

121

E3

FALL, 2016

SOLNS

① 10pts

$$(x-2)^3 (x-(6+2i))^2 (x-(6-2i))^2 (x+3)$$

② 10pts

$$\begin{array}{r} -2 \overline{) 7 \quad -2 \quad 11 \quad 1 \quad -173 \quad -4} \\ \underline{-14 \quad 32 \quad -86 \quad 170 \quad 6} \\ 7 \quad -16 \quad 43 \quad -85 \quad -3 \end{array}$$

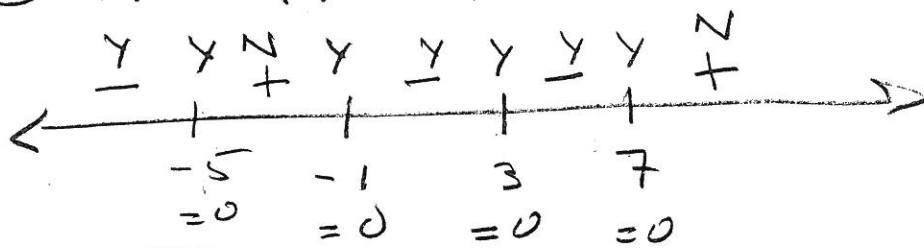
$$2 = P(-2)$$

③ 5pts

$$7x^5 - 2x^4 + 11x^3 + x^2 - 173x - 4$$

$$= (x+2)(7x^4 - 16x^3 + 43x^2 - 85x - 3) - 2$$

④ a)  $(x-3)^2 (x+5)(x-7)(x+1) = f(x) \leq 0$  10pts

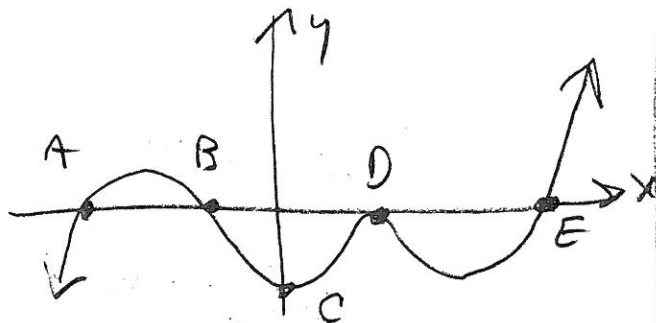


$$\leq 0$$

10pts

$$(-\infty, -5] \cup [-1, 7]$$

b)



$$A = (-5, 0)$$

$$B = (-1, 0)$$

$$C = (0, -315)$$

$$D = (3, 0)$$

$$E = (7, 0)$$

10pts

(2)

(4) (c) 5pts  $\frac{(x-3)^2(x+5)}{(x-7)(x+1)} \geq 0$

←  $\begin{array}{ccccccc} \text{N} & \text{Y} & \text{Y} & \text{N} & \text{N} & \text{Y} & \text{N} & \text{Y} & \text{Y} \\ \hline & & & & & & & & \\ & -5 & & -1 & & 3 & & 7 & \\ & = 0 & & \star & & = 0 & & \star & \end{array}$  →  $\geq 0$

$$\boxed{[-5, -1) \cup \{-1, 3\} \cup (7, \infty)}$$

(5)  $f(x) = 4x^5 - 9x^3 + 8x^2 - 9x + 6$

(a) 4 sign changes = 4, 2, 0 positive roots.

5pts  $f(-x) = -4x^5 + 9x^3 + 8x^2 + 9x + 6$

1 sign change

1 negative root

(b)  $\frac{p}{q} \leftrightarrow \frac{6}{4}$  5pts

$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{3}{2}, \pm \frac{6}{2},$

$\pm \frac{1}{4}, \pm \frac{2}{4}, \pm \frac{3}{4}, \pm \frac{6}{4} = \pm \frac{3}{2}$

(5) (c) BONUS

$$\begin{array}{r}
 2 \overline{) 4 \quad 0 \quad -9 \quad 8 \quad -9 \quad 6} \\
 \underline{\phantom{2} 8 \quad 16 \quad 14 \quad 44 \quad 70} \\
 4 \quad 8 \quad 7 \quad 22 \quad 35 \quad 76
 \end{array}$$

$\nearrow$  All positive in Bottom Row  $\rightarrow$   
 $x=2$  is upper bound on positive zeros

