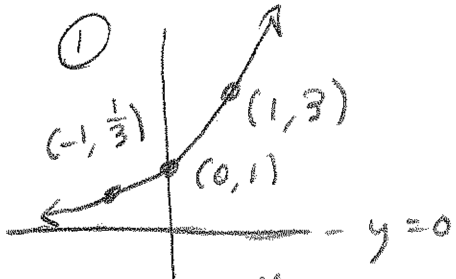
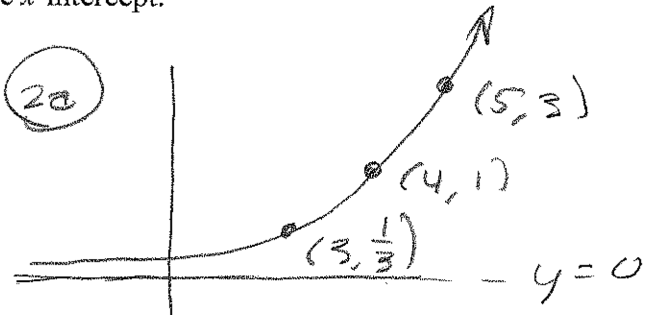
MAT 121-G11 | Test 4 – Fall, 2013 | (5 pts) Name KEY

100 Points | Covers Chapter 4, plus 8.3 | NO GRAPHING CALCULATORS!!!

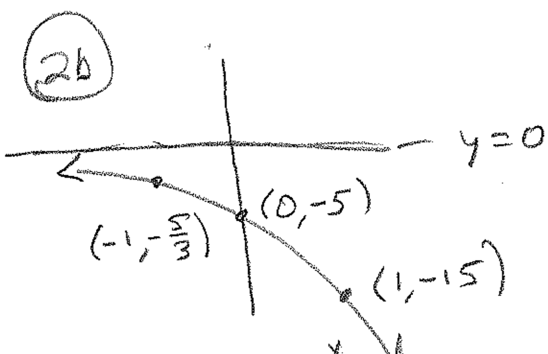
1. (20 pts) Starting with $f(x)=3^x$, sketch the graph of $g(x)=-5\cdot 3^{x-6}+7$ in 4 steps (counting $f(x)=3^x$ as the first step). Use $x = -1$, $x = 0$, and $x = 1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Your final graph should also show the y-intercept and, for 5 bonus points, the x-intercept.



