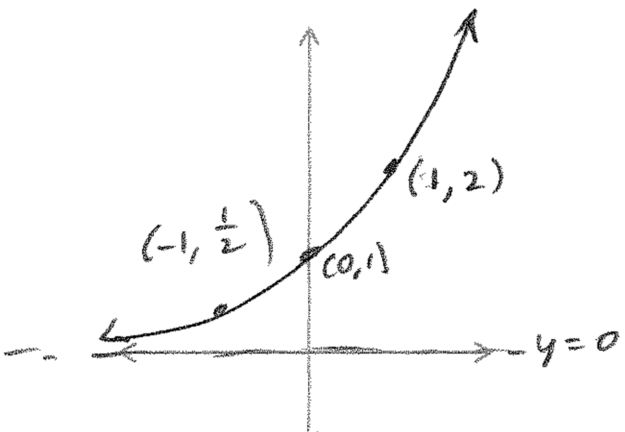
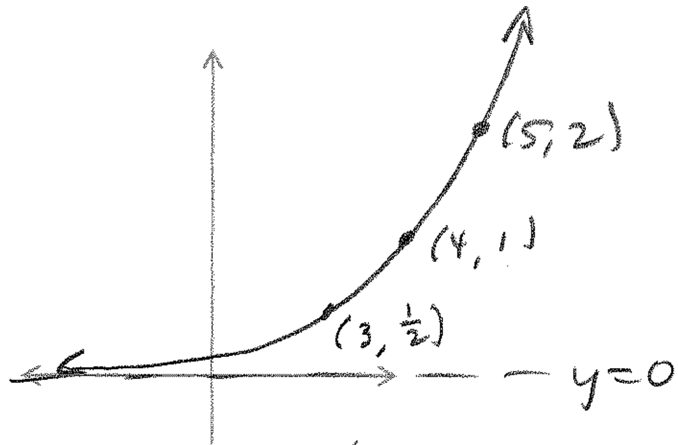
MAT 121-G81 Test 4 – Fall, 2013 (5 pts) Name________________

100 Points Covers Chapter 4, plus 7.1, 7.3 NO GRAPHING CALCULATORS!!!

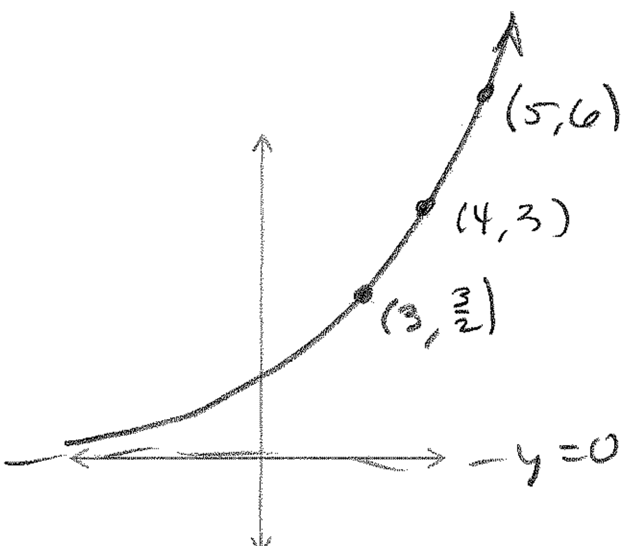
1. (20 pts) Starting with $f(x)=2^x$, sketch the graph of $g(x)=3\cdot 2^{x-4}-7$ in 4 steps (counting $f(x)=5^x$ as the first step). Use $x=-1$, $x=0$, and $x=1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Your final graph should also show the y-intercept and, for 5 bonus points, the x-intercept.



