

You may take advantage of up to 15 points of bonus problems on this test.

1. (10 pts) Write the augmented matrix corresponding to the system of linear equations. Take your time and get it right, because #2 depends on your accuracy.

$$\begin{aligned}x + y + 3z &= 8 \\2x + 3y + 8z &= 17 \\3x + 3y + 10z &= 26\end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ 2 & 3 & 8 & 17 \\ 3 & 3 & 10 & 26 \end{array} \right]$$

2. (10 pts) Use your augmented matrix from #1 to solve the system.

R_1
 $-2R_1 + R_2$
 $-3R_1 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$z = 2$

$$y + 2z = 1$$
$$y + 2(2) = 1$$

$y = -3$

$$x + y + 3z = 8$$
$$x - 3 + 6 = 8$$

$x = 5$

1
2
3

3. (5 pts) Check your answer to #2 by multiplying two matrices together. I'll give you the first one. You can get #2 wrong and still get this one 100% right. Make up 3 numbers if you have to.

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 8 \\ 3 & 3 & 10 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 - 3 + 6 \\ 10 - 9 + 16 \\ 15 - 9 + 20 \end{bmatrix} = \begin{bmatrix} 8 \\ 17 \\ 26 \end{bmatrix}$$

4. (15 pts) How much 20% alcohol must be mixed with 74% alcohol to obtain 50 liters of 65% alcohol?

	Vol	Pure Alcohol	
20%	x	.2x	3.25
74%	y	.74y	25.10
65%	50	.65(50)	32.5

Let x = Amt of 20% alcohol (in liters)

y = " " 74% " " " "

$$x + y = 50$$

$$.2x + .74y = 32.5$$

$$\left[\begin{array}{cc|c} 1 & 1 & 50 \\ .2 & .74 & 32.5 \end{array} \right]$$

$$(-.2)(50) = -10$$

$$\left[\begin{array}{cc|c} 1 & 1 & 50 \\ 0 & .54 & 22.5 \end{array} \right]$$

$$.54y = 22.5$$

$$y = \frac{22.5}{.54} = 41.\bar{6} = 41 + \frac{2}{3} = \frac{125}{3} = y$$

$$x + y = 50$$

$$x + \frac{125}{3} = 50 = \frac{150}{3}$$

$$\boxed{x = \frac{25}{3} \text{ liters}}$$

5. (10 pts) Set up the System and the Augmented Matrix for the word problem.

Janice invested in 3 different accounts: Money Market, Stock Market, and Junk Bonds. Her total investment was \$23,000. She invested 5 times as much in the Money Market as she invested in Junk Bonds. Money Market account pays 7% interest, Stock Market paid her a 5% return, and Junk Bonds paid 8% on her investment in them. If her combined earnings on all the investments was \$1,410, how much did she invest in each account?

Let $x =$ amt invested in Money Mkt (\$) (7%)
 $y =$ " " " Stock " (\$) (5%)
 $z =$ " " " Junk Bonds " (8%)

$$x + y + z = 23000$$

$$x - 5z = 0$$

$$.07x + .05y + .08z = 1410$$

MM is twice as much as Junk

$$x = 5z$$

$$x - 5z = 0$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 1 & 1 & 1 & 23000 \\ .07 & .05 & .08 & 1410 \end{array} \right]$$

(5 Points Bonus) - Solve the system you obtained in #5.

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & -6 & 23000 \\ 0 & .05 & .43 & 1410 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & 6 & 23000 \\ 0 & .13 & & 260 \end{array} \right]$$

$$\begin{aligned} &(-.07)(-5) + .08 \\ &= +.35 + .08 \\ &= \underline{-.27} \rightarrow .43 \end{aligned}$$

$$\begin{aligned} &(-.05)(6) + .43 \\ &= -.3 + .43 \\ &= .13 \end{aligned}$$

$$\begin{aligned} &(-.05)(23000) + 1410 \\ &= -1150 + 1410 = 260 \\ &-.05 \end{aligned}$$

