Test 2 Fall '16

MAT 121
100 Points Covers Chapter 2

Name KEY
NO GRAPHING CALCULATORS!!!

Do your work on separate paper, provided. Only write on ONE SIDE of each sheet.

All that you should put on THIS sheet is your NAME. One final warning about leaving margins at the top, especially where the staple goes!!! No foolin'. If it's not formatted the way I specify, I'm giving zero credit.

LEAVE ROOM BETWEEN AND AROUND PROBLEMS. USE JUST ONE COLUMN. DON'T CRAM 2 OR 3 COLUMNS ONTO YOUR WORK SHEETS.

1. Consider the relation $f = \{(1,3),(2,7),(1,5)(3,-2)\}$.

   a. (5 pts) Is f a function? If not, why not?

   b. (5 pts) What is the domain of f ?

   c. (5 pts) What is the range of f ?

   d. (5 pts) Is f one-to-one? If not, explain why not.

$2\,|\,180$
$2\,|\,90$
$3\,|\,45$
$3\,|\,15$
5

$5(x-2)^2-9$
$= 5(x^2-4x+4)-9$
$= 5x^2-20x+11$
$a=5,\ b=-20,\ c=11$
$b^2-4ac = (-20)^2-4(5)(11)$
$= 400-220 = 180$
$x = \dfrac{20 \pm 6\sqrt{5}}{10}$
$= \dfrac{10 \pm 3\sqrt{5}}{5}$

3. Let $f(x)=\sqrt{x+4}$ and $g(x)=\dfrac{7x^2-17x+\pi}{x+5}$.

   a. (5 pts) Write the function $\dfrac{f}{g}$. Do not simplify.

   b. (5 pts) What is the domain of $\dfrac{f}{g}$?

   c. (5 pts) Write the function $f \circ g$. Do not simplify.

   d. (5 pts) What is the domain of $f \circ g$?

4. (5 pts) Simplify the difference quotient for $f(x)=2x^2-3x$.

Bonus (5 pts) Pass to the limit as h approaches zero, and show me some calculus to go with #4.

5. (5 pts) Draw a picture for the difference quotient for $f(x)=\sqrt{x}$. Describe what the difference quotient represents, in words. Do not simplify your difference quotient. That's a bonus problem, later on.

6. Let $g(x)=2\sqrt{-7x+21}-11$.

   a. (10 pts) Sketch the graph of $g(x)$, by transforming the basic function $f(x)=\sqrt{x}$. I want to see 2 points labeled in the graph of g – preferably starting with $(0,0)$, $(1,1)$ and $(4,2)$ – and track where those points are moved to after every step, as demonstrated in video. This will take 5 graphs, counting the first graph of $f(x)=\sqrt{x}$ as the first.

   b. (5 pts) State the domain and range of $g(x)$, based on your final graph.

   c. (5 pts) Find the x- and y-intercept of $g(x)$, and label them, clearly, on the graph.

7. Let $r(x)=5(x-2)^2-9$.

   a. (10 pts) Sketch the graph of $r(x)=5(x-2)^2-9$ by transforming the basic function $f(x)=x^2$. I want to see 3 points labeled in the graph of f, and I want you to track where those points are moved to after every step, as demonstrated in videos.

   b. (5 pts) Find the x- and y-intercepts and add them to your final sketch, above. For x-intercept, leave final answer in simplified radical form.

8. (5 pts) Prove that $\dfrac{x-6}{x+6}$ is one-to-one.

9. (5 pts) Suppose y is jointly proportional to the cube of x and the square root of z, and inversely proportional to u. Write an equation for this relationship between y, x, z, and u.

10. (5 pts) Explain why $x=y^2-2$ does *not* define y as a function of x.

Answer two of the following for **Bonus** (5 pts each)

B1: Simplify the difference quotient for the function $f(x)=\sqrt{x}$. Then pass to the limit, as h approaches zero.

B2: Complete the square to re-write the function $h(x)=7x^2-3x+2$ in the form $a(x-h)^2+k$. What is the vertex?

B3: What is the domain of $r(x)=\dfrac{x-3}{x^2-5x+6}$?

B4: What is the domain of $w(x)=\dfrac{x^{77}-5x^{12}+17x}{\sqrt{5-10x}}$

B5: Prove that $g(x)=-\sqrt{10-5x}+7$ is 1-to-1.

B6: Given $g(x)=-\sqrt{10-5x}+7$, find what $g^{-1}(x)$ is. $\left(\text{Hint}: (-x+7)^2=(x-7)^2\right)$

B7: Given $g(x)=-\sqrt{10-5x}+7$, find the domain and range of $g^{-1}(x)$.