$f(x)=2^x$



$f(x-4)=2^{x-4}$

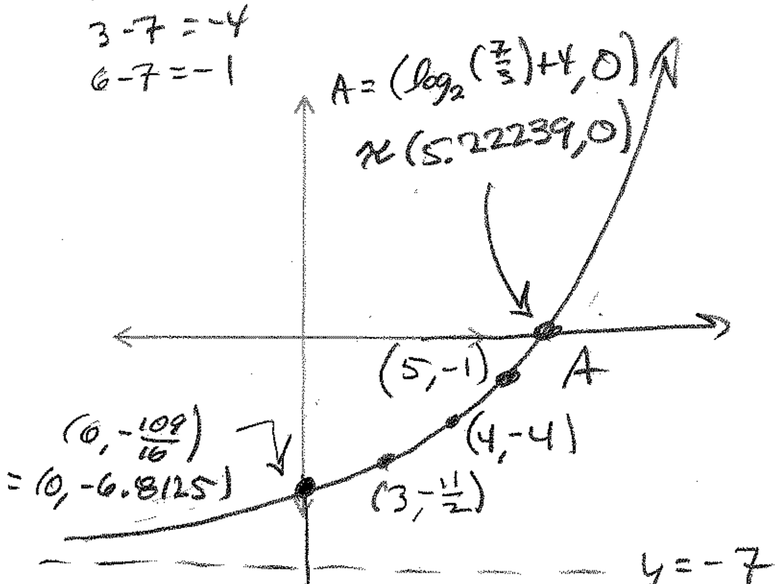


$3f(x-4)=3\cdot 2^{x-4}$

$\frac{3}{2}-\frac{14}{2}=-\frac{11}{2}=$

$3-7=-4$

$6-7=-1$



$3f(x-4)-7=g(x)=3\cdot 2^{x-4}-7$

$g(0)=3\cdot 2^{-4}-7 \approx -6.8125$

FIND A's $=\frac{3}{16}-7=\frac{3-112}{16}=-\frac{109}{16}$

$3\cdot 2^{x-4}=7$

$2^{x-4}=\frac{7}{3}$

$x-4=\log_2(7/3)$

$x=\log_2(7/3)+4=\frac{\ln(7/3)}{\ln(2)}+4 \approx 5.222392421$

2. Let $f(x)=\sqrt{2x-4}$ and $g(x)=\dfrac{1}{x-5}$.

a. (8 pts) What is the domain of f ?

$2x-4 \geq 0$
$2x \geq 4$

$x \geq 2$
$\{x \mid x \geq 2\} = [2,\infty)$

b. (7 pts) What is the domain of g ?

$\{x \mid x \neq 5\} = (-\infty,5) \cup (5,\infty)$

c. Determine the following composite functions. You don't need to simplify. In fact, I recommend you do not.

i) (5 pts) $(f \circ g)(x) = \sqrt{2\left(\frac{1}{x-5}\right) - 4}$

ii) (5 pts) $(g \circ f)(x) = \dfrac{1}{\sqrt{2x-4} - 5}$

HINT: It's an interval of length $\frac{1}{2}$

d. (5 pts) What is the domain of $(f \circ g)(x)$? (Now, you should simplify $(f \circ g)(x)$).

$\mathcal{D} = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \neq 5 \text{ and } \frac{1}{x-5} \geq 2\}$

