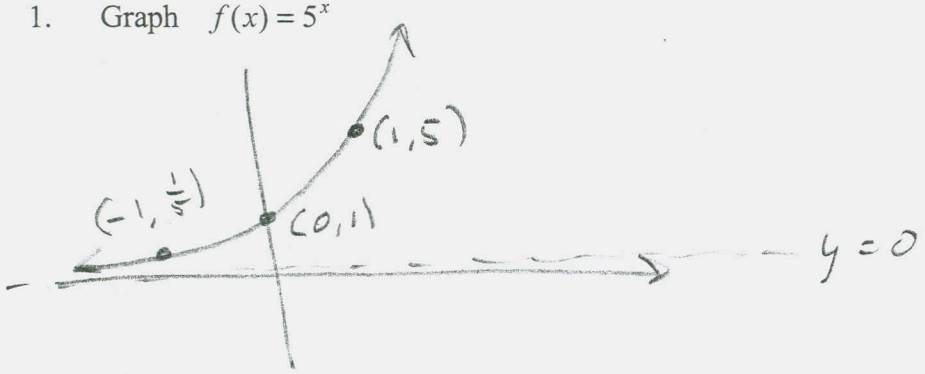
MAT 121 100 Points Fall, 2010

Test 4, Chapter 4 Name KEY

Work 10 of the following 12 problems. Omit two (2). If you omit a problem, write OMIT in the space provided. Otherwise, I'll grade the first 10 problems I come to, whether you work them or not.

1. Graph $f(x) = 5^x$



2. Graph $g(x) = -5^{1-x} + 7$ by transforming the basic function $f(x) = 5^x$

① $f(x) = 5^x$ See above

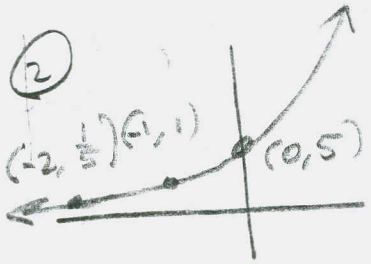
② $f(x+1) = 5^{x+1}$

③ $f(-x+1) = 5^{-x+1}$

$= 5^{1-x} = f(1-x)$

④ $-f(1-x) = -5^{1-x}$

⑤ $-f(1-x) + 7 = -5^{1-x} + 7$



5^{x+1}

$f(x+1)$

Alternate:

