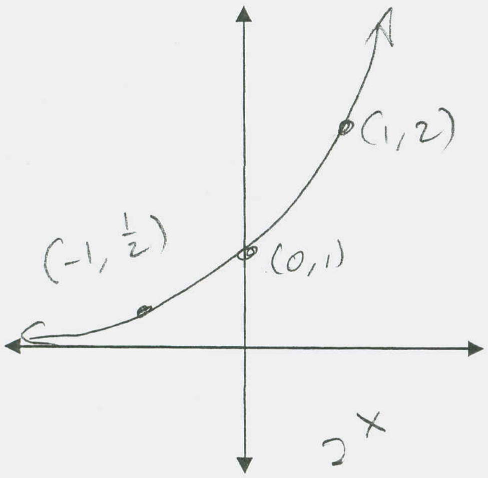
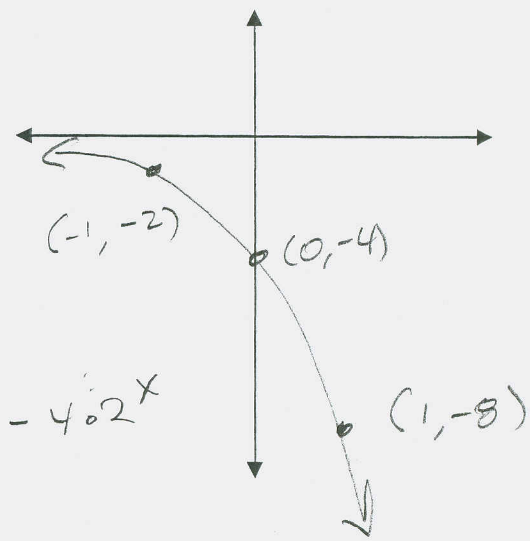
MAT 121 100 Points Fall, 2010

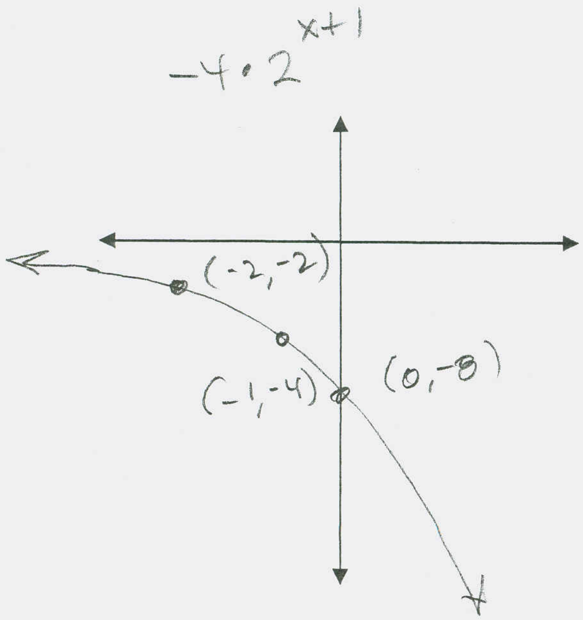
Test 4, Chapter 4 Name KEY

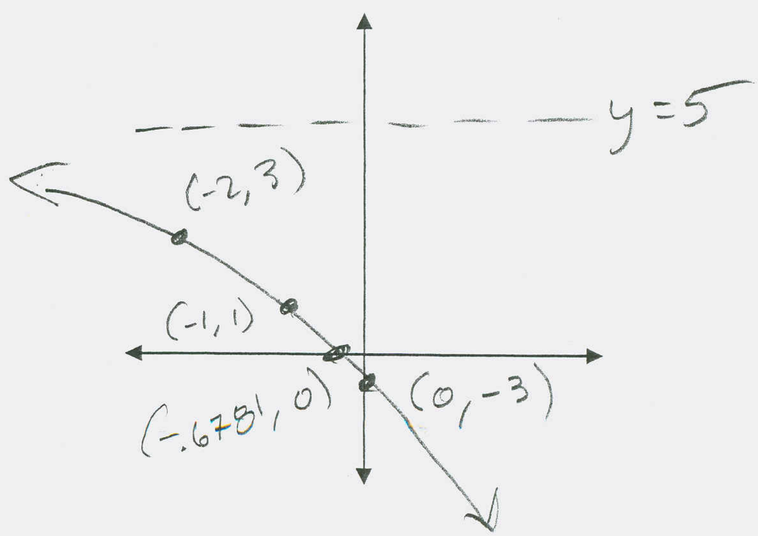
Work 10 of the following 12 problems. Omit two (2). If you omit a problem, write OMIT in the space provided. Otherwise, I'll grade the first 10 problems I come to, whether you work them or not.