B8: $|-5x+7| \geq 8$ & $|-5x+7| \geq -8$

Test 2 Fall '16

FALL '16

$f = \{(1,3), (2,7), (1,5), (3,-2)\}$

) No $(1,3), (1,5)$ pair $x=1$ w/ 3 & 5

(b) $\mathcal{D}(f) = \{1, 2, 3\}$

$\mathcal{D}(f) = [-4, \infty)$

$\mathcal{D}(g) = \mathbb{R} \setminus \{-5\}$

(c) $\mathcal{R}(f) = \{3, 7, 5, -2\}$

(d) No. F isn't a function!

(3) $f(x) = \sqrt{x+4}$ $g(x) = \dfrac{7x^2 17x + \pi}{x+5}$

(a) $\dfrac{f}{g} = \dfrac{\sqrt{x+4}}{\dfrac{7x^2 17x + \pi}{x+5}}$

(b) $\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid$ UGH!

(c) $f \circ g = \sqrt{\dfrac{7x^2 17x + \pi}{x+5} + 4}$

(4) BIG

121 TEST 2

Test 2 Fall '16

(4) $f(x) = 2x^2-3x \longrightarrow$

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2-3(x+h)-(2x^2-3x)}{h}$$

$$= \frac{2(x^2+2xh+h^2)-3x-3h-2x^2+3x}{h}$$

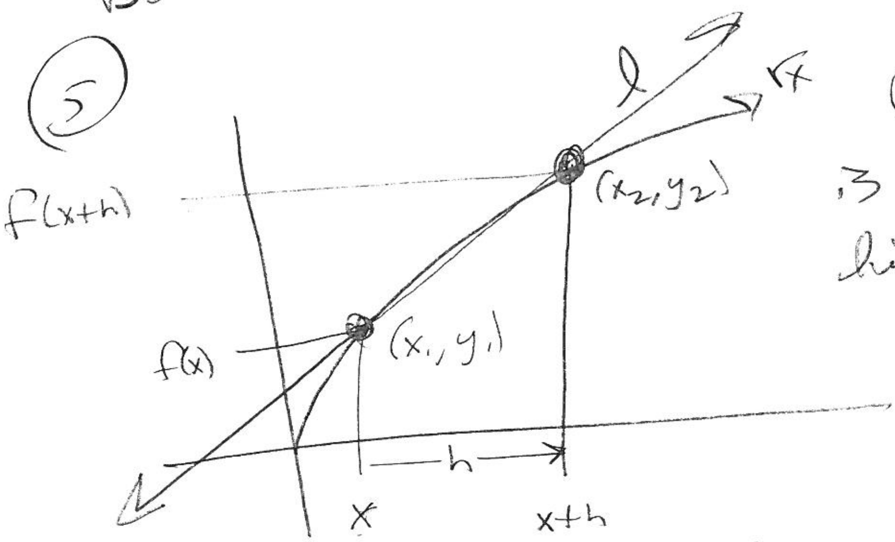
$$= \frac{2x^2+4xh+2h^2-3x-3h-2x^2+3x}{h}$$

$$= \frac{4xh+2h^2-3h}{h} = \frac{h(4x+2h-3)}{h}$$

$$= \boxed{4x+2h-3}$$

Bonus: $\xrightarrow{h \to 0}$ $\boxed{4x-3 = f'(x)}$

(5)



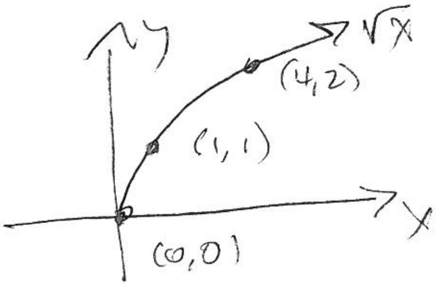
Difference Quotient is slope of the secant line, l.