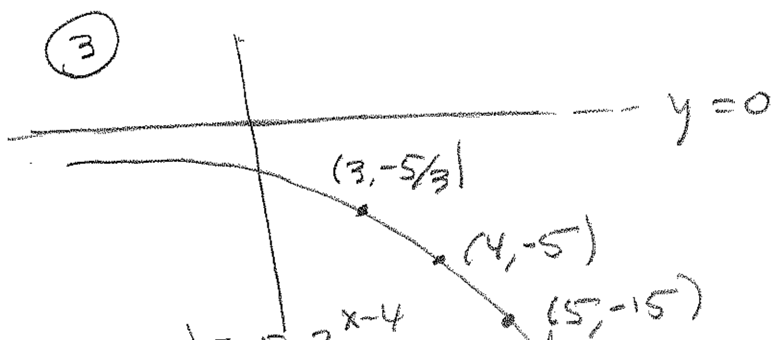
$f(x)=3^x$



$f(x-4)=3^{x-4}$

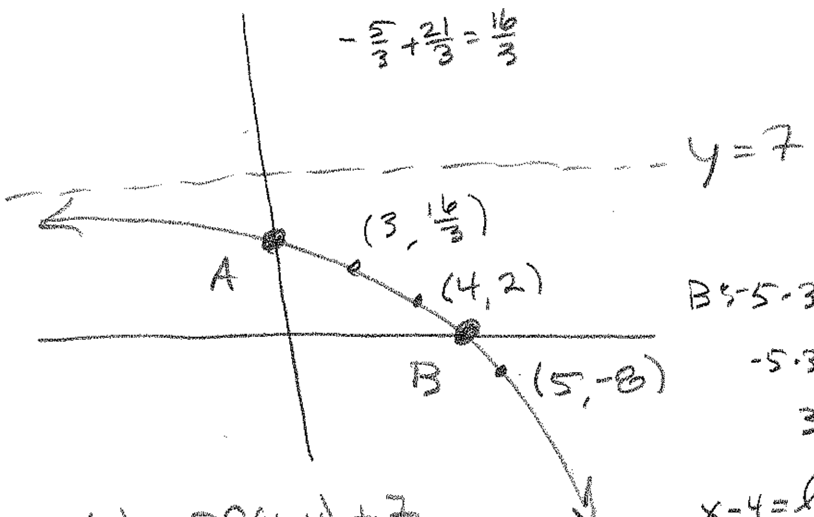


$-5f(x)=-5\cdot 3^x$



$-5f(x-4)=-5\cdot 3^{x-4}$

$-\frac{5}{3}+\frac{21}{3}=\frac{16}{3}$



$$g(x)=-5f(x-4)+7$$
$$=-5\cdot 3^{x-4}+7$$
$$=g(x)$$

A: $g(0)=-5\cdot 3^{-4}+7$

$=\frac{-5}{81}+\frac{567}{81}$

$=\frac{562}{81} \rightarrow \boxed{A=\left(0,\frac{562}{81}\right)}$

$\approx (0, 6.93827)$

6.938271605

B: $-5\cdot 3^{x-4}+7=0$

$-5\cdot 3^{x-4}=-7$

$3^{x-4}=\frac{7}{5}$

$x-4=\log_3(7/5)$

$x=4+\log_3\left(\frac{7}{5}\right)\approx 4+\frac{\ln(7/5)}{\ln(3)}\approx 4.30627028$

$B=\left(4+\frac{\ln(7/5)}{\ln(3)},\ 0\right) \boxed{\approx (4.30627, 0) \approx B}$

2. Let $f(x)=\sqrt{3x-9}$ and $g(x)=\dfrac{1}{x-5}$.

a. (8 pts) What is the domain of f ?

$\{x \mid x \geq 3\} = [3, \infty)$

b. (7 pts) What is the domain of g ?

$\{x \mid x \neq 5\} = (-\infty, 5) \cup (5, \infty)$

c. Determine the following composite functions. You don't need to simplify. In fact, I recommend you do not.

i) (5 pts) $(f \circ g)(x) = \sqrt{3\left(\frac{1}{x-5}\right) - 9}$

ii) (5 pts) $(g \circ f)(x) = \dfrac{1}{\sqrt{3x-9} - 5}$

d. (5 pts) What is the domain of $(f \circ g)(x)$? Now, you should simplify $(f \circ g)(x)$. Hint: The final domain is an interval of length ½. Very small domain.

$\mathcal{D} = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \neq 5 \text{ and } \frac{1}{x-5} \geq 3\}$

$= \{x \mid x \neq 5 \text{ and } 5 < x \leq \frac{16}{3}\}$

$= \{x \mid 5 < x \leq \frac{16}{3}\}$

$= \boxed{\left(5, \frac{16}{3}\right]}$

$\dfrac{1}{x-5} \geq 3$

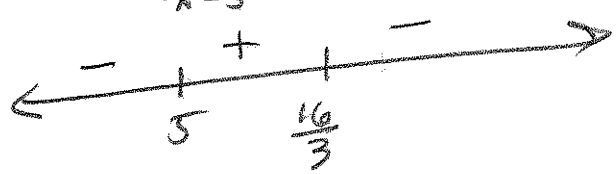
$\dfrac{1}{x-5} - \dfrac{3(x-5)}{x-5} \geq 0$