1. (20 pts) Starting with $f(x) = 2^x$, sketch the graph of $g(x) = -4 \cdot 2^{x+1} + 5$ in 4 steps (counting $f(x) = 2^x$ as the first step). Use $x = -1$, $x = 0$, and $x = 1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Your final graph should also show the $y$-intercept and, for 5 bonus points, the $x$-intercept (See #5b).



2. (10 pts) Find the inverse of the function $g(x) = -3^{1-x} + 7$

$-3^{1-y} + 7 = x$

$-3^{1-y} = x - 7$

$3^{1-y} = 7 - x$

$1 - y = \log_3(7-x)$

$-y = \log_3(7-x) - 1$

$y = \boxed{1 - \log_3(7-x) = f^{-1}(x)}$

Check:

$1 - \log_3(7 - (-3^{1-x} + 7))$

$= 1 - \log_3(7 + 3^{1-x} - 7)$

$= 1 - \log_3(3^{1-x})$

$= 1 - (1-x) = x$ ✓

3. (10 pts) Solve $\ln(x-4) + \ln(x+1) = \ln(6)$ for $x$.

$\ln((x-4)(x+1)) = \ln(6)$

$(x-4)(x+1) = 6$

$x^2 - 3x - 4 = 6$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x \in \{-2, 5\}$.

$x = -2 \notin D \Rightarrow$ Soln Set is $\boxed{\{5\}}$

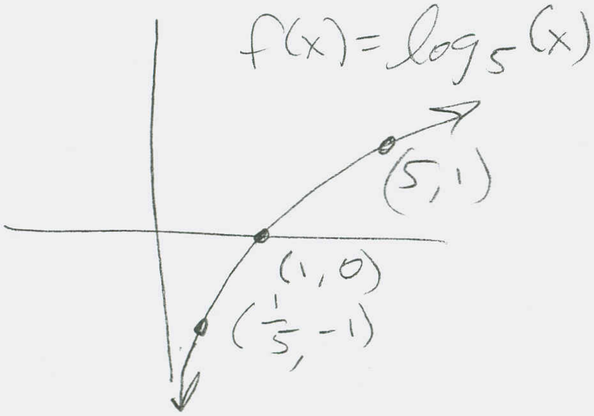
4. (10 pts) Graph $h(x) = \log_5(-2x-8)$. You can do it in 3 graphs (counting $f(x) = \log_5(x)$ as the first), if you combine the horizontal stretch/shrink with the horizontal reflection. If you do the stretch/shrink and the reflection separately, it will take 4 graphs. Use the same 3 key points that are used in class.

