MAT 121 – Spring, 2011 Test 2
Chapter 2 (5 pts) Name KEY 1

1. (10 pts) $f = \{(1,-1),(2,4),(3,2),(4,4)\}$

   a. Function? (Yes/no) Yes

   b. If not, why not?

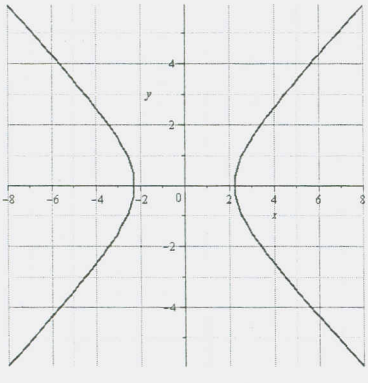
   c. If it *is* a function, is it 1-to-1? (Yes/no) No

   d. If it is *not* 1-to-1, why not? $f(2) = f(4) = 4$

   e. What's the domain? $\{1, 2, 3, 4\}$

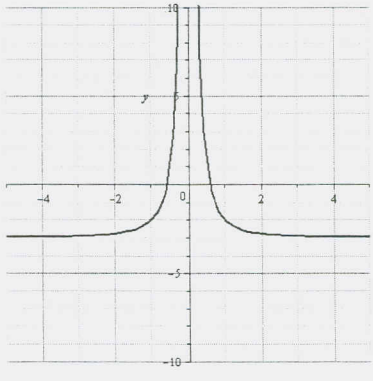
   f. What's the range? $\{-1, 4, 2\}$

2. (10 pts) For each of the following graphs, determine if the relation is a function. If it is a function, state whether or not it is 1-to-1.



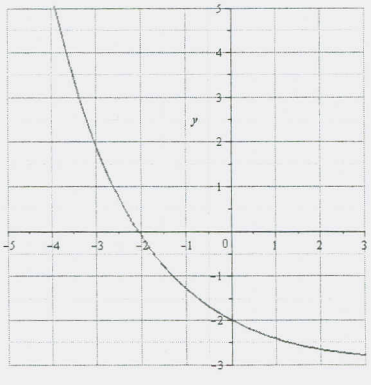
Is it a function? No

If it is a function, is it 1-to-1?



Is it a function? Yes

If it is a function, is it 1-to-1? No



Is it a function? Yes

If it is a function, is it 1-to-1? Yes

3. (5 pts) Determine whether or not $|y+3|-2x=5$ defines y as a function of x. If it does not, show/explain why not. (Solve for y and look at how many solutions you get.)

$|y+3|-2x=5$

$|y+3|=2x+5$

$y+3=\pm(2x+5)$

$y=-3\pm(2x+5)$

Two values for y, given one x input.

MORE DETAIL:

$y+3=2x+5$ OR $y+3=-(2x+5)$

$y=2x+2$ OR $y=-2x-8$

$x=0 \rightarrow y=2$ OR $y=-8$ Not Func.

4. (10 pts) Let $f(x)=x^2+3$. Simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+3-(x^2+3)}{h}$$

$$= \frac{x^2+2xh+h^2+3-x^2-3}{h}$$

$$= \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h} = \boxed{2x+h} \quad (h\neq 0)$$

5. Let $f(x)=\dfrac{x-2}{x-5}$ and $g(x)=\sqrt{x-2}$.

a. (5 pts) What is the domain of f? $(-\infty,5)\cup(5,\infty)=\{x\mid x\neq 5\}$

b. (5 pts) What is the domain of g? $[2,\infty)=\{x\mid x\geq 2\}$

c. (5 pts) Find $(f\circ g)(x)$. (Do not simplify.)

$$(f\circ g)(x)=\frac{\sqrt{x-2}-2}{\sqrt{x-2}-5}$$

d. (5 pts) What is the domain of $(f\circ g)(x)$?

$\mathcal{D}=\{x\mid x\in\mathcal{D}(g) \text{ and } g(x)\in\mathcal{D}(f)\}$

$=\{x\mid x\geq 2 \text{ and } g(x)\neq 5\}$

$=\{x\mid x\geq 2 \text{ and } x\neq 27\}$

$=[2,27)\cup(27,\infty)$

SCRATCH:

$g(x)=5$

$\sqrt{x-2}=5$

$x-2=25$

$x=27$

e. Determine each of the following functions (without simplifying) and state the domain of each in *interval notation*.

i. (5 pts) $(f+g)(x)=\dfrac{x-2}{x-5}+\sqrt{x-2}$

$\mathcal{D}=\{x\mid x\in\mathcal{D}(f) \text{ and } x\in\mathcal{D}(g)\}$

$=\{x\mid x\neq 5 \text{ and } x\geq 2\}=\boxed{[2,5)\cup(5,\infty)}$

ii. (5 pts) $\left(\dfrac{g}{f}\right)(x)=\dfrac{\sqrt{x-2}}{\frac{x-2}{x-5}}$

$\mathcal{D}=\{x\mid x\in\mathcal{D}(f) \text{ and } f(x)\neq 0 \text{ and } x\in\mathcal{D}(g)\}$

