MAT 121 – Spring, 2011 Test 2
Chapter 2 (5 pts) Name KEY 2

1. (10 pts) $f = \{(1,-1),(2,4),(3,2),(4,5)\}$

   a. Function? (Yes/no) Yes

   b. If not, why not?

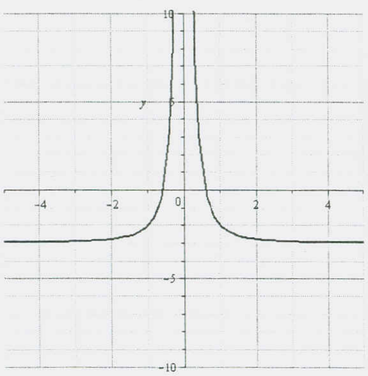
   c. If it *is* a function, is it 1-to-1? (Yes/no) Yes

   d. If it is *not* 1-to-1, why not?

   e. What's the domain? $\{1, 2, 3, 4\}$

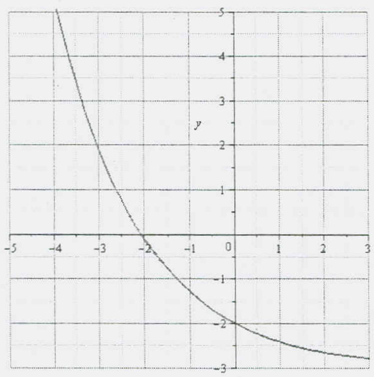
   f. What's the range? $\{-1, 4, 2, 5\}$

2. (10 pts) For each of the following graphs, determine if the relation is a function. If it *is* a function, state whether or not it is 1-to-1.



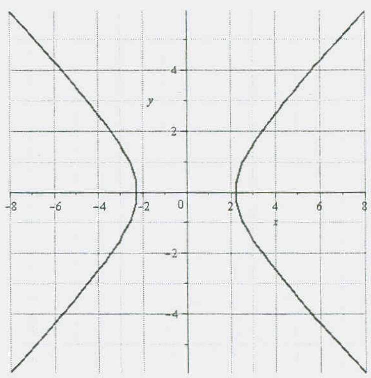
Is it a function? Yes

Is it 1-to-1? NO



Is it a function? Yes

Is it 1-to-1? Yes



Is it a function? NO

Is it 1-to-1?

3. (5 pts) Determine whether or not $|x+3|-2y=5$ defines y as a function of x. If it does not, show/explain why not. (Solve for y and look at how many solutions you get.)

$-2y = 5-|x+3|$

$y = -\frac{1}{2}(5-|x+3|)$

Yes.

4. (10 pts) Let $f(x)=x^2+3$. Simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$.

$\frac{(x+h)^2+3-(x^2+3)}{h} =$

$= \frac{x^2+2xh+h^2+3-x^2-3}{h}$

$= \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h}$

$= \boxed{2x+h}$

$h \neq 0$

5. Let $f(x)=\dfrac{x+2}{x+3}$ and $g(x)=\sqrt{x+5}$.

a. (5 pts) What is the domain of f? $\{x \mid x \neq -3\} = (-\infty,-3)\cup(-3,\infty)$

b. (5 pts) What is the domain of g? $\{x \mid x \geq -5\} = [-5,\infty)$

c. (5 pts) Find $(f\circ g)(x)$. (Do not simplify.)

$$\frac{\sqrt{x+5}+2}{\sqrt{x+5}+3}$$

SCRATCH:
$g(x) \neq -3$
$\sqrt{x+5} \neq -3$
is always true.

d. (5 pts) What is the domain of $(f\circ g)(x)$?

$$\mathcal{D}=\{x \mid x\in\mathcal{D}(g) \text{ and } g(x)\in\mathcal{D}(f)\}$$
$$=\{x \mid x\geq -5 \text{ and } g(x)\neq -3\}$$
$$=\{x \mid x\geq -5\} = [-5,\infty)$$

e. Determine each of the following functions (without simplifying) and state the domain of each in *interval notation*.

i. (5 pts) $(f+g)(x)$

$$=\frac{x+2}{x+3}+\sqrt{x+5}$$
$$\mathcal{D}=\{x \mid x\in\mathcal{D}(f) \text{ and } x\in\mathcal{D}(g)\}=\{x \mid x\neq -3 \text{ and } x\geq -5\}$$
$$=\boxed{[-5,-3)\cup(-3,\infty)}$$

ii. (5 pts) $\left(\dfrac{g}{f}\right)(x)$

$$=\frac{\sqrt{x+5}}{\frac{x+2}{x+3}}$$

$\mathcal{D}$ = Same AND need $f(x)\neq 0$

$$=\{x \mid x\neq -3 \text{ and } x\geq -5 \text{ \underline{and} } x\neq -2\}$$
$$=\boxed{[-5,-3)\cup(-3,-2)\cup(-2,\infty)}$$

