MAT 121-G11 – Fall, 2011 Test 2
Chapter 2 Name KEY

1. (10 pts) $f = \{(1,-1),(2,4),(3,2),(4,4)\}$

   a. Function? (Yes/no) Yes

   b. If not, why not?

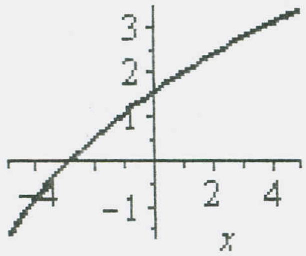
   c. What's the domain?

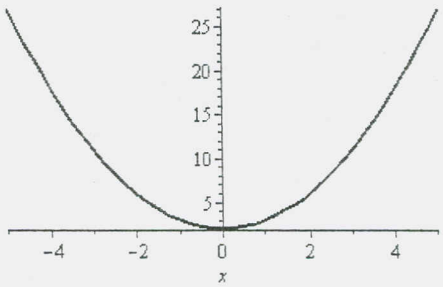
   $\{1,2,3,4\}$

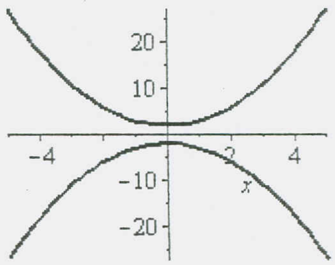
   d. What's the range?

   $\{-1,4,2\}$

2. (10 pts) For each of the following graphs, determine if the relation is a function. If it is a function, state whether or not it is 1-to-1.







Is it a function? Yes

If it is a function, is it 1-to-1?

Yes

Is it a function? Yes

If it is a function, is it 1-to-1?

No

Is it a function? No

If it is a function, is it 1-to-1?

3. (10 pts) Let $f(x) = x^2 + 3$. Simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+3-(x^2+3)}{h}$$

$$= \frac{x^2+2xh+h^2+3-x^2-3}{h} = \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h}$$

$$= \boxed{2x+h}$$

4. (5 pts) Determine whether or not $y^2 - 2x = 5$ defines y as a function of x. If it does not, show/explain why not. (Solve for y and look at how many solutions you get.)

$y^2 = 2x+5$

$y = \pm\sqrt{2x+5}$

let $x=1$

$y=\sqrt{7}$ OR $y=-\sqrt{7}$

$(1,\sqrt{7})$ and $(1,-\sqrt{7})$ have $x=1$ paired

with $y=\pm\sqrt{7}$.

5. Let $f(x) = \dfrac{x+5}{x-6}$ and $g(x) = \sqrt{x+8}$.

a. (5 pts) What is the domain of f?

$\{x \mid x \neq -6\} = (-\infty, 6) \cup (-6, \infty)$

b. (5 pts) What is the domain of g?

$\{x \mid x \geq -8\} = [-8, \infty)$

c. (5 pts) Find $(f \circ g)(x)$. (Do not simplify.)

$$\frac{\sqrt{x+8}+5}{\sqrt{x+8}+6}$$

d. (5 pts) What is the domain of $(f \circ g)(x)$?

$\{x \mid x \geq -8 \text{ AND } \sqrt{x+8} \neq +6\} = \{x \mid x \geq -8 \text{ and } x \neq 28\}$

$= [-8, 28) \cup (28, \infty)$

$\sqrt{x+8} = 6$

$x+8 = 36$

$x = 28$

Still working with $f(x)=\dfrac{x+5}{x-6}$ and $g(x)=\sqrt{x+8}$.

e. Determine each of the following functions (without simplifying) and state the domain of each in *interval notation*.

$\{x \mid x \geq -8 \text{ and } x \neq 6\}$

i. (5 pts) $(f+g)(x)$

$$\frac{x+5}{x-6}+\sqrt{x+8}$$

$$\mathcal{D}=[-8,\infty)\cap\left((-\infty,6)\cup(6,\infty)\right)=[-8,6)\cup(6,\infty)$$

ii. (5 pts) $\left(\dfrac{f}{g}\right)(x) = \dfrac{\frac{x+5}{x-6}}{\sqrt{x+8}}$

$\mathcal{D}=(-8,6)\cup(6,\infty)$

Can't let $g(x)=0$.

6. (5 pts) Answer *one* of the following:

a. Show that $f(x)=\dfrac{x+3}{x-1}$ is 1-to-1, algebraically.

b. Let $f(x)=\dfrac{x+3}{x-1}$. Find $f^{-1}(x)$.

(a) $\ni$ $f(x_1)=f(x_2)$. Then

$$\frac{x_1+3}{x_1-1}=\frac{x_2+3}{x_2-1}$$

$$(x_2-1)(x_1+3)=(x_2+3)(x_1-1)$$

$$x_2x_1+3x_2-x_1-3=x_2x_1-x_2+3x_1-3$$

$$3x_2-x_1=-x_2+3x_1$$

$$-4x_1=-4x_2$$

$$x_1=x_2 \quad \blacksquare$$

(b) $x=\dfrac{y+3}{y-1}$

$$x(y-1)=y+3$$

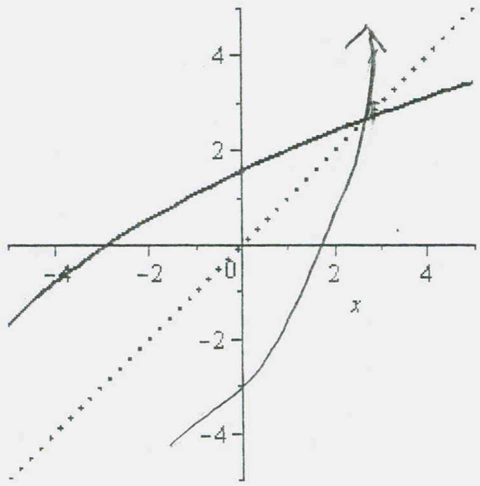
$$xy-x=y+3$$

$$xy-y=x+3$$

$$y(x-1)=x+3$$

$$y=\boxed{\frac{x+3}{x-1}=f^{-1}(x)}$$

7. (5 pts) The graph of f is given. Sketch the graph of f^{-1}.



