MAT 121 – Fall, 2010 Test 2
Chapter 2

Name KEY

1. (10 pts) $f = \{(2,-1),(3,-2),(4,2),(3,4)\}$

   a. Function? (Yes/no) No

   b. If not, why not? $x=3$ is paired w/ $y=-2$ AND $y=4$

   c. If it *is* a function, is it 1-to-1? (Yes/no) X

   d. If it is *not* 1-to-1, why not? X

   e. Domain? $\mathcal{D} = \{2,3,4,3\}$

   f. Range? $\mathcal{R} = \{-1,-2,2,4\}$

2. (5 pts) $g = \{(2,-2),(4,6),(3,2),(11,5)\}$

   a. Function? (Yes/no) Yes

   b. If not, why not? X

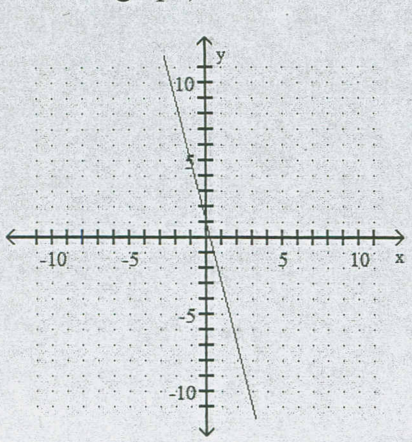
   c. If it *is* a function, is it 1-to-1? (Yes/no) Yes

   d. If it is *not* 1-to-1, why not? X

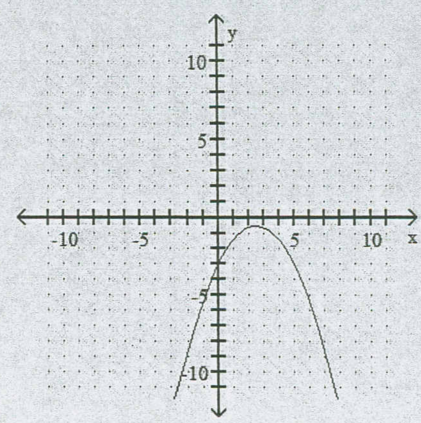
   e. Domain? $\mathcal{D} = \{2,4,3,11\}$

   f. Range? $\mathcal{R} = \{-2,6,2,5\}$

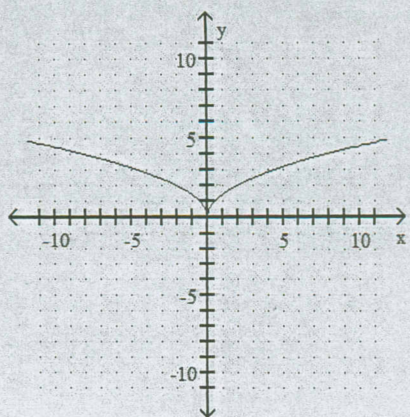
3. (5 pts) Use the horizontal line test to determine if the following functions are 1-to-1 (Yes/No for each graph).



Yes



No



No

4. (5 pts) Determine whether or not $|y-2|+x=7$ defines y as a function of x. If it does not, show/explain why not.

No. $|y-2| = -x+7$

$y-2=-x+7$ OR $y-2=x-7$

$\underbrace{y=-x+9 \quad \text{OR} \quad y=x-5}$

Two different y-values for single x-value, e.g.

$x=0 \Longrightarrow y=9$ OR $y=-5$

This gives $(0,9)$, $(0,-5)$ in the relation

5. (10 pts) Let $f(x)=x^2-5$. Simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-5-(x^2-5)}{h} =$$

$$= \frac{x^2+2xh+h^2-5-x^2+5}{h}$$

$$= \frac{2xh+h^2}{h}$$

$$= \frac{h(2x+h)}{h}$$

$$= \boxed{2x+h}$$

6. Let $f(x) = 3x - 5$ and $g(x) = \sqrt{x+7}$.

a. (5 pts) What is the domain of f? (Use Interval Notation.)

$(-\infty, \infty)$

b. (5 pts) What is the domain of g? (Use Interval Notation.)