5^x, 5^{-x}, $5^{-(x-1)}$

$-5^{-(x-1)}$, $-5^{-(x-1)} + 7$

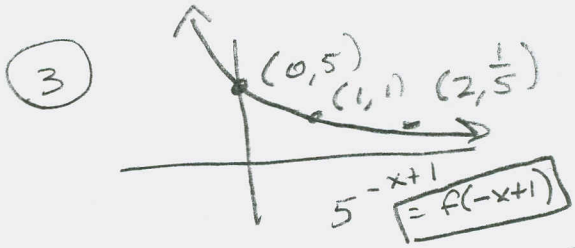
Flip ↔

Right 1

Flip ↕

up 7

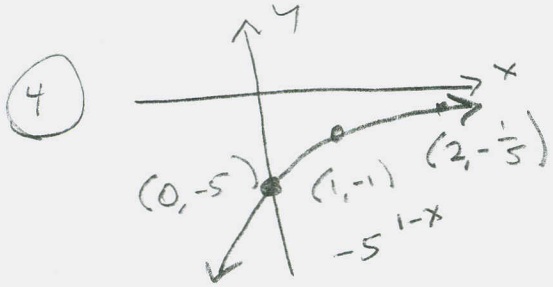
5^{-x+1}



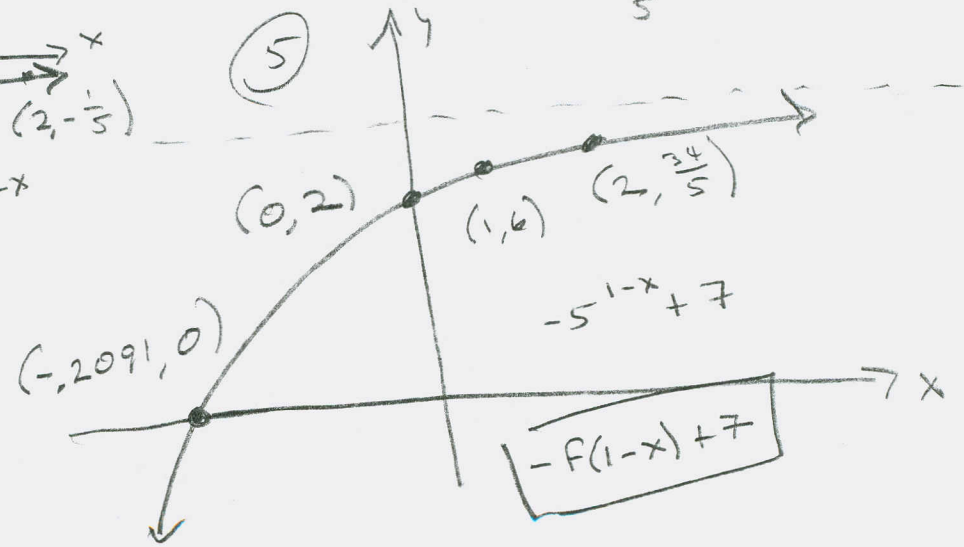
5^{-x+1}

$= f(-x+1)$

$-\frac{1}{5} + 7 = \frac{-1+35}{5} = \frac{34}{5}$



$-f(-x+1)$



$-f(1-x) + 7$

3. Find the inverse of the function $g(x) = -5^{1-x} + 7$

$x = -5^{1-y} + 7 = x$

$-5^{1-y} = x - 7$

$5^{1-y} = -x + 7$

$\log_5(5^{1-y}) = \log_5(-x+7)$

$1 - y = \log_5(7-x)$

$-y = \log_5(7-x) - 1$

$y = -\log_5(7-x) + 1$

$= f^{-1}(x)$

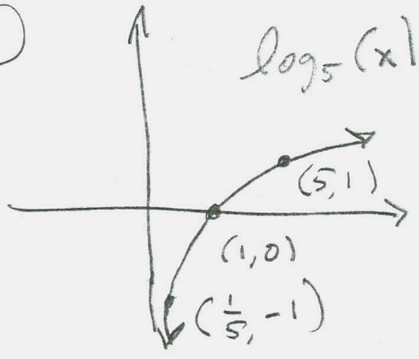
4. Graph $h(x) = -\log_5(x-3)$

① $\log_5(x)$

② $\log_5(x-3)$

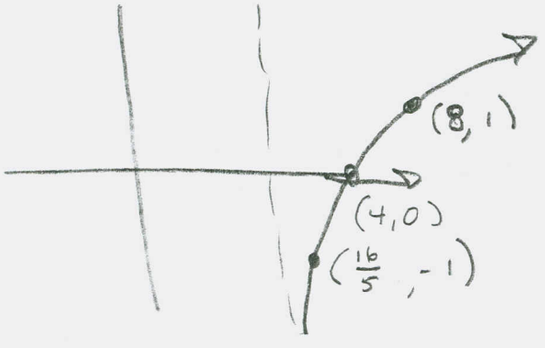
③ $-\log_5(x-3)$

① $\log_5(x)$



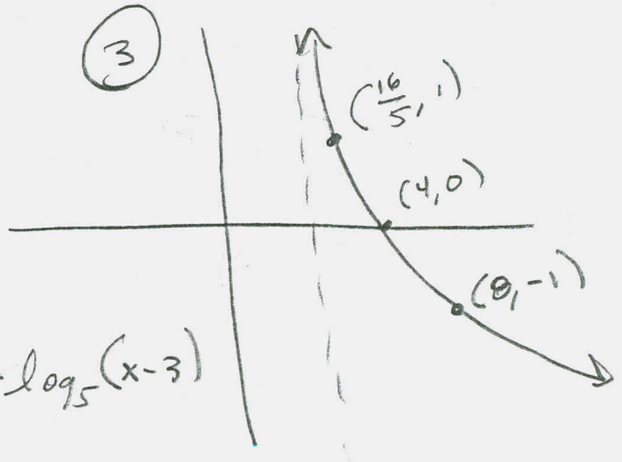
$\frac{1}{5} + 3 = \frac{1+15}{5} = \frac{16}{5}$