$$+.13z = 260$$

$$z = \frac{260}{.13} = \boxed{2000 = z}$$

$$y + 6(2000) = 23000$$

$$y = 23000 - 12000 = \boxed{11000 = y}$$

$$x + 11000 + 2000 = 23000$$

$$\boxed{x = 10000}$$

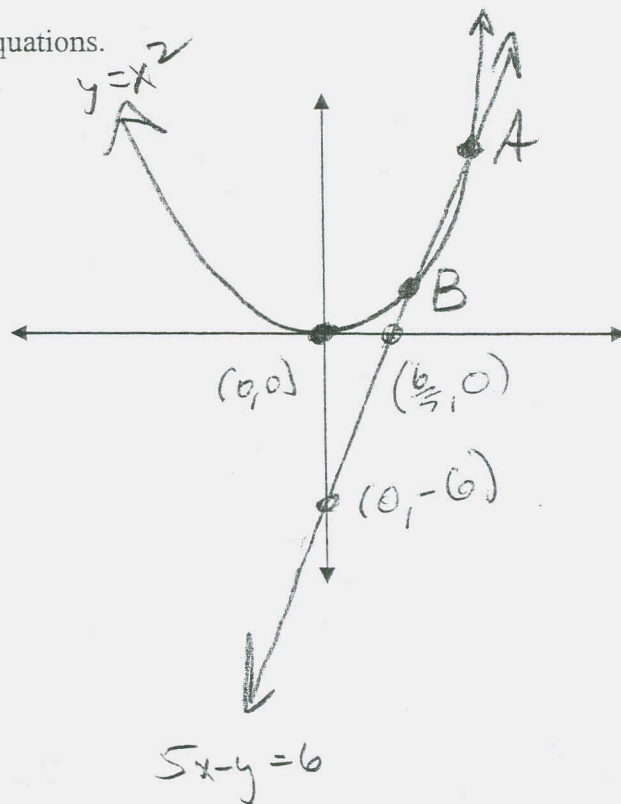
6. (15 pts) Sketch the graph of the system of nonlinear equations.

$$5x - y = 6$$

$$y = x^2$$

$$5x - y = 6$$

0	y
6	-6
6	0
5	



7. (15 pts) Solve the nonlinear system of equations.

$$5x - y = 6$$

$$y = x^2$$

$$5x - x^2 = 6$$

$$-x^2 + 5x - 6 = 0$$

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

$$x = 2, 3$$

$$y = x^2 \Rightarrow y = 4, 9, \text{ respectively, giving}$$

$$B = (2, 4), (3, 9) = A$$

8. (5 pts) Label the graph in #6 with your solutions from #7.

9. (15 pts) Sketch the solution set of the following system of inequalities. However you decide to shade (good stuff or bad stuff), I expect to see you indicate the "good stuff" with the phrase "good stuff."