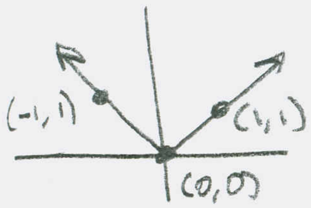
8. (5 pts) If f varies jointly as x and w and inversely with the cube of r, write the equation describing this relationship.

$$f = k\,\frac{xw}{r^3}$$

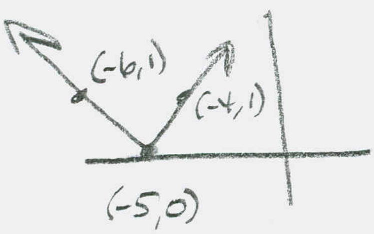
9. Graph each of the following functions using techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages in separate sketches. Track 3 key points through the transformations.

a. (5 pts) $g(x) = 2|x+5| + 4$

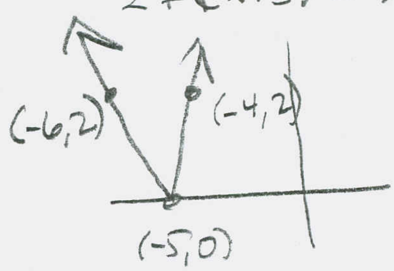
$f(x) = |x|$



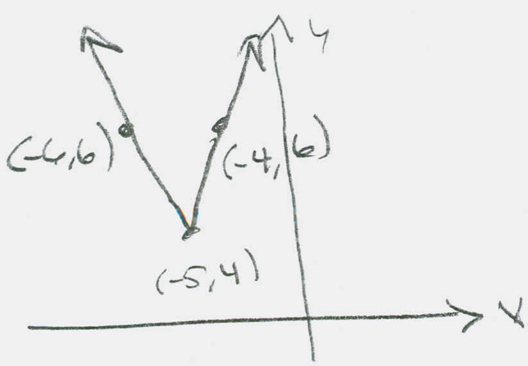
$f(x+5) = |x+5|$



$2f(x+5) = 2|x+5|$



$2\cdot f(x+5) + 4 = 2|x+5| + 4 = g(x)$



$-\frac{4}{3}+3$

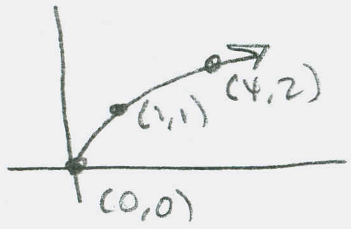
$-\frac{1}{3}+3=\frac{8}{3}$

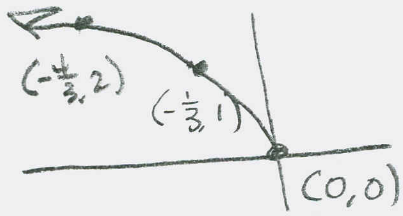
#9, continued... Graph using transformations.

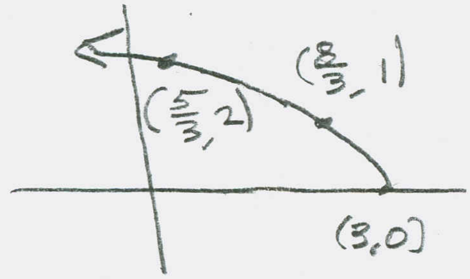
b. (5 pts) $h(x)=2\sqrt{9-3x}$ Hint: $9-3x=-3x+9=-3(x-3)$

$= 2\sqrt{-3(x-3)}$

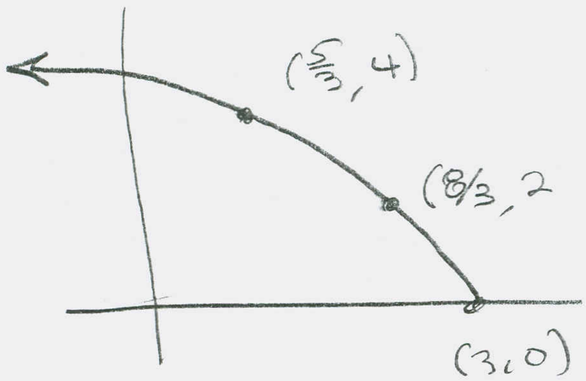
$f(x)=\sqrt{x}$ $f(-3x)=\sqrt{-3x}$ $f(-3(x-3))=\sqrt{-3(x-3)}$







$2f(-3(x-3))=2\sqrt{-3(x-3)}=g(x)$



10. Solve the absolute value inequalities:

a. $|2x-3|-1>5$

$|2x-3|>6$

$2x-3>6$ OR $2x-3<-6$

$2x>9$ OR $2x<-3$

$\{x \mid x>\frac{9}{2}$ OR $x<-\frac{3}{2}\}$

$(-\infty,-\frac{3}{2})\cup(\frac{9}{2},\infty)$

b. $|2x-3|-1\le -5$

$|2x-3|\le -4$

$\emptyset$