6. (5 pts) Answer *one* of the following:

a. Show that $f(x)=\dfrac{x-1}{x+2}$ is 1-to-1, algebraically.

b. Let $f(x)=\dfrac{x-1}{x+2}$. Find $f^{-1}(x)$.

a. $\dfrac{x_1-1}{x_1+2}=\dfrac{x_2-1}{x_2+2}$

$(x_1-1)(x_2+2)=(x_2-1)(x_1+2)$

$x_1x_2+2x_1-x_2-2=x_2x_1+2x_2-x_1-2$

$2x_1-x_2=2x_2-x_1$

$3x_1=3x_2$

$x_1=x_2$ ■

b. $x=\dfrac{y-1}{y+2}$

$x(y+2)=y-1$

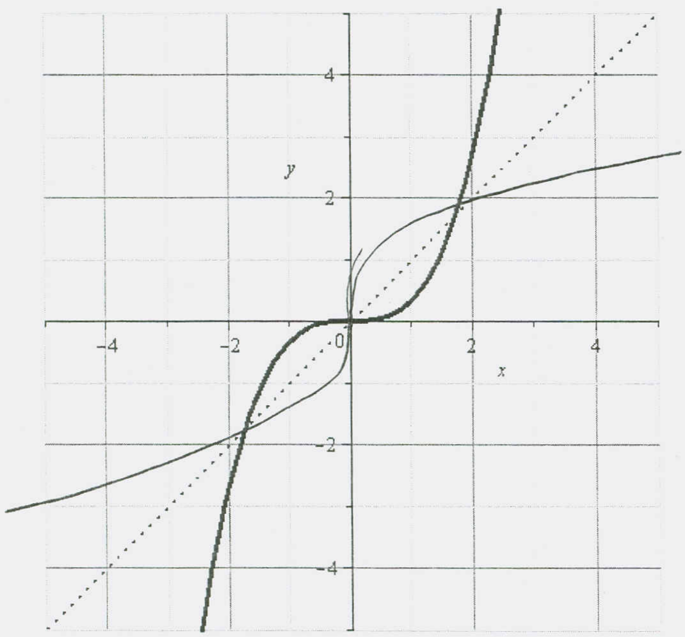
$xy+2x=y-1$

$xy-y=-2x-1$

$y(x-1)=-2x-1$

$y=\boxed{\dfrac{-2x-1}{x-1}=f^{-1}(x)}$

7. (5 pts) The graph of f is given. Sketch the graph of f^{-1}.



8. (5 pts) If f varies jointly as q^2 and h, and $f = -36$ when $q = 3$ and $h = 2$, find f when $q = 4$ and $h = 2$.

$f = kq^2h$

$-36 = k \cdot 3^2 \cdot 2$

$k = \frac{-36}{18} = -2$

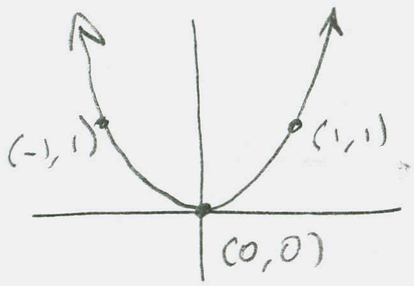
$f = (-2)(4^2)(2)$

$= \boxed{-64 = f}$

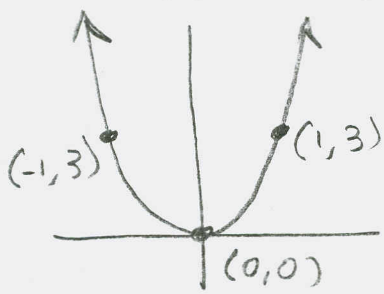
9. Graph each of the following functions using techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages in separate sketches. Track 3 key points through the transformations.