$[-7, \infty)$

Need $x + 7 \geq 0$

$x \geq -7$

$[-7, \infty)$

c. Determine each of the following functions and state the domain of each in interval notation.

i. (5 pts) $(f+g)(x)$

$\boxed{= 3x - 5 + \sqrt{x+7} \quad \mathcal{D} = [-7, \infty)}$

ii. (5 pts) $\left(\frac{g}{f}\right)(x)$

$\boxed{= \frac{\sqrt{x+7}}{3x-5}}$

$\boxed{[-7, \tfrac{5}{3}) \cup (\tfrac{5}{3}, \infty)}$

Need $x \geq -7$

and $3x - 5 \neq 0$

$\Rightarrow x \neq \frac{5}{3}$

iii. (5 pts) $(f \circ g)(x)$

$= f(g(x)) =$

$= f(\sqrt{x+7}) =$

$\boxed{= 3\sqrt{x+7} - 5}$

$\mathcal{D} = [-7, \infty)$

iv. (5 pts) $(g \circ f)(x)$

$= g(f(x))$

$= g(3x-5)$

$= \sqrt{3x - 5 + 7}$

$\boxed{= \sqrt{3x+2}}$

Need $3x + 2 \geq 0$

$3x \geq -2$

$x \geq -\frac{2}{3}$

$\boxed{\mathcal{D} = [-\tfrac{2}{3}, \infty)}$

7. (5 pts) Show that $f(x)=\dfrac{x+1}{x-3}$ is 1-to-1.

S $f(x_1)=f(x_2)$

Then $\dfrac{x_1+1}{x_1-3}=\dfrac{x_2+1}{x_2-3}$

$(x_1+1)(x_2-3)=(x_2+1)(x_1-3)$

$x_1x_2-3x_1+x_2-3=x_1x_2-3x_2+x_1-3$

$-3x_1+x_2=-3x_2+x_1$

$-4x_1=-4x_2$

$x_1=x_2$ $\blacksquare$

8. (5 pts) Let $f(x)=\dfrac{x+1}{x-3}$. Find $f^{-1}(x)$.