$\dfrac{1-3x+15}{x-5} \geq 0$

$\dfrac{-3x+16}{x-5} \geq 0$

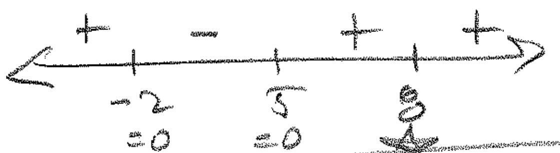
$-3x = -16$

$x = \frac{16}{3}$



3. (5 pts) What is the domain of $\sqrt{\dfrac{(x-5)(x+2)^3}{(x-8)^2}}$?

$$\frac{(x-5)(x+2)^3}{(x-8)^2} \geq 0$$



$$\boxed{x \in (-\infty, -2] \cup [5, 8) \cup (8, \infty)}$$

4. (5 pts) Let $f(x) = 5^{2x-5} - 3$. Find $f^{-1}(x)$.

$$5^{2y-5} - 3 = x$$

$$5^{2y-5} = x + 3$$

$$2y - 5 = \log_5 (x-3)$$

$$2y = \log_5 (x-3) + 5$$

$$y = \boxed{\frac{\log_5 (x-3) + 5}{2} = f^{-1}(x)}$$

5. Find the geometric sums:

a. (10 pts) $5+10+20+40+......+320$

$a=5,\ r=\frac{10}{5}=2$

$5\cdot 2^{n-1}=5\cdot 2^{6}$

$n-1=6$

$n=7$

$S=a\left(\frac{r^n-1}{r-1}\right)=5\left(\frac{2^7-1}{2-1}\right)=\frac{5(127)}{1}=635$

2 | 320
2 | 160
2 | 80
2 | 40
2 | 20
2 | 10
5

b. (5 pts) $\sum_{n=1}^{\infty} 3\cdot\left(\frac{5}{7}\right)^{n-1}$

$a=3,\ r=\frac{5}{7}<1$

$a\left(\frac{1}{1-r}\right)=3\left(\frac{1}{1-\frac{5}{7}}\right)=3\left(\frac{1}{\frac{2}{7}}\right)=3\left(\frac{7}{2}\right)=\boxed{\frac{21}{2}}$

$$\begin{array}{r} 280 \\ -28 \\ \hline 252 \end{array} \qquad \begin{array}{r} 18 \\ 14 \\ \hline 72 \\ 180 \\ \hline 252 \end{array}$$

6. (5 pts) Solve $\log_2(x+14)+\log_2(x+18)=5$.

$\log_2((x+14)(x+18))=5$

$x^2+32x+252=2^5=32$

$x^2+32x+220=0$

$x^2+32x+16^2=-220+256$

$(x+16)^2=36$

$x+16=\pm 6$

$x=-16\pm 6$ → -10, -22

$\log_2(-10+14)+\log_2(-10+18)$

$\log_2(4)+\log_2(8)$

$2+3=5$ ✓

$x=-22\notin D$

$\boxed{x\in\{-10\}}$

7. Suppose the half-life of C-14 is 5200 years. (It isn't, quite, but just suppose…).

   a. (10 pts) Derive the exponential decay model, $A(t) = A_0 e^{kt}$. The trick is to use the half-life to find the relative decay rate, k.

$$A_0 e^{5200k} = \frac{1}{2} A_0$$

$$e^{5200k} = \frac{1}{2}$$

$$5200k = \ln(1/2) = -\ln(2)$$

$$\boxed{k = \frac{-\ln 2}{5200}}$$

$$\boxed{A(t) = A_0 e^{kt}}$$

   b. (5 pts) How old is a sample of charcoal from a prehistoric fire pit, if 65% of the C-14 has decayed (i.e., 35% is left.) ?

$$A_0 e^{kt} = .35 A_0$$

$$e^{kt} = .35$$

$$kt = \ln(.35)$$

$$t = \frac{\ln(.35)}{k} = \frac{\ln(.35)}{\frac{-\ln(2)}{5200}} = \frac{5200 \ln(.35)}{-\ln(2)}$$

$$\approx 7875.780499$$

$$\approx \boxed{7,876 \text{ yrs}}$$