a. (5 pts) $h(x) = 3(x+5)^2 + 1$

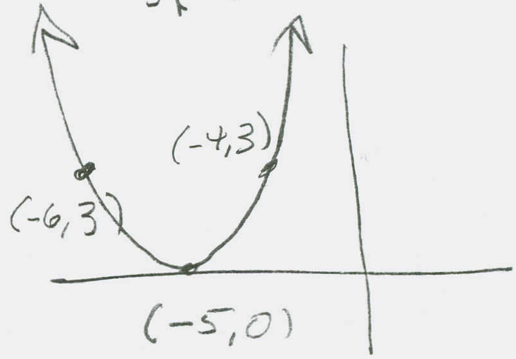
$f(x) = x^2$

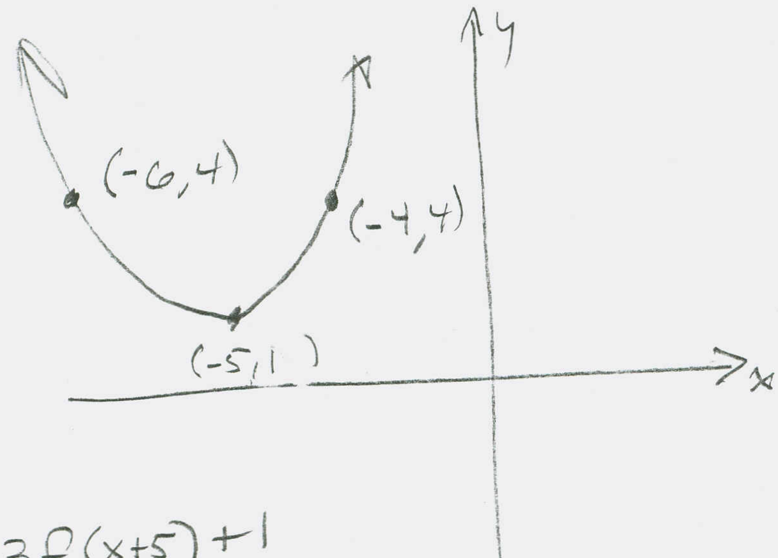


$3f(x) = 3x^2$



$3f(x+5) = 3(x+5)^2$





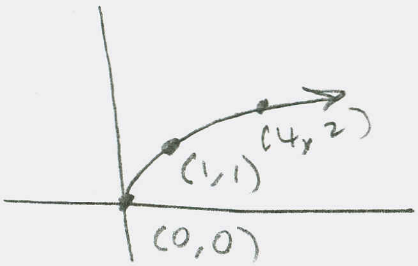
$3f(x+5)+1$

$= 3(x+5)^2 + 1$

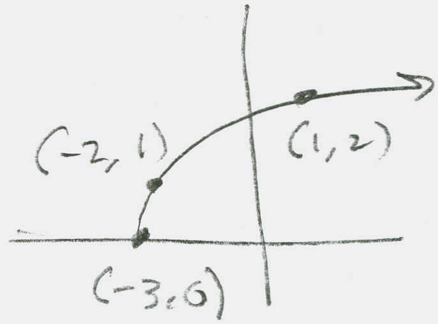
#9, continued... Graph using transformations.

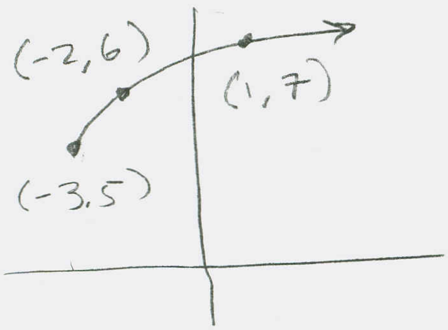
b. (5 pts) $g(x)=\sqrt{3-x}+5$ (Hint: $3-x=-x+3$ is one way. $3-x=-(x-3)$ is another.)

$f(x)=\sqrt{x}$

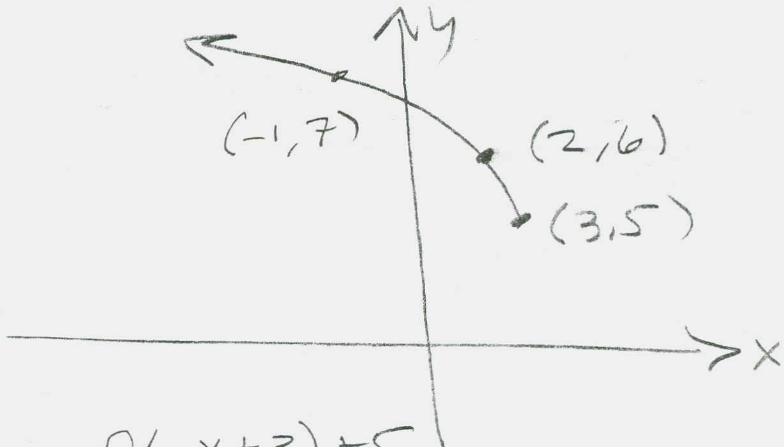


$f(x+3)=\sqrt{x+3}$





$f(x+3)+5$



$f(-x+3)+5$

$=\sqrt{3-x}+5=g(x)$

10. (5 pts) Find the inverse of $f(x)=3x-7$

$x=3y-7$

$x+7=3y$

$y=\boxed{\dfrac{x+7}{3}=f^{-1}(x)}$