~~$f(x) = -4x^5 + 9x^3 + 8x^2 + 9x + 6$~~

$$\begin{array}{r}
 -3 \overline{) 4 \quad 0 \quad -9 \quad 8 \quad -9 \quad 6} \\
 \underline{\phantom{-3} 12 \quad 12 \quad 36 \quad -81 \quad 219 \quad -630} \\
 -4 \quad -12 \quad 27 \quad -73 \quad 210 \quad -624
 \end{array}$$

Signs alternate  $\Rightarrow$   
 $x=-3$  is lower bound on roots

(4)

(6)

$$\begin{array}{r}
 \downarrow 4 \quad 0 \quad -9 \quad 8 \quad -9 \quad 6 \\
 \quad 4 \quad 4 \quad -5 \quad 3 \quad 6 \\
 \hline
 \downarrow 4 \quad 4 \quad -5 \quad 3 \quad -6 \quad 0 \\
 \quad 4 \quad 8 \quad 3 \quad -6 \quad 0 \\
 \hline
 -2 \downarrow 4 \quad 8 \quad 3 \quad 6 \quad 0 \\
 \quad -8 \quad 0 \quad -6 \quad 0 \\
 \hline
 4 \quad 0 \quad 3 \quad 0
 \end{array}$$

10 P's

$$x=1, m=2$$

$$x=-2, m=1$$

$4x^2+3$  is irreducible over the reals.

$$f(x) = (x-1)^2(x+2)(4x^2+3)$$

(7)

$$4x^2+3=0$$

$$4x^2 = -3$$

$$x^2 = -\frac{3}{4}$$

$$x = \pm \sqrt{-\frac{3}{4}} = \pm i \frac{\sqrt{3}}{2}$$

$$f(x) = 4(x-1)^2(x+2)\left(x - \frac{\sqrt{3}}{2}i\right)\left(x + \frac{\sqrt{3}}{2}i\right)$$

5 P's

(5) ~~(5)~~

(8)