$x=\dfrac{y+1}{y-3}$

$x(y-3)=y+1$

$xy-3x=y+1$

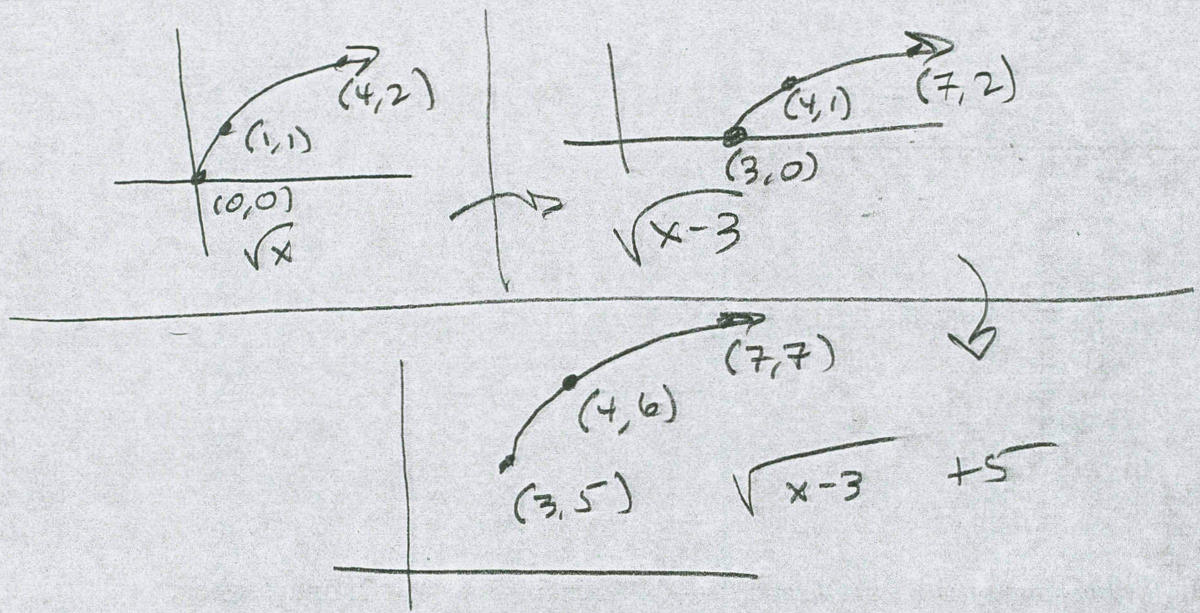
$xy-y=3x+1$

$y(x-1)=3x+1$

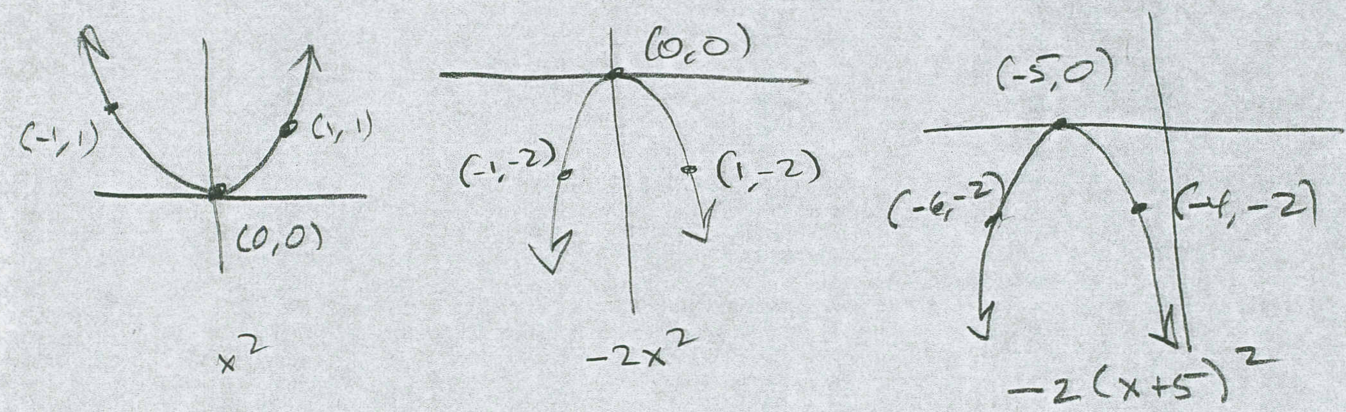
$f^{-1}(x)=\dfrac{3x+1}{x-1}$

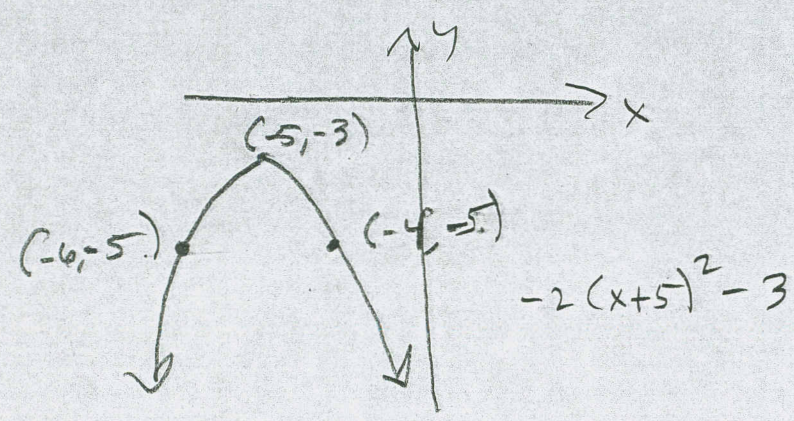
9. Graph each of the following functions using techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages in separate sketches. Track 3 key points through the transformations.

a. (5 pts) $g(x) = \sqrt{x-3} + 5$

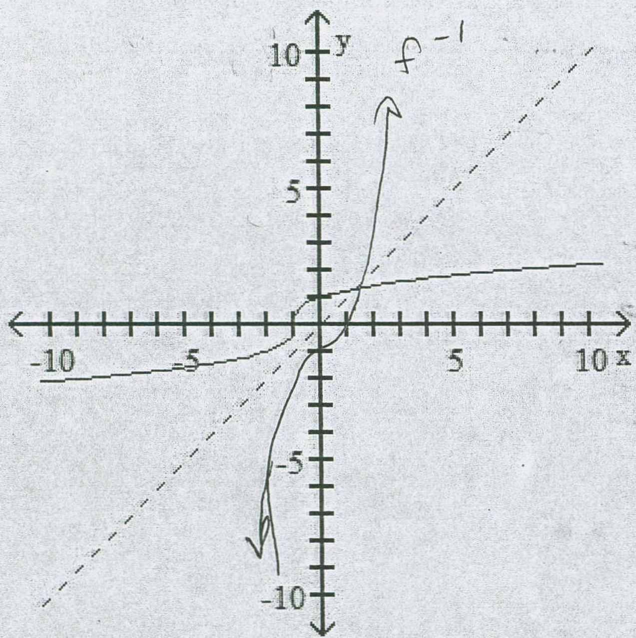


b. (5 pts) $h(x) = -2(x+5)^2 - 3$





10. (5 pts) The graph of f is given. Sketch the graph of f^{-1}.



11. (5 pts) If f varies jointly as q^2 and h, and $f = -36$ when $q = 3$ and $h = 2$, find f when $q = 4$ and $h = 2$.

$f = k\,q^2 h$

$-36 = k(3)^2(2)$

$-36 = 18k$

$-2 = k$

$\boxed{f = -2q^2h}$

$f = -2(4)^2(2)$

$\boxed{f = -64}$

12. (5 pts) Find the inverse of $f(x) = 3x - 7$

$x = 3y - 7 = x$

$3y = x + 7$

$y = \frac{x+7}{3}$

$\boxed{f^{-1}(x) = \frac{1}{3}x + \frac{7}{3}}$