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(1) $f = \{(3,-7), (4,-1), (2,5), (9,-1)\}$ is a function.

Each x is paired with no more than one y

x ← 1st coord.

y ← 2nd coord.

(2) $\mathcal{D}(f) = \{3, 4, 2, 9\}$

(3) $R(f) = \{-7, -1, 5, -1\}$

(4) $f(x) = \dfrac{x+7}{x-2}$, $g(x) = \sqrt{x+1}$ ⟶

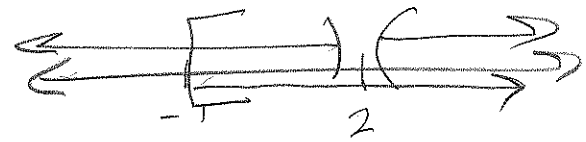
(a) $\mathcal{D}(f) = \mathbb{R} \setminus \{2\} = (-\infty, 2) \cup (2, \infty) = \{x \mid x \neq 2\}$

(b) $\mathcal{D}(g) = \{x \mid x \geq -1\} = [-1, \infty)$

(c)(i) $(f+g)(x) = \dfrac{x+7}{x-2} + \sqrt{x+1}$

"AND" "$\cap$"

(ii) $\mathcal{D}(f+g) = \mathcal{D}(f) \cap \mathcal{D}(g)$



$= [-1, 2) \cup (2, \infty)$

$= \{x \mid x \neq 2 \text{ AND } x \geq -1\}$

(d)(ii) $(f \circ g)(x) = \dfrac{\sqrt{x} + 7}{\sqrt{x} - 2}$

(e) $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ AND } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \geq -1 \text{ AND } \sqrt{x+1} \neq 2\}$

$\sqrt{x+1} = 2$

$x+1 = 4$

$x = 3$

$= \{x \mid x \geq -1 \text{ AND } x \neq 3\}$

$= [-1, 3) \cup (3, \infty)$

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(5) $f(x) = x^2 - 5x \Longrightarrow \dfrac{f(x+h) - f(x)}{h}$

$$= \frac{(x+h)^2 - 5(x+h) - (x^2 - 5x)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h}$$

$$= \frac{2xh + h^2 - 5h}{h}$$

BONUS

$$= 2x + h - 5 \; \boxed{\xrightarrow{h \to 0} 2x - 5 = f'(x)}$$

(6) $x^2 + y^2 = 49$

$y^2 = 49 - x^2$

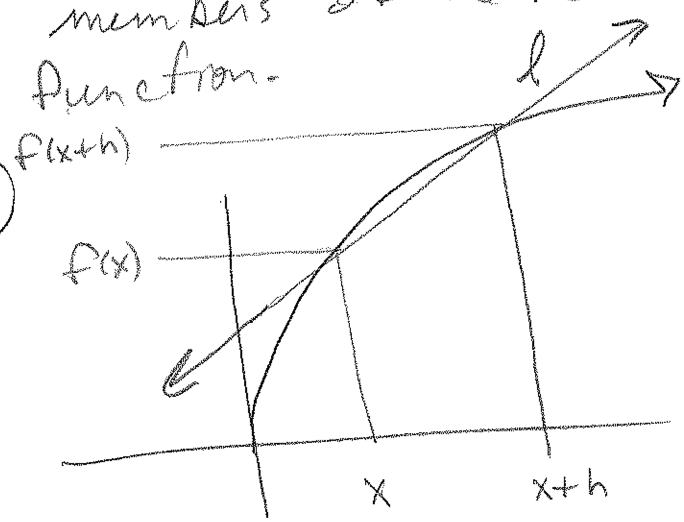
$y = \pm\sqrt{49 - x^2}$

Therein lies the prob. The "$\pm$"

2 y-values for some x, prob'ly:

$x = 0 \Longrightarrow y = \pm 7 \Longrightarrow (0,7), (0,-7)$ are members of the relation; hence, not a function.

(7)



$$\frac{f(x+h) - f(x)}{h} =$$

Average slope of $f(x)$ on $[x, x+h]$

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Bonus $f(x)=\sqrt{x} \Rightarrow \frac{f(x+h)-f(x)}{h}$

$$= \frac{\sqrt{x+h}-\sqrt{x}}{h} = \left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$= \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}}$$

~~Bonus~~ pts,

$h \to 0 \Rightarrow \frac{1}{2\sqrt{x}}$ Megabonus.

(8).

(a) $f(x)=\frac{2}{3}x-7$ is 1 to 1

$f(x_1)=f(x_2)$. Then

$$\frac{2}{3}x_1-7=\frac{2}{3}x_2-7$$

$$\frac{2}{3}x_1=\frac{2}{3}x_2$$

$$x_1=x_2 \;\blacksquare$$

(b) $f(x)=x^2-6x+7$

$$y^2-6y+7=x$$

$$y^2-6y=x-7$$

$$y^2-6y+3^2=x-7+9$$

$$(y-3)^2=x+2$$

$$y-3=\pm\sqrt{x-2}$$

$$f^{-1}(x)=3+\sqrt{x-2}$$

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(9) $f(x) = \frac{x+3}{x-1} \longrightarrow$

$$(f \circ f)(x) = \frac{\frac{x+3}{x-1} + 3}{\frac{x+3}{x-1} - 1} = \frac{\frac{x+3+3(x-1)}{x-1}}{\frac{x+3-1(x-1)}{x-1}}$$

$$= \frac{\frac{x+3+3x-3}{x-1}}{\frac{x+3-x+1}{x-1}} = \frac{\frac{4x}{x-1}}{\frac{4}{x-1}} = \frac{4x}{x-1} \cdot \frac{x-1}{4} = x$$

$\longrightarrow f = f^{-1}$!

(10) $\boxed{y = k \frac{m_1 m_2}{r^2}}$