$$2x^2 + x - 6 = (2x - 3)(x + 2)$$

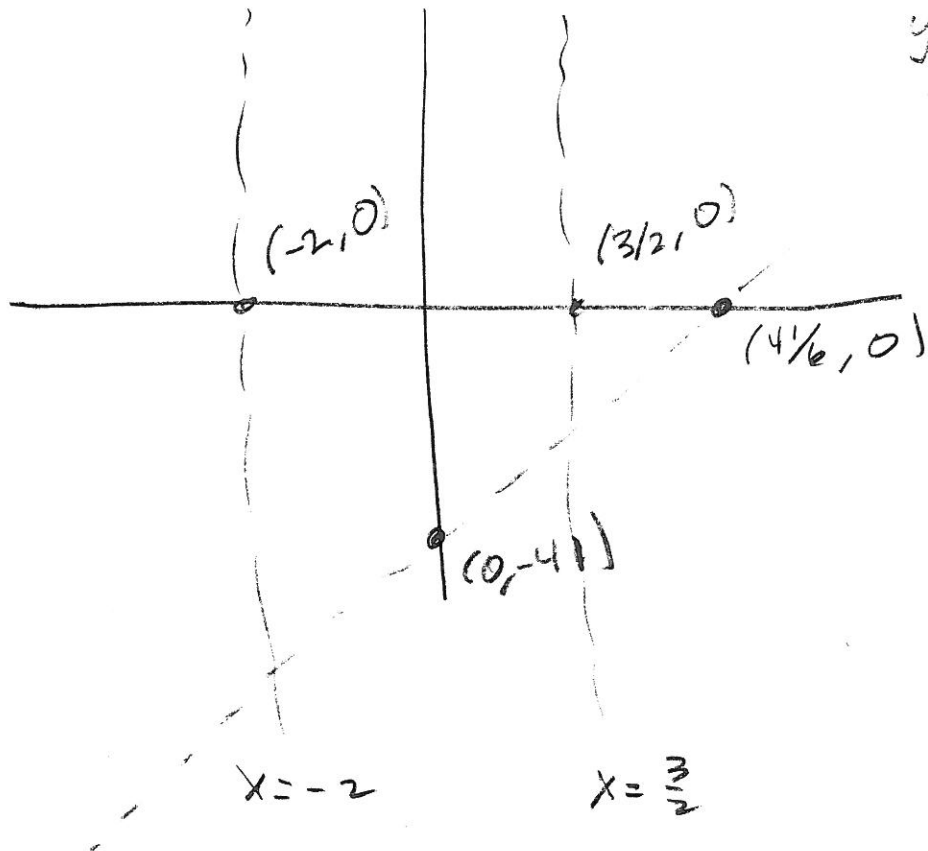
sp 13

$$x = \frac{3}{2}, x = -2 \text{ V.A.}$$

$$\begin{array}{r}
 2x^2 + x - 6 \quad \overline{) \quad 6x^3 - 35x^2 + 64x - 35} \\
 \underline{-(6x^3 + 3x^2 - 18x)} \phantom{- 35} \\
 -38x^2 + 82x - 35 \\
 \underline{-38x^2 - 19x} \\
 \phantom{-38x^2 - 19x} - 101x + 35
 \end{array}$$

$$y = 6x - 41$$

is oblique  
asymptote



(0) (A)

$$\textcircled{9} F(x) = \frac{5x^2 + 2x - 39}{x^2 + 2x - 8} = \frac{(5x - 13)(x + 3)}{(x + 4)(x - 2)}$$

$$\begin{aligned} 5x^2 + 2x - 39 &= 5x^2 + 15x - 13x - 39 \\ &= 5x(x + 3) - 13(x + 3) \\ &= (x + 3)(5x - 13) \end{aligned}$$

$$\begin{aligned} a &= 5, b = 2, c = -39 \\ b^2 - 4ac &= 4 - 4(5)(-39) \\ &= 4 + 780 \\ &= 784 \\ \sqrt{784} &= 28 \end{aligned}$$

$$x^2 + 2x - 8 = (x + 4)(x - 2)$$

$$a = 1, b = 2, c = -8$$

$$b^2 - 4ac = 2^2 - 4(1)(-8) = 4 + 32 = 36$$

$$\sqrt{36} = 6$$

$$D = \mathbb{R} \setminus \{-4, 2\}$$

$$\text{V.A. : } x = -4, x = 2$$

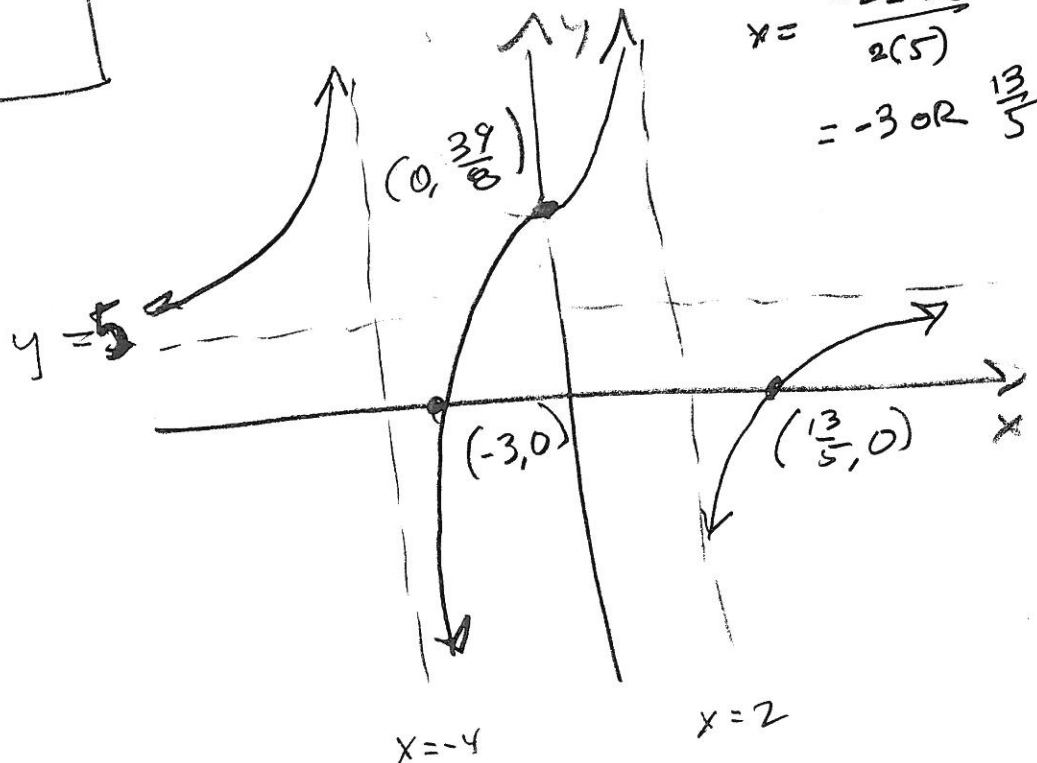
$$x\text{-int: } \left(\frac{13}{5}, 0\right), (-3, 0)$$