$$m_{sec} = \frac{y_2-y_1}{x_2-x_1} = \frac{f(x+h)-f(x)}{h}$$

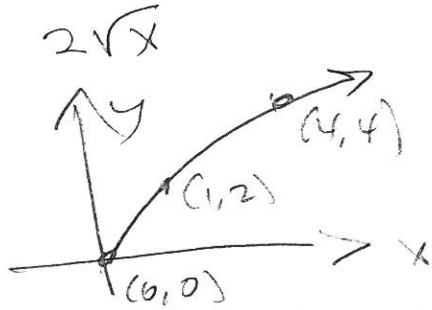
121 TEST 2

Test 2 Fall '16

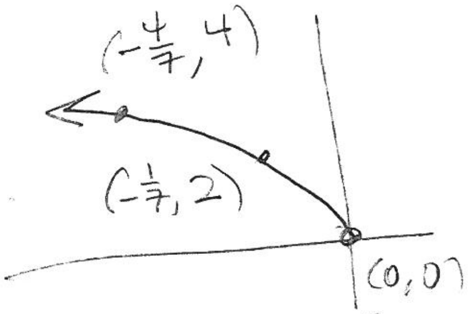
6 (a) $g(x) = 2\sqrt{-7x+21} - 11 = 2\sqrt{-7(x-3)} - 11$



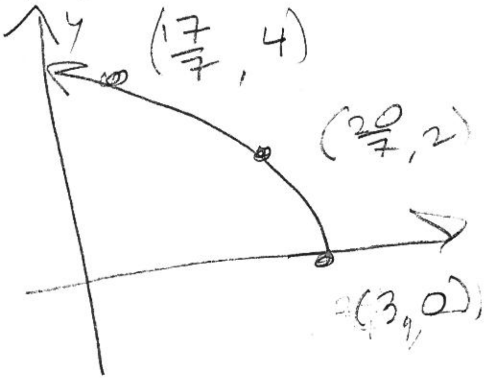
(2) $2\sqrt{x}$



(3) $2\sqrt{-7x}$

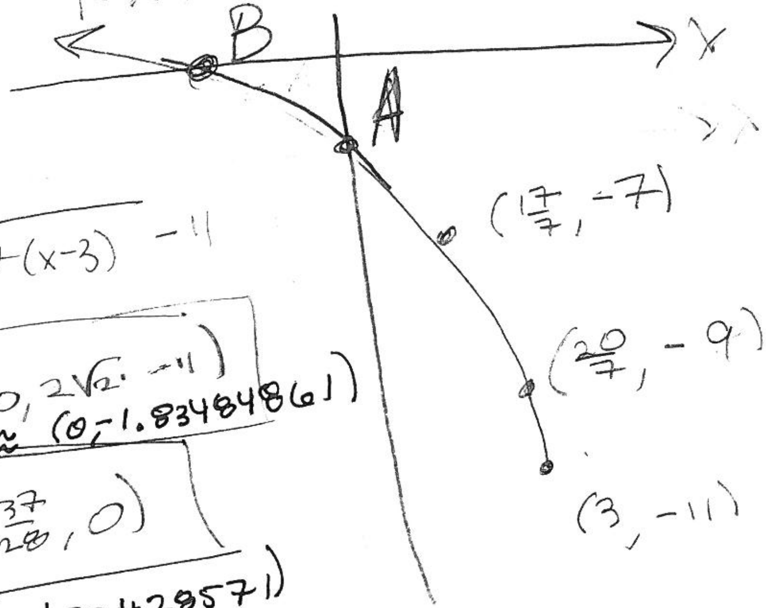


(4)



$-\frac{1}{7} + 3 =$

$-\frac{4}{7} + \frac{21}{7} =$



$2\sqrt{21} - 11$

(2)(4.x) − 11

is negative

$2\sqrt{-7(x-3)} - 11$

$A = (0, 2\sqrt{21} - 11)$

$\approx (0, -1.834848 61)$

$B = (-\frac{37}{28}, 0)$

$\approx (-1.32142857 1)$

FIND B

$2\sqrt{-7x+21} =$

$2^2(-7x+21) = 11^2$

$-28x + 84 = 121$

$-28x = 37$

$x = -\frac{37}{28}$?