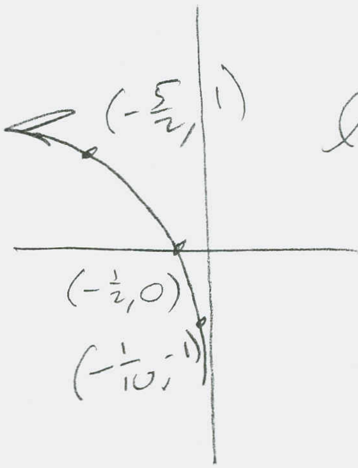
$-2x-8 = -2(x+4)$

$x \longrightarrow -2x \longrightarrow -2(x+4)$

$\log_5(x) \longrightarrow \log_5(-2x) \longrightarrow \log_5(-2(x+4))$

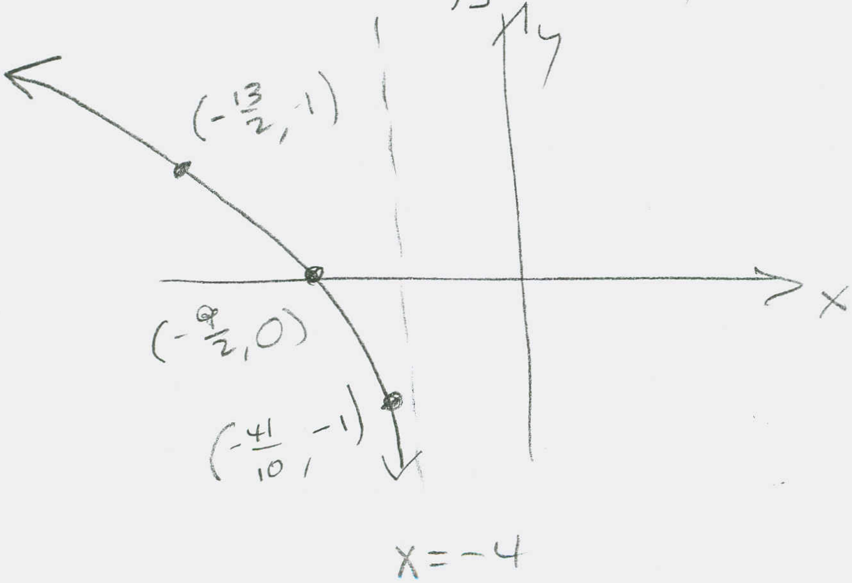
$-\frac{1}{2}$ times x    left 4




$\log_5(-2x)$
$= f(-2x)$

$h(x) = \log_5(-2x-8)$




$-\frac{1}{2} - 4 = -\frac{9}{2}$

$-\frac{5}{2} - 4 = -\frac{13}{2}$

$-\frac{1}{10} - 4 = -\frac{41}{10}$

5. Solve for $x$:

a. (10 pts) $9^{2x-3} = 3^{-3x+2}$.

$$\left(3^2\right)^{2x-3} = 3^{-3x+2}$$

$$3^{4x-6} = 3^{-3x+2}$$

$$4x-6 = -3x+2$$

$$7x = 8$$

$$\boxed{x = \frac{8}{7}}$$

b. (10 pts) $\cancel{-5^{1-x}+7} = 0$ (Solving this equation has a lot to do with the 1st question.)

$$-4 \cdot 2^{x+1} + 5 = 0$$

$$-4 \cdot 2^{x+1} = -5$$

$$2^{x+1} = \frac{-5}{-4} = \frac{5}{4}$$

$$x+1 = \log_2(5/4)$$

$$\boxed{x = \log_2\left(\frac{5}{4}\right) - 1} = \frac{\ln\left(\frac{5}{4}\right)}{\ln(2)} - 1 \approx -.6780719051$$

$$\approx \boxed{-.6781}$$

c. (10 pts) $5^{x-1} = 3^x$ for $x$. Give an exact answer and then round your answer to 4 decimal places.

$$x-1 = \log_5(3^x)$$

$$x-1 = x\log_5(3) = ax,$$

where $a = \log_5(3) \Longrightarrow$

$$x-1 = ax$$

$$x - ax = 1$$

$$x(1-a) = 1$$

$$x = \frac{1}{1-a} = \frac{1}{1-\log_5(3)} = \frac{1}{1-\ln(3)/\ln(5)} \approx 3.150660103$$

$$\approx \boxed{3.1507}$$

6. (10 pts) Radioactive Wieligminium-12.5 has a half-life of 250 years. What's its decay rate? Write the function modeling the amount of radioactive Wieligminium-12.5 remaining in a sample after $t$ years.

$$A_0 e^{-kt} = A(t)$$

$$A_0 e^{-250k} = \frac{1}{2} A_0$$

$$e^{-250k} = \frac{1}{2}$$

$$-250k = \ln\left(\frac{1}{2}\right)$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{-250} = \frac{\ln(2)}{250}$$

$$\boxed{A(t) = A_0 e^{-\frac{\ln(2)}{250} t}}$$

7. (10 pts) Using your work from the previous problem (No double jeopardy – go with what you have (or make something up!)), a sample of radioactive Wieligminium decayed from 20 grams to 5 grams. How old is the sample?

$$A_0 e^{-kt} = 20 e^{-kt} = 5 \longrightarrow$$

$$e^{-kt} = \frac{1}{4}$$

$$-kt = \ln\left(\frac{1}{4}\right)$$

$$t = \frac{\ln\left(\frac{1}{4}\right)}{-k} = \frac{\ln(4)}{k} = \frac{\ln(4)}{\frac{\ln(2)}{150}} = \frac{150 \ln(4)}{\ln(2)}$$

$$= \boxed{300 \text{ yrs}}$$