$$y\text{-int: } \left(0, \frac{39}{8}\right)$$

$$\text{H.A. : } y = 2$$

10 pts

$$x = \frac{-2 \pm 6}{2(1)} = -3 \text{ or } \frac{13}{5}$$



(7) (A)

B1

$$(x - (3 - \sqrt{5})) (x - (3 + \sqrt{5})) (x - (2 + 7i))^2 (x - (2 - 7i))^2 (x + 5)^{17}$$

B2

$$|3x - 7| - 2 \geq 8$$

$$|3x - 7| \geq 10$$

$$3x - 7 \geq 10 \quad \text{OR} \quad 3x - 7 \leq -10$$

$$3x \geq 17$$

$$3x \leq -3$$

$$\left\{ x \mid x \geq \frac{17}{3} \quad \text{OR} \quad x \leq -1 \right\}$$

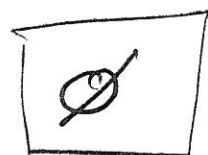
$$= (-\infty, -1] \cup \left[ \frac{17}{3}, \infty \right)$$



(b) sp's

$$|3x - 7| + 8 < 2$$

$$|3x - 7| < -6 \quad \text{Never!}$$

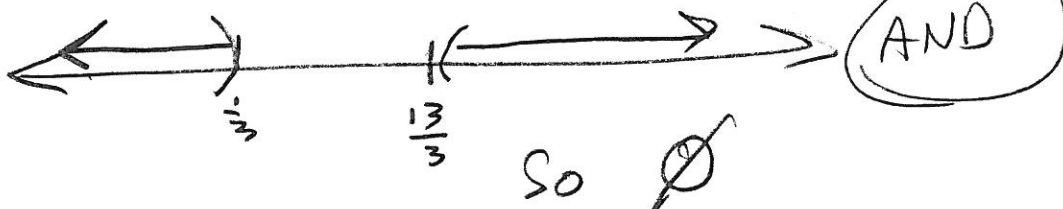


FOR THE SLOW:

$$3x - 7 < -6 \quad \text{AND} \quad 3x - 7 > 6$$

$$3x < 1 \quad \text{AND} \quad 3x > 13$$

$$\left\{ x \mid x < \frac{1}{3} \quad \text{AND} \quad x > \frac{13}{3} \right\} = \emptyset!$$



B3

$$R(x) = \frac{6x^3 - 35x^2 + 64x - 35}{2x^2 + x - 6} = \frac{(x-1)(2x-5)(3x-7)}{(2x-3)(x+2)}$$

$$\begin{array}{r} \underline{1} \quad 6 \quad -35 \quad 64 \quad -35 \\ \quad \quad 6 \quad -29 \quad 35 \\ \hline 6 \quad -29 \quad 35 \end{array}$$