(b) $D(g) = (-\infty, 3]$

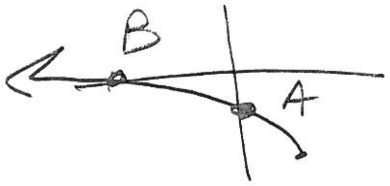
$R(g) = [-11, \infty)$

121 TEST 2

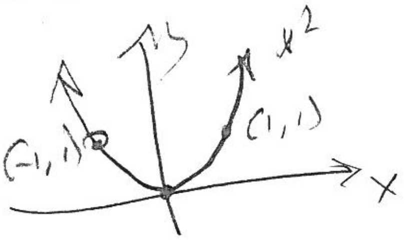
Test 2 Fall '16

(6) (c) See graph $A = \left(-\frac{37}{28}, 0\right)$

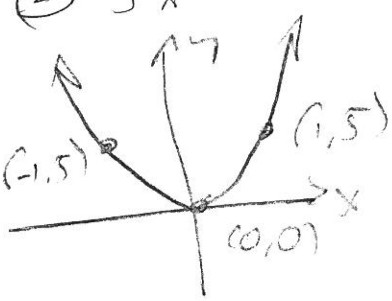
$B = \left(0, 2\sqrt{21} - 11\right)$



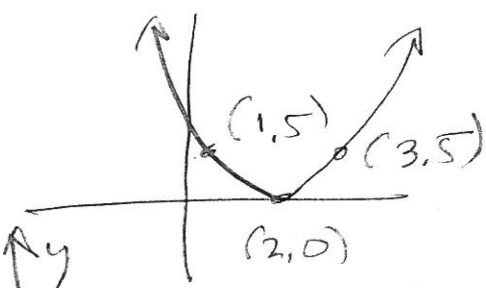
(7) $5(x-2)^2 - 9$

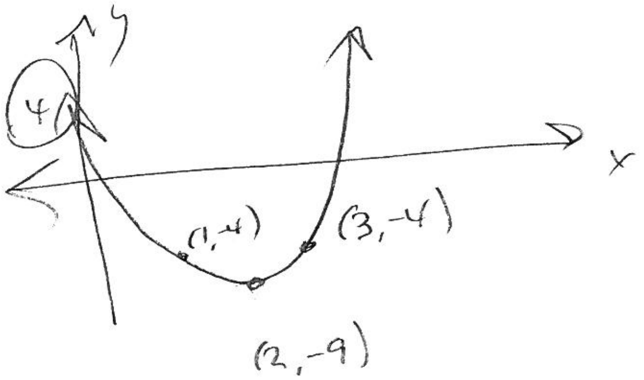


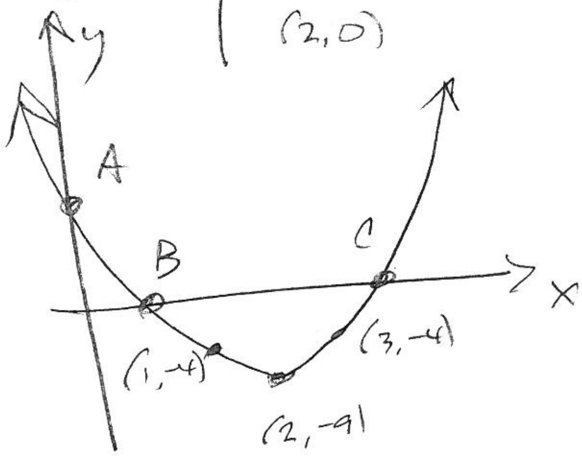
(2) $5x^2$



(3) $5(x-2)^2$







A: $(0, 5(-2)^2 - 9)$

$= (0, 20 - 9) = \boxed{(0, 11) = A}$

B: $5(x-2)^2 - 9 = 0$

$5(x-2)^2 = 9$

$(x-2)^2 = \frac{9}{5}$

$x - 2 = \pm\sqrt{\frac{9}{5}} = \pm\frac{3}{\sqrt{5}} = \pm\frac{3\sqrt{5}}{5}$

$\Rightarrow x = 2 \pm \frac{3\sqrt{5}}{5}$

$\boxed{B = \left(2 - \frac{3\sqrt{5}}{5}, 0\right), C = \left(2 + \frac{3\sqrt{5}}{5}, 0\right)}$

121 TEST 2

Test 2 Fall '16

(8) $f(x) = \frac{x-6}{x+6}$ is 1-to-1

<u>Proof</u> Suppose $f(x_1) = f(x_2)$.

Then $\frac{x_1 - 6}{x_1 + 6} = \frac{x_2 - 6}{x_2 + 6}$

$\Rightarrow (x_1 - 6)(x_2 + 6) = (x_2 - 6)(x_1 + 6)$

$\Rightarrow x_1 x_2 + 6x_1 - 6x_2 - 36 = x_1 x_2 + 6x_2 - 6x_1 - 36$

$\Rightarrow 6x_1 - 6x_2 = 6x_2 - 6x_1$

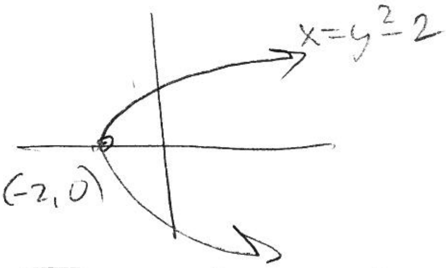
$\Rightarrow 12x_1 = 12x_2$

$\Rightarrow x_1 = x_2$ ■

(9) $\boxed{y = K \frac{x^3 \sqrt{z}}{u}}$

(10) $\boxed{m1}$

$x = y^2 - 2$



FAILS VERTICAL LINE TEST

$y^2 - 2 = x$

$y^2 = x - 2$

$y = \pm\sqrt{x-2}$

$y = \sqrt{x-2}$

$y = -\sqrt{x-2}$

Need one y for one x, but we're getting 2!