② $\log_5(x-3)$



$x = 3$

③



$x = 3$

5. Solve $\log_5(x-4)+\log_5(x+2)=\log_5(7)$ for x.

$\log_5(x^2-2x-8)=\log_5(7)$

$x^2-2x-8=7$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x\in\{-3,5\}$

$-3\notin D$

$\boxed{x\in\{5\}}$

6. Solve for t: $A=P\left(1+\frac{r}{m}\right)^{mt}$.

$P\left(1+\frac{r}{m}\right)^{mt}=A$

$\left(1+\frac{r}{m}\right)^{mt}=\frac{A}{P}$

$mt\ln\left(1+\frac{r}{m}\right)=\ln\left(\frac{A}{P}\right)$

$\boxed{t=\frac{\ln\left(\frac{A}{P}\right)}{m\ln\left(1+\frac{r}{m}\right)}}$

7. Solve $-5^{1-x}+7=0$ for x. Give an exact answer and then round to 4 decimal places. If you use this to supply the x-intercept for the appropriate graph on Page 1, it's worth a couple bonus points.

$$-5^{1-x} = -7$$

$$5^{1-x} = 7$$

$$1-x = \log_5(7)$$

$$-x = \log_5(7) - 1$$

$$\boxed{x = -\log_5(7) + 1}$$

$$= -\frac{\ln(7)}{\ln(5)} + 1$$

$$\approx -.2090619551$$

$$\approx -.2091$$

8. Solve $5^{x-1} = 3^x$ for x. Give an exact answer and then round your answer to 4 decimal places.

$$(x-1)\ln(5) = x\ln(3)$$

$$a = \ln(5),\ b = \ln(3)$$

$$a(x-1) = bx$$

$$ax - a = bx$$

$$ax - bx = a$$

$$(a-b)x = a$$

$$x = \frac{a}{a-b}$$

$$= \boxed{\frac{\ln(5)}{\ln(5)-\ln(3)} = x}$$

$$\approx 3.150660103$$

$$\approx \boxed{3.1507 \approx x}$$

9. Radioactive Wieligminium-12.5 has a half-life of 100 years. What's its decay rate?

$$Pe^{-(k)(100)} = \frac{1}{2}P$$

$$e^{-100k} = \frac{1}{2}$$

$$-100k = \ln\left(\frac{1}{2}\right)$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{-100} = \frac{\ln(2)}{100} \approx .006931471 8$$

Decay rate is approximately

.693147180%

10. Using your work from the previous problem, a very old sample of radioactive Wieligminium decayed from 14 grams to 5 grams. To the nearest *day*, how old is the sample?

$$14e^{-kt} = 5$$

$$e^{-kt} = \frac{5}{14}$$

$$-kt = \ln\left(\frac{5}{14}\right)$$

$$t = \frac{\ln\left(\frac{5}{14}\right)}{-k} = \frac{\ln\left(\frac{14}{5}\right)}{k}$$

$$= \frac{\ln\left(\frac{14}{5}\right)}{\frac{\ln(2)}{100}} = \frac{100\ln\left(\frac{14}{5}\right)}{\ln(2)} \approx 148.5426827$$

$$(.5426827 \text{ yrs})\left(\frac{365 \text{ days}}{1 \text{ yr}}\right)$$

$$\approx 198.0791917$$

148 yrs, 198 days

.5426827 yrs

11. Solve $(\log(x))^2 = \log(x^2)$ for x.

$(\log(x))^2 = 2\log(x)$

$u^2 = 2u$

$u^2 - 2u = 0$

$u(u-2) = 0$

$u = 0$ OR $u = 2$

$\log x = 0$ $\log x = 2$

$x = 1$ $x = 100$

$x \in \{1, 100\}$

12. What's the future value of \$5,000 invested at 4% APR, if interest is compounded... 5 yrs

a. ... monthly? 6104.98297

$5000\left(1 + \frac{.04}{12}\right)^{12(5)} \approx \$6104.98$

b. ... daily? 6106.94657

$5000\left(1 + \frac{.04}{365}\right)^{365(5)} \approx \$6106.95$

c. ... continuously? 6107.013791

$5000e^{.04(5)} \approx \$6107.01$