MAT 121 Fall, 2011
100 Points
Test 3 Makeup Name KEY

1. (10 pts) Form a polynomial in factored form with *real* coefficients with the given zeros and degree. Please do not expand the polynomial.

Zeros: -2, multiplicity 3; -5, multiplicity 2. Degree 5.

$(x+2)^3(x+5)^2$

2. (10 pts) Expand $(x-3+2i)(x-3-2i)$

$x^2-3x-2ix-3x+9+6i+2ix-6i-4i^2$

$= x^2-6x+9+4 = \boxed{x^2-6x+13}$

$P(x) = (x-3)(2x^4+6x^3+18x^2+52x+160) + 479$

3. (10 pts) Use synthetic division to find $P(3)$ if $P(x) = 2x^5-2x^2+4x-1$.

$$\begin{array}{r|rrrrrr} 3 & 2 & 0 & 0 & -2 & 4 & -1 \\ & & 6 & 18 & 54 & 156 & 480 \\ \hline & 2 & 6 & 18 & 52 & 160 & \boxed{479 = P(3)} \end{array}$$

4. (10 pts) Divide $f(x) = 3x^4 - x^3 + 3x^2 - 4$ by $d(x) = x^2 - 2$. Then write the result in the form *Dividend = Divisor · Quotient + Remainder*.

$$\begin{array}{r|l} & 3x^2 - x + 9 \\ \hline x^2-2 & 3x^4 - x^3 + 3x^2 + 0x - 4 \\ & -(3x^4 \qquad - 6x^2) \\ \hline & \quad -x^3 + 9x^2 + 0x - 4 \\ & \quad -(-x^3 \qquad + 2x) \\ \hline & \qquad 9x^2 - 2x - 4 \\ & \qquad -(9x^2 \qquad - 18) \\ \hline & \qquad\quad -2x + 14 \end{array}$$

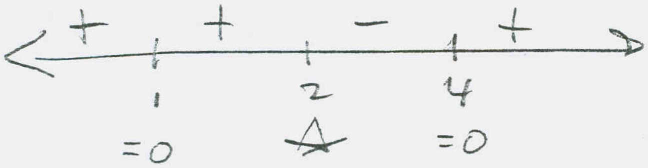
$f(x) = (x^2-2)(3x^2-x+9) + -2x+14$

5. Solve the inequalities:

a. (10 pts) $(x-1)^2(x-2)(x-4)^3 \le 0$

$$\boxed{x \in \{1\} \cup [2,4]}$$

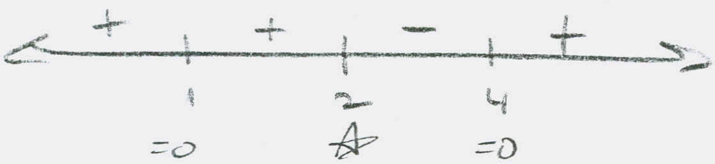
b. (10 pts) $\dfrac{(x-1)^2(x-4)^3}{(x-2)} \le 0$



$$\boxed{x \in \{1\} \cup (2,4]}$$

c. (5 pts) What is the domain of $f(x) = \sqrt{\dfrac{(x-1)^2(x-4)^3}{(x-2)}}$?

Need $\dfrac{(x-1)^2(x-4)^3}{x-2} \ge 0$



$$\boxed{(-\infty,2) \cup [4,\infty)}$$

d. (5 pts) What is the domain of $f(x) = \log_3\left(\dfrac{(x-1)^2(x-4)^3}{(x-2)}\right)$?

Need $\dfrac{(x-1)^2(x-4)^3}{x-2} > 0$

$$\boxed{x \in (-\infty,1) \cup (1,2) \cup (4,\infty)}$$

$-x^5-4x^4-2x^3+14x^2+23x+10$

6. (15 pts) Find all real zeros of $f(x)=x^5-4x^4+2x^3+14x^2-23x+10$. Factor $f(x)$ over the real number field. This will likely entail an irreducible quadratic factor that can not be split over the real number field

$\pm 1, \pm 2, \pm 5, \pm 10$ ONE NEG. ZERO.

$$\begin{array}{r|rrrrrr} -1 & 1 & -4 & 2 & 14 & -23 & 10 \\ & & -1 & 5 & -7 & -7 & 30 \\ \hline & 1 & -5 & 7 & 7 & -30 & \text{No} \end{array}$$

$$\begin{array}{r|rrrrrr} -2 & 1 & -4 & 2 & 14 & -23 & 10 \\ & & -2 & 12 & -28 & 28 & -10 \\ \hline 1 & 1 & -6 & 14 & -14 & 5 & 0 \\ & & 1 & -5 & 9 & -5 & \\ \hline 1 & 1 & -5 & 9 & -5 & 0 & \\ & & 1 & -4 & 5 & & \\ \hline & 1 & -4 & 5 & & & \end{array}$$

$x^2-4x+5=0$

$f(x)=(x+2)(x-1)^2(x^2-4x+5)$

$x=-2, 1$

$x=1$ has $m=2$

7. (15 pts) Use your work from #6 to find any *non*real zeros of $f(x)$. Then write $f(x)$ as the product of *linear* factors. That is, break $f(x)$ *all* the way down, with the nonreal zeros you find (plus the real zeros you already found from #6).

$x^2-4x+5=0$

$x^2-4x=-5$

$x^2-4x+2^2=-5+4$

$(x-2)^2=-1$

$x-2=\pm i$

$x=2\pm i$

$x=1,\ m=2$

$x=-2$

$x=2\pm i$

$f(x)=(x+2)(x-1)^2(x-(2+i))(x-(2-i))$