MAT 121-G11 Test 3 Take-Home – Fall, 2013 Name KEY

50 Points Some Chapter 3 Questions NO GRAPHING CALCULATORS!!!

This Take-Home is due Wednesday, October 23rd, at the *beginning* of class. Don't be late! If you're going to be late, hand it in *early*. NO LATE SUBMISSIONS ACCEPTED!!!

1. (5 pts) For each of the following polynomials, give an end behavior graphic, for instance, $\swarrow \cdots \searrow$

a. $f(x) = -3x^3 + 7x^2$

$\nwarrow \cdots \searrow$

b. $g(x) = 25x^4 - 15x^2 + 5$

$\nwarrow \cdots \nearrow$

Let $f(x) = 4x^5 - 12x^4 - 5x^3 + 21x^2 - 11x - 21$ for the remainder of this test.

2. (5 pts) What does Descartes' Rule of Signs tell you about positive and negative zeros (roots) of f?

3 or 1 positive

$-4x^5 - 12x^4 + 5x^3 + 21x^2 + 11x - 21$ 2 or 0 neg

3. (5 pts) Use the Rational Zeros (Roots) Theorem to list the possible rational zeros of f.

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}$

$\pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$

$\pm 7, \pm \frac{7}{2}, \pm \frac{7}{4}$

$\pm 21, \pm \frac{21}{2}, \pm \frac{21}{4}$

$P = 21$

$Q = 4$

$$\begin{array}{r|rrrrrr} 5 & 4 & -12 & -5 & 21 & -11 & -21 \\ & & 20 & 40 & 175 & 980 & 4845 \\ \hline & 4 & 8 & 35 & 196 & 969 & 4824 \end{array}$$

$\frac{1960}{2} = 980$

4. (5 pts) Show that $x = 5$ is an upper bound on real zeros for f.

$$\begin{array}{r|rrrrrr} 5 & 4 & -12 & -5 & 21 & -11 & -21 \\ & & 20 & 40 & 175 & 980 & \text{HUGE} \\ \hline & 4 & 8 & 35 & 196 & \text{BIG} & \text{humongous} \end{array}$$

All positives in bottom row

5. (5 pts) Find all real and nonreal zeros of $f(x) = 4x^5 - 12x^4 - 5x^3 + 21x^2 - 11x - 21$. Show the breakdown by synthetic divisions, step by step. Do your work on separate paper, and only show *me* the guesses that *worked*. Neatness counts. No credit for sloppy work.

-1	4	-12	-5	21	-11	-21
		-4	16	-11	-10	21
-1	4	-16	11	10	-21	0
		-4	20	-31	21	
3	4	-20	31	-21	0	
		12	-24	21		
	4	-8	7	0		

$4x^2 - 8x + 7 = 0$

$a=4, b=-8, c=7$

$b^2 - 4ac = (-8)^2 - 4(4)(7)$

$= 64 - 112$

$= -48$

$\sqrt{-48} = \sqrt{16 \cdot 3}\ i$

$= 4i\sqrt{3}$

$\begin{array}{r} 4 \\ 16 \\ \times\ 7 \\ \hline 112 \end{array}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{8 \pm 4i\sqrt{3}}{2(4)} = \dfrac{4(2 \pm i\sqrt{3})}{2(4)} = \boxed{\dfrac{2 \pm i\sqrt{3}}{2}}$

$x = -1, m = 2$

$x = 3, m = 1$

6. (5 pts) Factor f over the *REAL* number field. (Involves an *irreducible* quadratic factor.)

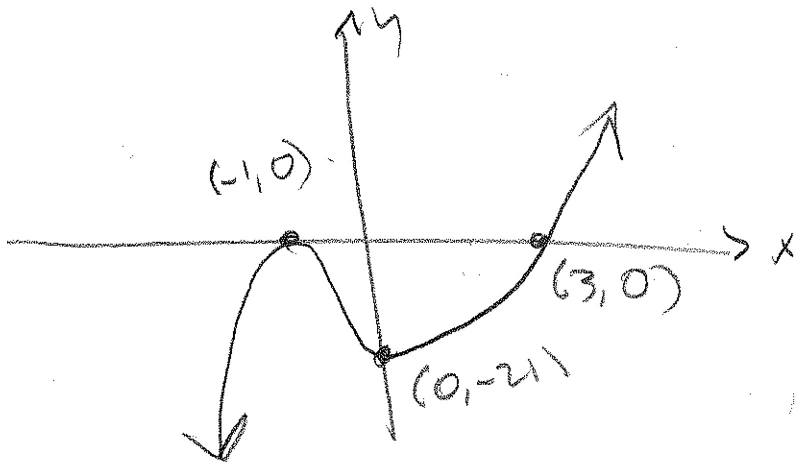
$$4(x+1)^2(x-3)\left(x-\left(\frac{2+i\sqrt{3}}{2}\right)\right)\left(x-\left(\frac{2-i\sqrt{3}}{2}\right)\right)$$ oops!

7. (5 pts) Factor f over the *COMPLEX* number field. (All linear factors.)

$$4(x+1)^2(x-3)(4x^2-8x+7)$$

switch

8. (5 pts) Use the work you've done to sketch the graph of f showing all intercepts. A *smooth* graph is the goal, here. I'm looking for the essence of the thing.



9. (5 pts) Discuss how you used your work to help build the graph. I'm particularly interested in behavior near x-intercepts and end behavior.

$x=-1$ touches $(m=2)$

$x=3$ crosses $(m=1)$

$(0,-21)$ from constant term.

Also, $4x^5$ is ↙ ... ↗ end behavior.

10. (5 pts) Sketch the graph of $g(x)=\dfrac{x^3-7x+6}{x^2-5x+4}$. It has an oblique asymptote. I expect you to find that asymptote and include it in your graph of g.

$x^3-7x+6=(x-1)(x^2+x-6)=(x-1)(x+3)(x-2)$

$x^2-5x+4=(x-1)(x-4)$

$g(x)=\dfrac{(x-1)(x+3)(x-2)}{(x-1)(x-4)}=\dfrac{(x+3)(x-2)}{x-4},\ x\neq 1$

$\mathcal{D}=\mathbb{R}\setminus\{1,4\}$

Hole: $x=1$, $y=\dfrac{(1+3)(1-2)}{1-4}=\dfrac{4(-1)}{-3}=\dfrac{4}{3}\rightsquigarrow (1,\tfrac{4}{3})$ $\boxed{\text{HOLE}}$

vertical Asymptote: $\boxed{x=4}$ V.A.

Oblique Asymptote

$$\begin{array}{r|rrr} 4 & 1 & 1 & -6 \\ & & 4 & 20 \\ \hline & 1 & 5 & 14 \end{array}$$

$\boxed{y=x+5}$ O.A.

y-int: $(0,\tfrac{6}{4})=\boxed{(0,\tfrac{3}{2})}$ y-int

$$\underset{-3}{-}\ \ +\ \ \underset{2}{}\ \ -\ \ \underset{4}{}\ \ +$$

$\boxed{\text{x-int: } (-3,0),\ (2,0)}$