121 TEST 2

Test 2 Fall '16

B1 $f(x) = \sqrt{x} \longrightarrow$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h} =$$

$$= \left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$= \frac{(x+h)-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= \boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}} \xrightarrow{h \to 0} \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}} = f'(x)$$

B2 $7x^2-3x+2 =$

$$7\left(x^2 - \frac{3}{7}x \qquad\right) + 2$$

$$= 7\left(x^2 - \frac{3}{17}x + \left(\frac{3}{14}\right)^2\right) + 2 - 7\left(\frac{9}{196}\right)$$

$$= 7\left(x - \frac{3}{14}\right)^2 + 2 - \frac{9}{28}$$

$$\frac{56-9}{28} = \frac{47}{28}$$

$$= 7\left(x - \frac{3}{14}\right)^2 + \frac{47}{28}$$

$$(h,k) = \left(\frac{3}{14}, \frac{47}{28}\right)$$

121 TEST 2

Test 2 Fall '16

B3 $r(x) = \dfrac{x-3}{x^2-5x+6}$

$$\mathcal{D}(r) = \{x \mid x^2-5x+6 \neq 0\} = \{x \mid x \neq 2 \text{ and } x \neq 3\}$$

$$= (-\infty, 2) \cup (2,3) \cup (3, \infty)$$

$$= \mathbb{R} \setminus \{2,3\}$$

$x^2-5x+6 = 0$

$(x-3)(x-2) = 0$

$x \in \{2,3\}$

B4 $w(x) = \dfrac{\text{Stuff}}{\sqrt{\text{☺}}}$

$$\mathcal{D}(w) = \{x \mid \text{☺} \geq 0 \text{ and } \text{☺} \neq 0\}$$

$$= \{x \mid \text{☺} > 0\}$$

$5 - 10x > 0$

$-10x > -5$

$x < \dfrac{-5}{-10} = \dfrac{1}{2}$

$$\mathcal{D}(w) = \{x \mid x < \tfrac{1}{2}\}$$

$$= (-\infty, \tfrac{1}{2})$$

Test 2 Fall '16

121 TEST 2

B5 $-\sqrt{10-5x}+7$ is 1-to-1

Pf Suppose $g(x_1)=g(x_2)$

Then $-\sqrt{10-5x_1}+7=-\sqrt{10-5x_2}+7$

$\Rightarrow -\sqrt{10-5x_1}=-\sqrt{10-5x_2}$

$\Rightarrow \sqrt{10-5x_1}=\sqrt{10-5x_2}$

$\Rightarrow 10-5x_1=10-5x_2$

$\Rightarrow -5x_1=-5x_2$

$\Rightarrow x_1=x_2$ ■

B6 g^{-1}: $-\sqrt{10-5y}+7=x$

$-\sqrt{10-5y}=x-7$

$10-5y=(x-7)^2=x^2-14x+49$

$-5y=x^2-14x+39$

$y=-\frac{1}{5}(x^2-14x+39)=g^{-1}(x)$

121 TEST 2

Test 2 Fall '16

B7 $\mathcal{D}$ & $\mathcal{R}$ of g^{-1}

well, $g(x) = -10\sqrt{10-5x} + 7$

$= -10\sqrt{-5(x-2)} + 7$

$\boxed{\begin{array}{l} \mathcal{D}(g) = (-\infty, 2] = \mathcal{R}(g^{-1}) \\ \mathcal{R}(g) = (-\infty, 7] = \mathcal{D}(g^{-1}) \end{array}}$

$(2,7)$

B8 $|-5x+7| \geq 8$

$-5x+7 \geq 8$ OR $-5x+7 \leq -8$

$-5x \geq 1$ $\qquad -5x \leq -15$

$\{x \mid x \leq -\frac{1}{5}$ OR $x \geq 3\} = \boxed{(-\infty, -\frac{1}{5}] \cup [3, \infty)}$

$|-5x+7| \geq -8 \Longrightarrow \boxed{(-\infty, \infty)}$