$$2x + 3y \leq 6$$

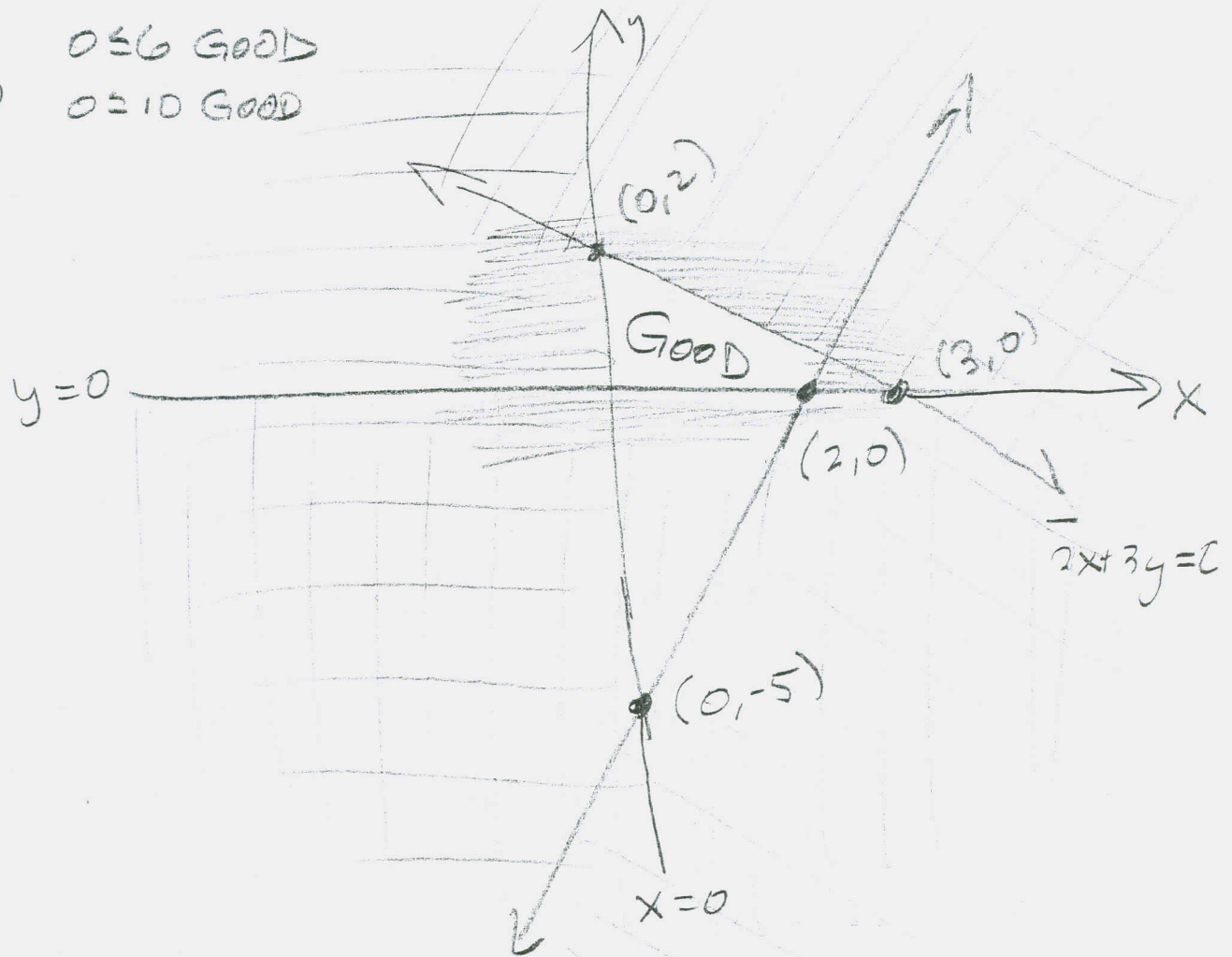
$0 \leq 6$ GOOD

$$5x - 2y \leq 10$$

$0 \leq 10$ GOOD

$$x \geq 0$$

$$y \geq 0$$



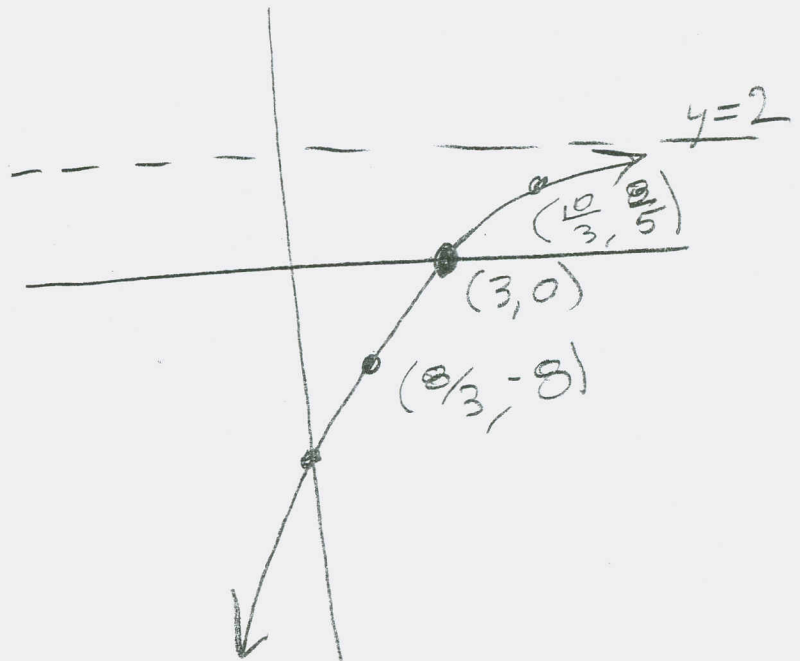
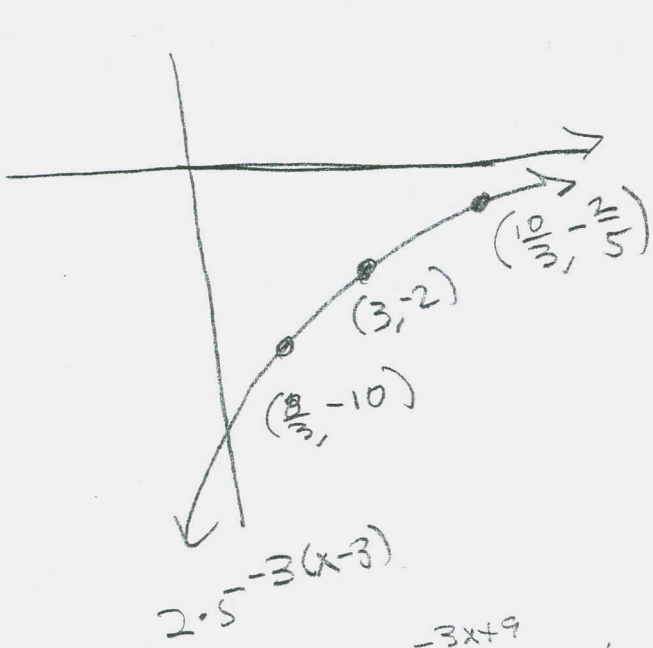
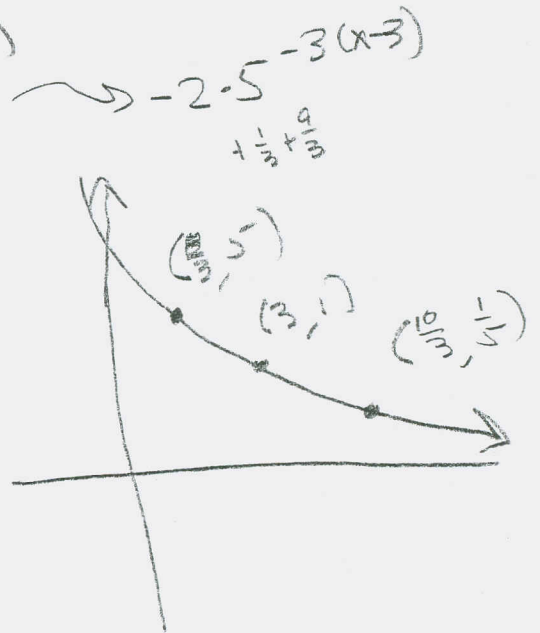
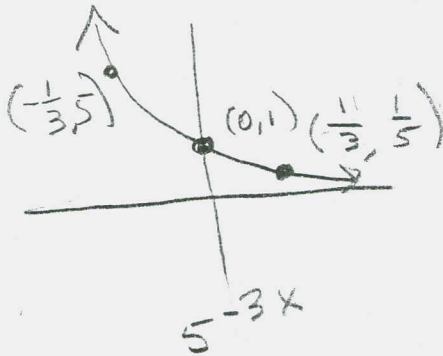
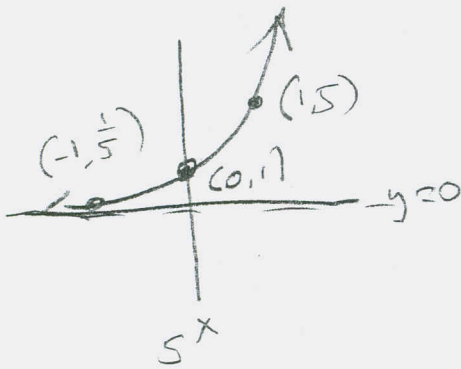
(10 pts bonus) Sketch the graph of $g(x) = -2 \cdot 5^{-3x+9} + 2$ by transforming the basic function $f(x) = 5^x$, as demonstrated on many occasions in lecture.

(5 pts bonus) Find the x -intercept, accurate to 5 decimal places in the x -coordinate. Label it clearly on your final sketch.

$$g(x) = -2 \cdot 5^{-3(x-3)} + 2$$

$$f(x) = 5^x \rightsquigarrow 5^{-3x} \rightsquigarrow 5^{-3(x-3)} \rightsquigarrow -2 \cdot 5^{-3(x-3)} + 2$$

$$\rightsquigarrow -2 \cdot 5^{-3(x-3)} + 2$$



$$5^{-3x+9} = 1$$

$$-3x+9 = 0$$

$$-3x = -9$$

$$x = 3$$

Too EASY!