121-G11 Trad. Lec.

MAT 121 – Fall, 2010 Test 1

Chapter 1 Name KEY

Solve the following equations. For each equation, state whether it is conditional, identity or inconsistent.

5pts 1. $\frac{3}{2}x+\frac{1}{3}=\frac{1}{4}x-\frac{1}{6}$ LCD = 24

$24\left(\frac{3}{2}x\right)+24\left(\frac{1}{3}\right)=24\left(\frac{1}{4}x\right)-24\left(\frac{1}{6}\right)$

$12(3x)+8(1)=6x-4(1)$

$36x+8=6x-4$

$30x=-12 \Rightarrow x=-\frac{12}{30}=\boxed{-\frac{2}{5}=x}$ $x\in\{-\frac{2}{5}\}$

5pts 2. $\frac{1}{x-1}-\frac{1}{x+1}=\frac{2}{x^2-1}$ LCD $=(x-1)(x+1)$

$(x-1)(x+1)\left(\frac{1}{x-1}\right)-(x-1)(x+1)\left(\frac{1}{x+1}\right)=\left(\frac{2}{(x-1)(x+1)}\right)(x-1)(x+1)$

$x+1-(x-1)=2$

$2=2$

IDENTITY

$x\in\boxed{\{x \mid x\neq\pm 1\}}$

3. (5 pts bonus) Give an example of an equation in the variable x that is...

a. ... an identity. $3x+2=3x+2$

b. ... inconsistent. $3x+2=3x-1$

10 +5

5pts 4. The old combine can harvest the crop in 96 hours, but a new one can do it in 72 hours. How many hours will it take the two of them operating at the same time to harvest the crop?

Let x = the # of hours it takes them to finish, if they work together.

Then $\frac{1}{96}x + \frac{1}{72}x = 1$

$288\left(\frac{1}{96}x\right) + 288\left(\frac{1}{72}x\right) = 288(1)$

$3x + 4x = 288$

$7x = 288$

$x = \frac{288}{7}$ hrs

≈ 41.14 hrs

96	72
2	2
48	36
2	2
24	18
2	3
12	9
2	3
6	3
3	

$LCD = 2^5 \cdot 3^2 = 288$

5pts 5. How much 75% antifreeze solutions should be added to 3 quarts of 30% antifreeze solution to yield a 50% antifreeze solution?

Let x = the amt of 75% antifreeze soln (quarts)

Then

$.75x + .3(3) = .5(x+3)$

$.75x + .9 = .5x + 1.5$

$.25x = .6$

$x = \frac{.6}{.25} = 2.4$ qts $= x$

5pts 6. Find the *exact* distance between (- 2, 5) and (6, 8).