V.A.  $x = -2, x = \frac{3}{2}$

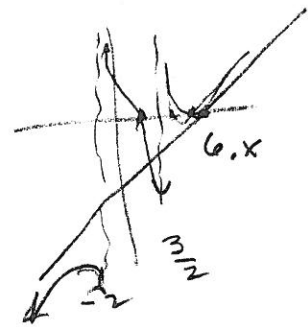
x-Int:  $-7$

$(1, 0), (\frac{7}{3}, 0), (\frac{5}{2}, 0)$

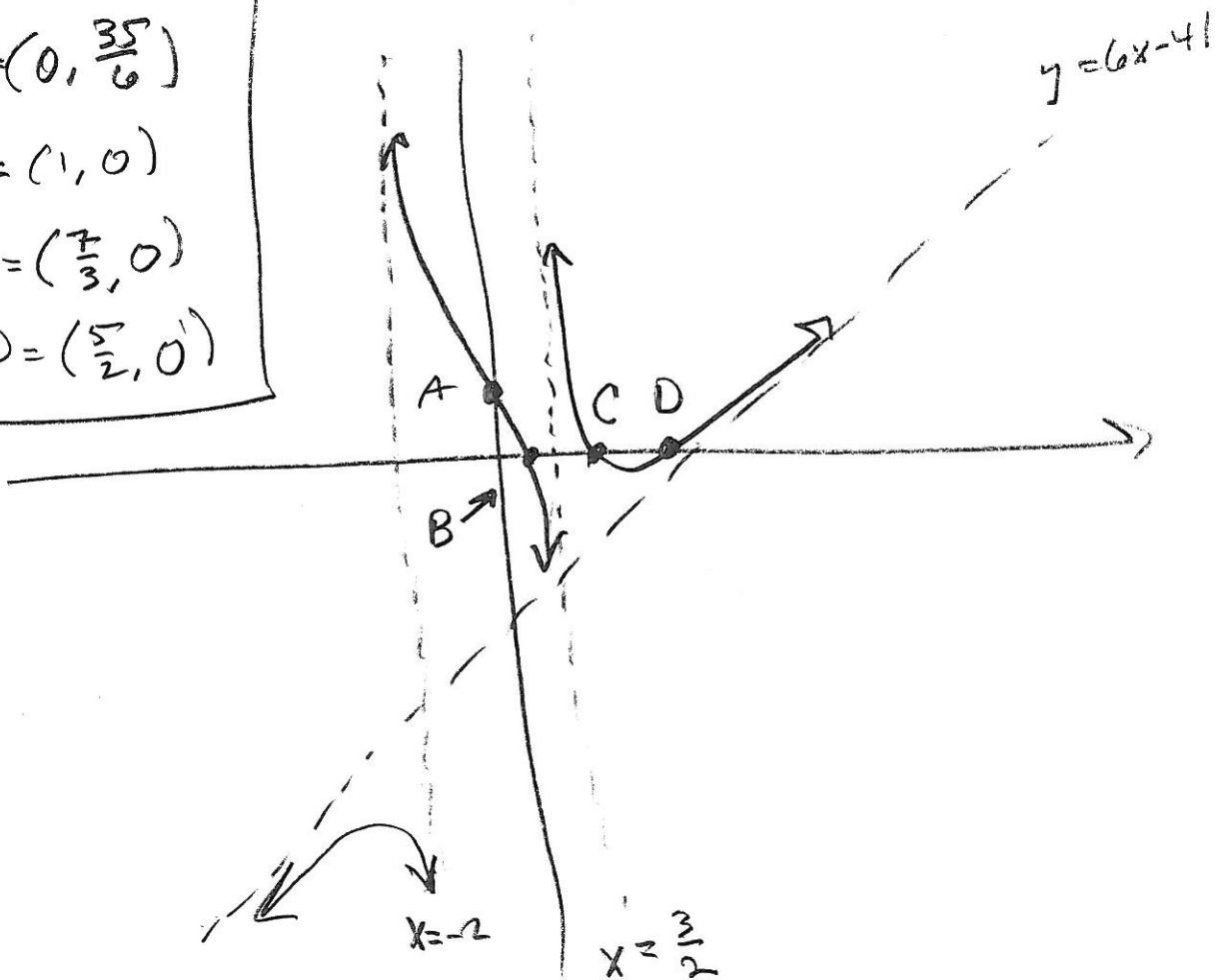
$(2)(3)(7)(5)$

$$\begin{aligned} 6x^2 &= 14x - 15x + 35 \\ &= 2x(3x-7) - 5(3x-7) \\ &= (2x-5)(3x-7) \end{aligned}$$

$\frac{41}{6} = 6 + \frac{5}{6}$



- $A = (0, \frac{35}{6})$
- $B = (1, 0)$
- $C = (\frac{7}{3}, 0)$
- $D = (\frac{5}{2}, 0)$