$=\{x\mid x\neq 5 \text{ and } x\neq 2 \text{ and } x\geq 2\}=\boxed{(2,5)\cup(5,\infty)}$

6. (5 pts) Answer *one* of the following:

a. Show that $f(x)=\dfrac{x-1}{x+2}$ is 1-to-1, algebraically.

b. Let $f(x)=\dfrac{x-1}{x+2}$. Find $f^{-1}(x)$.

a. $f(x_1)=f(x_2)$

$$\frac{x_1-1}{x_1+2}=\frac{x_2-1}{x_2+2}$$

$$(x_1-1)(x_2+2)=(x_2-1)(x_1+2)$$

$$x_1x_2+2x_1-x_2-2=x_2x_1+2x_2-x_1-2$$

$$2x_1-x_2=2x_2-x_1$$

$$3x_1=3x_2$$

$$x_1=x_2$$ ▨

b. $x=\dfrac{y-1}{y+2}$

$$x(y+2)=y-1$$

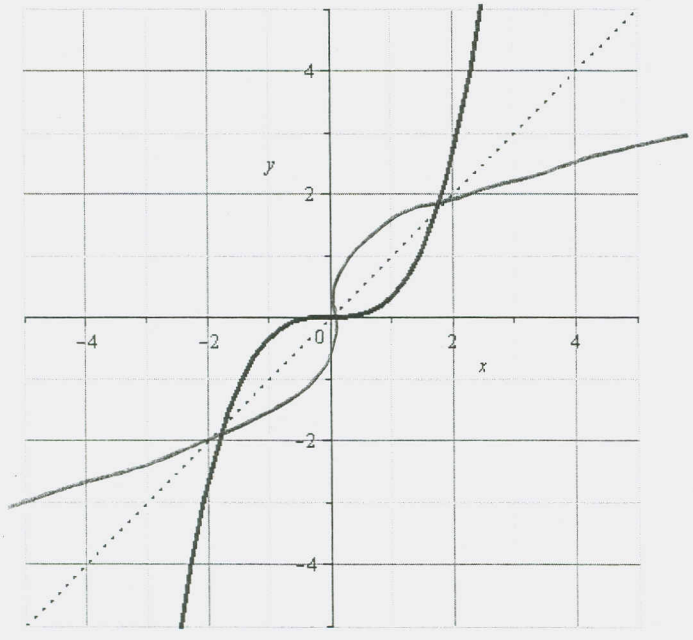
$$xy+2x=y-1$$

$$xy-y=-2x-1$$

$$y(x-1)=-2x-1$$

$$y=\boxed{\frac{-2x-1}{x-1}=f^{-1}(x)}$$

7. (5 pts) The graph of f is given. Sketch the graph of f^{-1}.

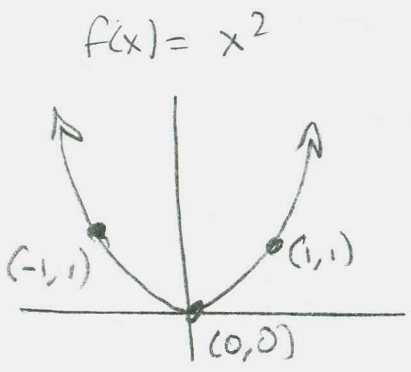


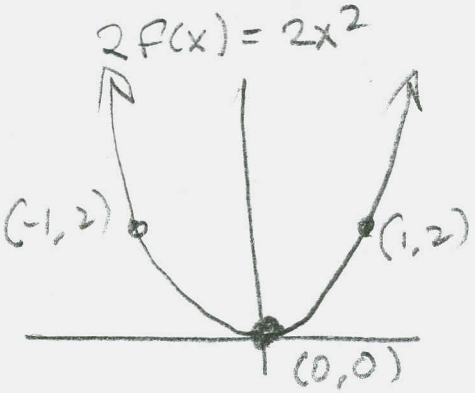
8. (5 pts) If f varies jointly as m_1 and m_2 and inversely with the square of r, write the equation describing this relationship.