$= \{x \mid x \neq 5 \text{ and } 5 \leq x \leq \frac{11}{2}\}$

$= \boxed{\{x \mid 5 < x \leq \tfrac{11}{2}\} = (5, \tfrac{11}{2}] = (5, 5.5]}$

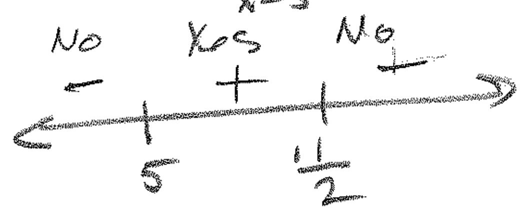
$\frac{1}{x-5} \geq 2$

$\frac{1}{x-5} - 2 \geq 0$

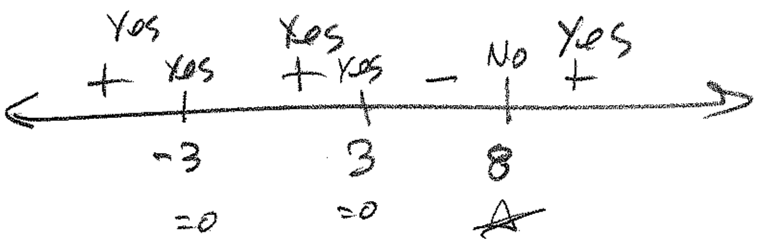
$\frac{1-2(x-5)}{x-5} \geq 0$

$\frac{1-2x+10}{x-5} \geq 0$

$\frac{-2x+11}{x-5} \geq 0$



3. (5 pts) What is the domain of $\sqrt{\dfrac{(x-3)(x+3)^2}{(x-8)}}$?



$\boxed{x \in (-\infty, 3] \cup (8, \infty)}$

4. (5 pts) Let $f(x) = 3^{2x-5} + 1$. Find $f^{-1}(x)$.

$3^{2y-5} + 1 = x$

$3^{2y-5} = x - 1$

$2y - 5 = \log_3(x-1)$

$2y = \log_3(x-1) + 5$

$y = \frac{1}{2}\left[\log_3(x-1) + 5\right]$

$= \dfrac{\log_3(x-1) + 5}{2}$

$= f^{-1}(x)$

5. Find the geometric sums:

a. (10 pts) $5+10+20+40+......+320$

$a=5$

$r=2$

$320 = 5\cdot 2^6 \Longrightarrow n-1=6$

$\Longrightarrow n=7$

$$\begin{array}{r|l} 2 & 320 \\ 2 & 160 \\ 2 & 80 \\ 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ & 5 \end{array}$$

$a\left(\frac{1-r^n}{1-r}\right)$

$=5\left(\frac{1-2^7}{1-2}\right)$

$=5\left(\frac{1-128}{-1}\right)$

$=5\left(\frac{-127}{-1}\right)$

$=\frac{1}{2}(1270)$

$=\boxed{635}$

b. (5 pts) $\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^{n-1}$

$a=1$

$r=\frac{2}{5}<1$ ✓

$a\left(\frac{1}{1-r}\right)=\frac{1}{1-\frac{2}{5}}=\frac{1}{\frac{3}{5}}=\frac{5}{3}$

6. (5 pts) Solve $\log(x)+\log(x+48)=2$.

$\log(x(x+48))=2$

$x^2+48x=10^2=100$

$x^2+48x-100=0$

$(x+50)(x-2)=0$

$x=-50 \notin D$

OR

$x=2$ Check $x=2$: $\log(2)+\log(2+48)$

$=\log(2(50))=\log(100)=2$

$x\in\{2\}$

7. Suppose the half-life of C-14 is 5800 years. (It isn't, quite, but just suppose…).

a. (10 pts) Derive the exponential decay model, $A(t) = A_0 e^{kt}$. The trick is to use the half-life to find the relative decay rate, k.

$$A(t) = A_0 e^{kt}$$

$$A(5800) = A_0 e^{5800k} = \frac{1}{2} A_0$$

$$e^{5800k} = \frac{1}{2}$$

$$5800k = \ln(1/2)$$

$$\boxed{k = \frac{\ln(1/2)}{5800}}$$

$$A(t) = A_0 e^{\frac{\ln(1/2)}{5800} t}$$

b. (5 pts) How old is a sample of charcoal from a prehistoric fire pit, if 80% of the C-14 has decayed (i.e., 20% is left.) ?

$$A(t) = A_0 e^{kt} = .2A_0$$

$$e^{kt} = .2$$

$$kt = \ln(.2)$$

$$t = \frac{\ln(.2)}{k} = \frac{\ln(.2)}{\left(\frac{\ln(1/2)}{5800}\right)} = \frac{5800 \ln(.2)}{\ln(1/2)}$$

$\approx$ 13467.18295 yrs

$\boxed{\approx 13{,}467 \text{ yrs old}}$