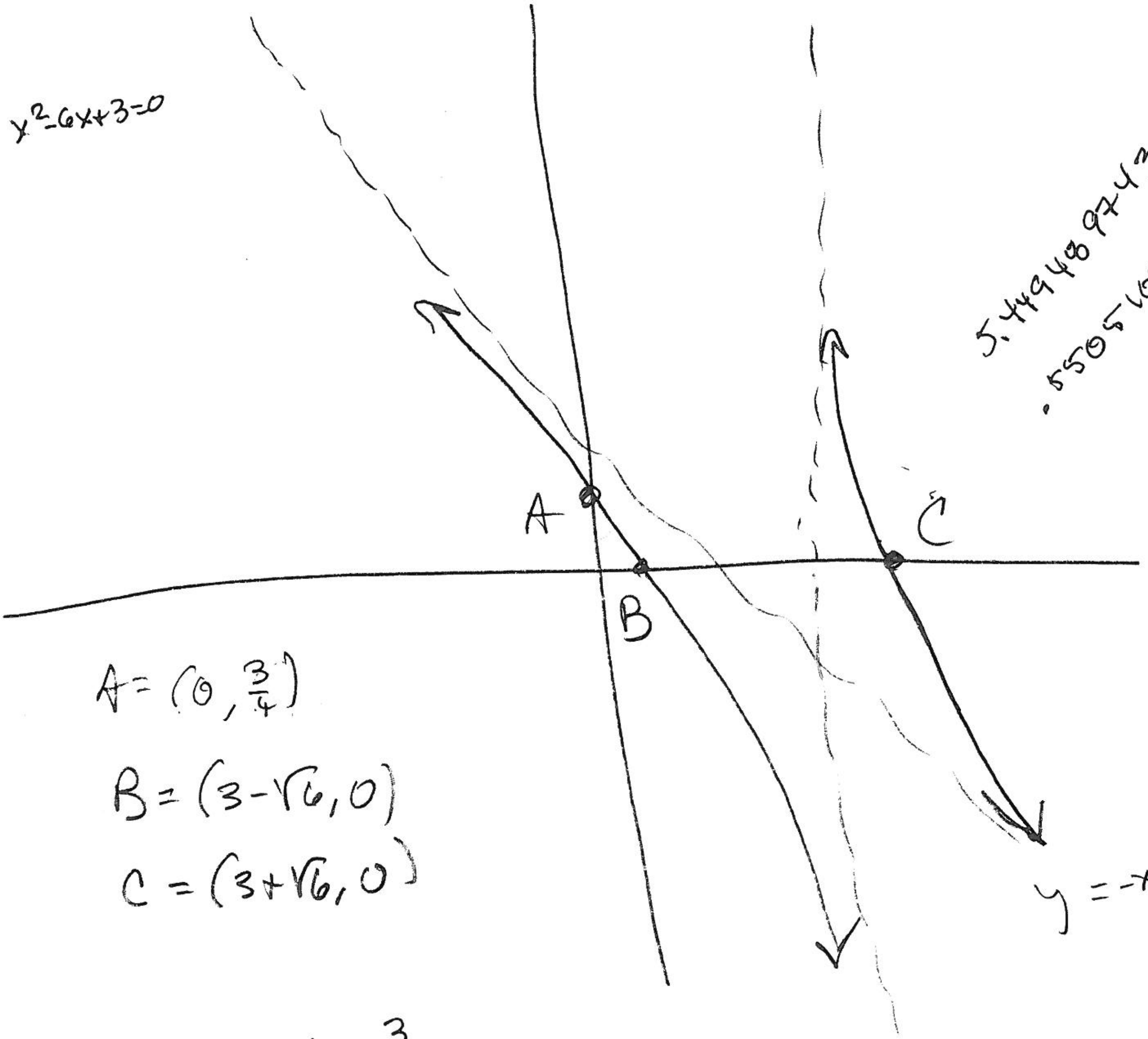
$x^2-6x+3=0$

5.449489743

.550510257



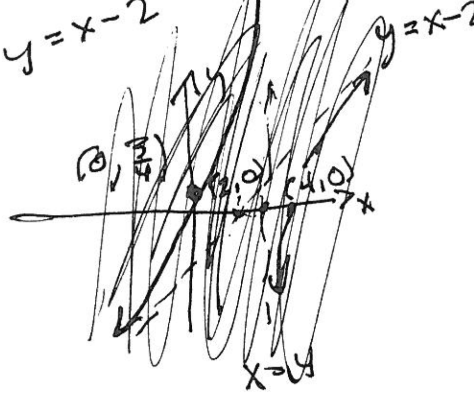
$A=(0,\frac{3}{4})$

$B=(3-\sqrt{6},0)$

$C=(3+\sqrt{6},0)$

$$\begin{array}{r|rrr} 4 & 1 & -6 & 3 \\ & & 4 & \\ \hline & 1 & -2 & \end{array}$$

$y=x-2$



$\frac{x^2-6x+3}{x-4}$

$(0,-\frac{3}{4})$

$A=(0,-\frac{3}{4})$

$B=(3-\sqrt{6},0)$

$C=(3+\sqrt{6},0)$