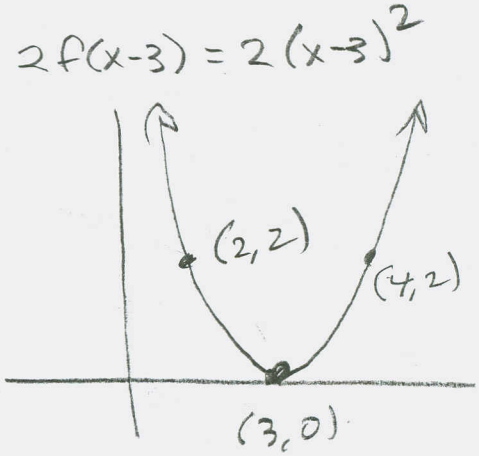
$$f = \frac{k m_1 m_2}{r^2}$$

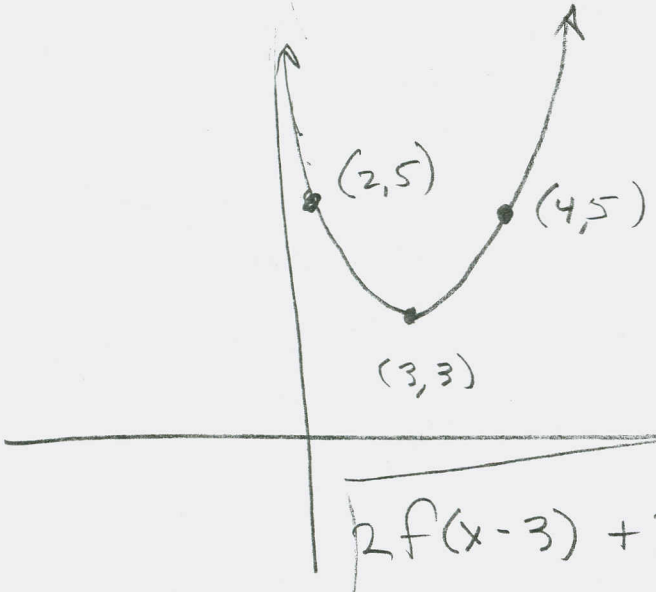
9. Graph each of the following functions using techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages in separate sketches. Track 3 key points through the transformations.

a. (5 pts) $h(x) = 2(x-3)^2 + 3$





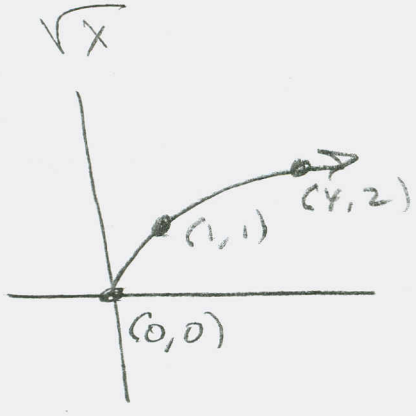


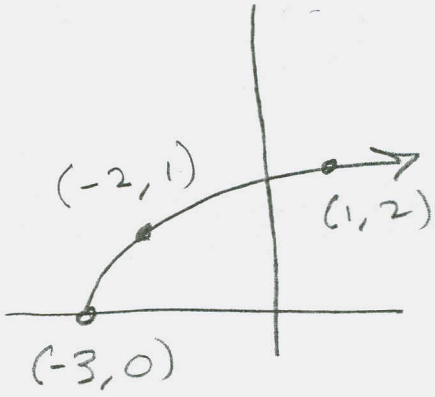


$2f(x-3) + 3 = h(x) = 2(x-3)^2 + 3$

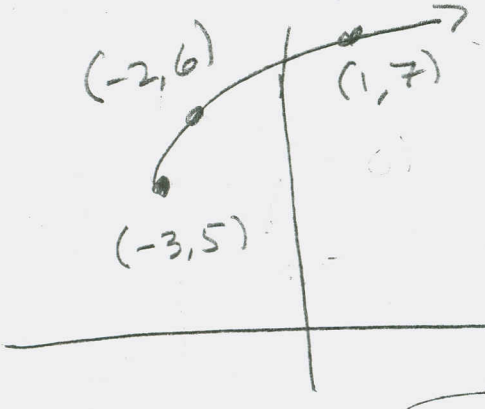
#9, continued... Graph using transformations.

b. (5 pts) $g(x) = \sqrt{3-x}+5$ (Hint: $3-x=-x+3$ is one way. $3-x=-(x-3)$ is another.)

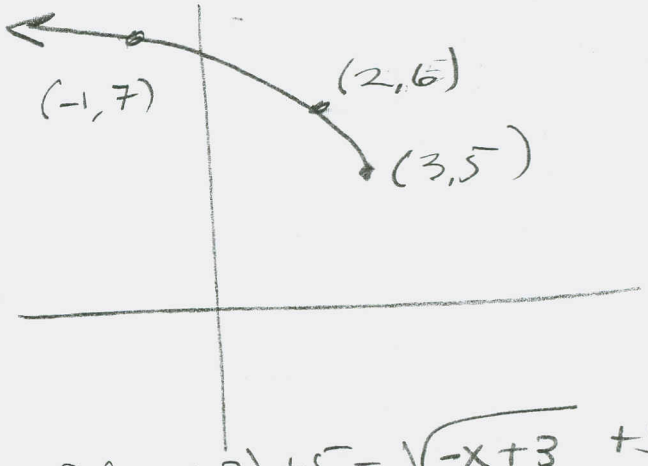




$\sqrt{x+3} = f(x+3)$



$f(x+3)+5 = \sqrt{x+3}+5$



$f(-x+3)+5 = \sqrt{-x+3}+5$

$= \sqrt{3-x}+5 = g(x)$

10. (5 pts) Find the inverse of $f(x) = 3x-7$

$x = 3y - 7$

$x+7 = 3y$

$\frac{x+7}{3} = y = f^{-1}(x)$