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B4 10 pts

9

$$x^3 - 12x + 16$$

$$= (x^2 + 2x - 8)(x - ?)$$

$$= (x+4)(x-2)(x-?)$$

But it's  $x-4$ , since  $(4)(-2)(x) = 16 \rightarrow x=4$

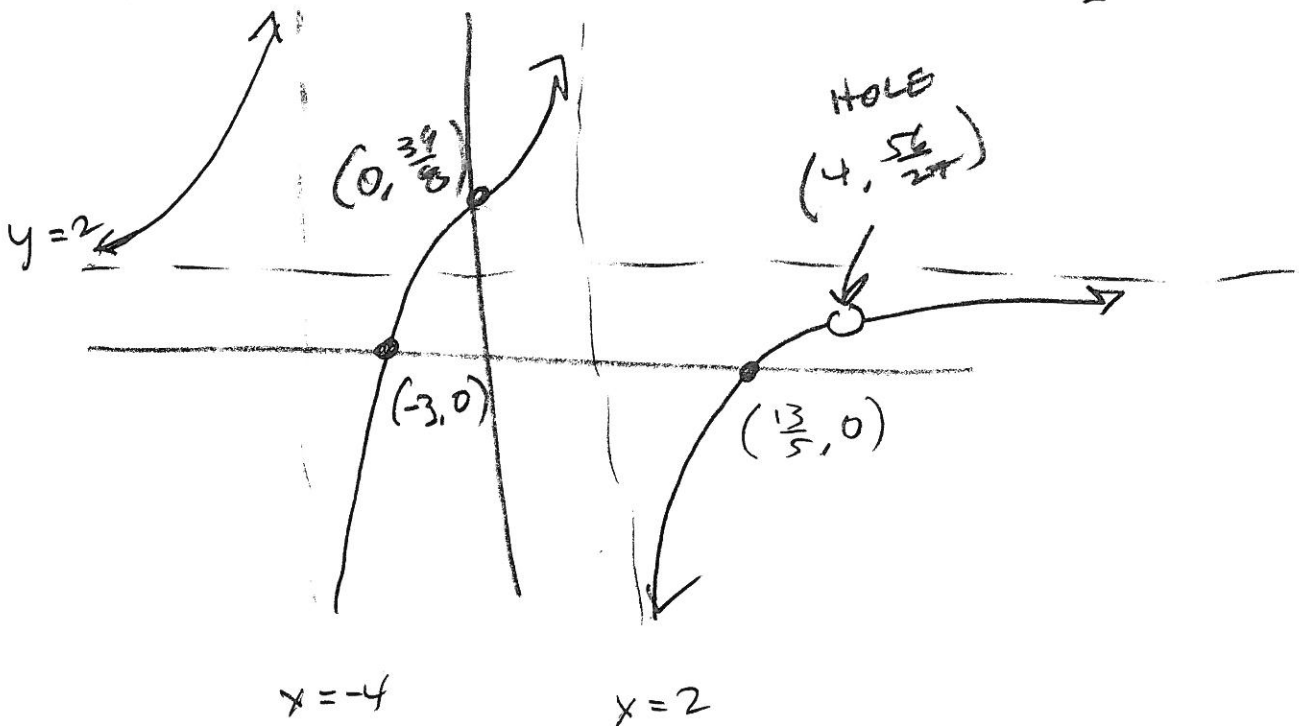
So,  $x=4$ ,  $\frac{(5x-13)(x+3)}{(x+4)(x-2)}$

$$\frac{(5(4)-13)(4+3)}{(4+4)(4-2)}$$

$$= \frac{(20-13)(7)}{(8)(2)}$$

$$= \frac{56}{27}$$

Hole:  $(4, \frac{56}{27})$



BS ~~10 pts~~

2) 5 pts

$$3x^2 - 5x + 10$$

$$= 3\left(x^2 - \frac{5}{3}x\right) + 10$$

$$= 3\left(x^2 - \frac{5}{3}x + \left(\frac{5}{6}\right)^2\right) + 10 - 3\left(\frac{25}{36}\right)$$

$$10 - \frac{25}{12} = \frac{120 - 25}{12} = \frac{95}{12}$$

10 ~~11~~

ANS To (b):

$$= 3\left(x - \frac{5}{6}\right)^2 + \frac{95}{12}$$

$$(h, k) = \left(\frac{5}{6}, \frac{95}{12}\right)$$

$$\text{SET } = 0 \Rightarrow 3\left(x - \frac{5}{6}\right)^2 = -\frac{95}{12}$$

$$\sqrt{\frac{95}{12}}$$

$$\left(x - \frac{5}{6}\right)^2 = -\frac{95}{36}$$

$$x - \frac{5}{6} = \pm \sqrt{-\frac{95}{36}} = \pm i \frac{\sqrt{95}}{6}$$

$$x = \frac{5 \pm \sqrt{95} i}{6}$$