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(6-(-2))^2 + (8-5)^2}$

$= \sqrt{8^2+3^2} = \sqrt{64+9} = \sqrt{73}$ sq

5

7. Determine the center and radius of the circle given by

5pts

~~$x^2 - 4x + 29 + y^2 + 10y = 49$~~ $x^2 - 4x + y^2 + 10y = 20$

$x^2 - 4x \quad + y^2 + 10y \quad = 20$

$x^2 - 4x + 2^2 + y^2 + 10y + 5^2 = 20 + 4 + 25$

$(x-2)^2 + (y+5)^2 = 49$

$(h, k) = (2, -5)$

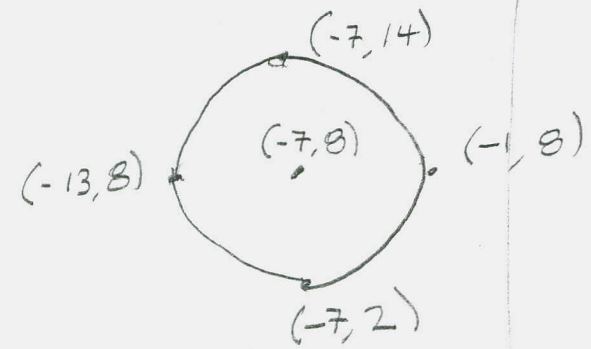
$r = 7$

8. Sketch the graph of $(x+7)^2 + (y-8)^2 = 36$.

$(h, k) = (-7, 8)$

$r = 6$

5pts

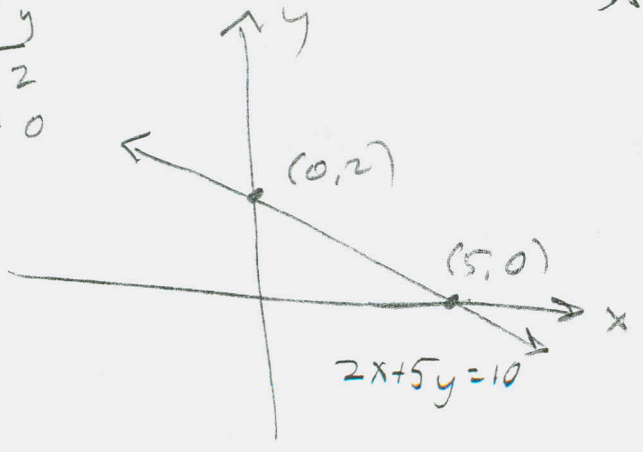


9. Sketch the graph of each of the following equations. Main points *I* want to see are the intercepts.

a. $2x + 5y = 10$

5pts

x	y
0	2
5	0



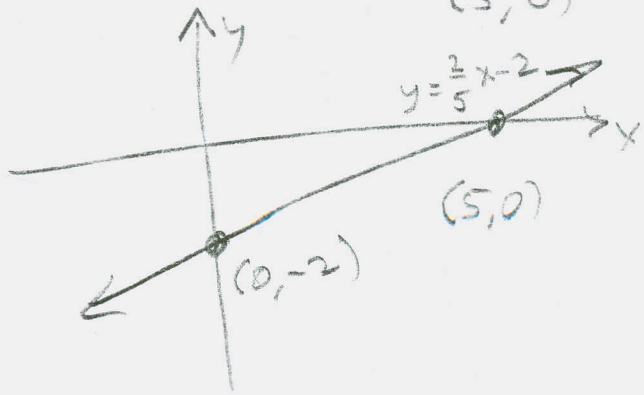
b. $y = \frac{2}{5}x - 2 \Rightarrow (0, -2)$

5pts

$y = 0 \Rightarrow \frac{2}{5}x = 2$

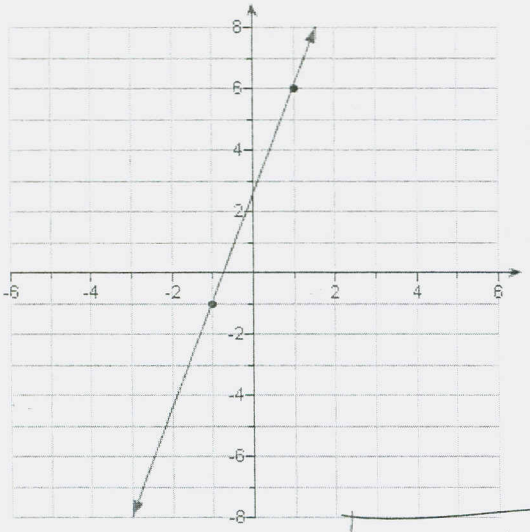
$2x = 10 \Rightarrow x = 5$

$(5, 0)$



20

10. Find the equation of the line from the graph



$(x_1, y_1) = (-1, -1)$

$(x_2, y_2) = (1, 6)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-1)}{1-(-1)} = \frac{7}{2}$ 3pts

$y = \frac{7}{2}(x-(-1)) - 1$

$= \frac{7}{2}x + \frac{7}{2} - 1$

$y = \frac{7}{2}x + \frac{5}{2}$ ⟶

$-\frac{7}{2}x + y = \frac{5}{2}$

$-7x + 2y = 5$

3pts a. Point-Slope form: $y = \frac{7}{2}(x-(-1)) - 1$

3pts b. Slope-Intercept form: $y = \frac{7}{2}x + \frac{5}{2}$

3pts c. Standard form (with integer coefficients): $-7x + 2y = 5$

11. Based on your answer to the previous question, find an equation of the line through (-1, -1) that is perpendicular to the one in the graph. Give your answer in slope-intercept form.

$m_\perp = -\frac{2}{7}$ $\quad y = -\frac{2}{7}(x-(-1)) - 1$

$= -\frac{2}{7}x - \frac{2}{7} - 1$

3pts $\quad y = -\frac{2}{7}x - \frac{9}{7}$

12. Compute the discriminant for each of the following quadratic equations, and determine the *nature* of the solutions (How many, and whether it/they is/are real or non-real):

3pts a. $5x^2 - 38x + 21$ $\quad b^2 - 4ac = (-38)^2 - 4(5)(21) = 1444 - 420$

$= 1024 \Rightarrow$ 2 real solns

3pts b. $9x^2 + 12x + 4$ $\quad b^2 - 4ac = 12^2 - 4(9)(4) = 144 - 144 = 0$

$\Rightarrow$ 1 real soln, repeated

3pts c. $x^2 + 12x + 4$ $\quad b^2 - 4ac = 12^2 - 4(1)(4) = 144 - 16 = 128$ ⟶

2 real solns

24

13. Find all real or non-real solutions of the following quadratic equations by completing the square AND by quadratic formula. Write in the lowest terms possible, e.g. $\sqrt{12} = 2\sqrt{3}$. Do *not* use decimal approximations via calculator.

3pts a. $x^2 - x + 1 = 0$

$b^2 - 4ac = (-1)^2 - 4(1)(1)$

$= 1 - 4 = -3$

$x = \frac{1 \pm \sqrt{-3}}{2} = \boxed{\frac{1 \pm i\sqrt{3}}{2} = x}$

3pts

$x^2 - x = -1$

$x^2 - x + \left(\frac{1}{2}\right)^2 = -1 + \frac{1}{4} = -\frac{3}{4}$

$\left(x - \frac{1}{2}\right)^2 = -\frac{3}{4}$

$x - \frac{1}{2} = \pm\sqrt{-\frac{3}{4}} = \pm i\frac{\sqrt{3}}{2}$

$\boxed{x = \frac{1 \pm i\sqrt{3}}{2}}$

3pts b. $x^2 - 2x - 2 = 0$

$b^2 - 4ac = (-2)^2 - 4(1)(-2)$

$= 4 + 8 = 12$

$x = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$

$= \frac{2(1 \pm \sqrt{3})}{2} = \boxed{1 \pm \sqrt{3} = x}$

3pts

$x^2 - 2x = 2$

$x^2 - 2x + 1^2 = 2 + 1^2$

$(x-1)^2 = 3$

$x - 1 = \pm\sqrt{3}$

$\boxed{x = 1 \pm \sqrt{3}}$

14. Solve the following inequalities:

3pts a. $3-5x<6$

$-5x<3$

$x>-\frac{3}{5}$

$\left(-\frac{3}{5},\infty\right)$

3pts b. $\frac{x}{5}-7>\frac{2}{3}$ LCD = 15

$15\left(\frac{x}{5}\right)-15(7)>15\left(\frac{2}{3}\right)$

$3x-105>10$

$3x>115$

$x>\frac{115}{3}$

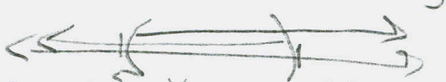
$\left(\frac{115}{3},\infty\right)$

3pts c. $|3x-2|<7$

$3x-2<7$ and $3x-2>-7$

$3x<9$ $3x>-5$

$x<3$ and $x>-\frac{5}{3}$



$\left(-\frac{5}{3},3\right)$

3pts d. $|3x-2|\le -7$

Nevah!

$\varnothing$

3pts e. $|3x-2|>-7$

Always!

$(-\infty,\infty)$

4pts f. $|3x-2|-10\ge -7$

$|3x-2|\ge 3$

$3x-2\ge 3$ OR $3x-2\le -3$

$3x\ge 5$ $3x\le -1$

$x\ge\frac{5}{3}$ OR $x\le -\frac{1}{3}$

$\left(-\infty,-\frac{1}{3}\right]\cup\left[\frac{5}{3},\infty\right)$