MAT 121-G11 Final Fall, 2010
200 Points

Name KEY

1. Solve $x^2 - 4x - 12 = 0$ in 3 ways:
   a. (10 pts) Factoring

$(x-6)(x+2)=0$

$x \in \{-2, 6\}$

   b. (10 pts) Completing the square

$x^2-4x = 12$

$x^2-4x+2^2 = 12+4$

$(x-2)^2 = 16$

$x-2 = \pm 4$

$x = 2 \pm 4 \rightarrow x=6, \quad x=-2$

$\{-2, 6\}$

   c. (10 pts) Quadratic formula

$b^2-4ac = (-4)^2 - 4(1)(-12)$

$= 16+48 = 64$

$x = \dfrac{4 \pm \sqrt{64}}{2(1)} = \dfrac{4 \pm 8}{2} = 2 \pm 4 \Longrightarrow \{-2, 6\}$

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2. Solve the following absolute value equations and inequalities:

a. (10 pts) $|2x-4| = 5$

$2x-4 = 5$ OR $2x-4 = -5$

$2x = 9$ $\qquad$ $2x = -1$

$x = \frac{9}{2}$ $\qquad$ $x = -\frac{1}{2}$

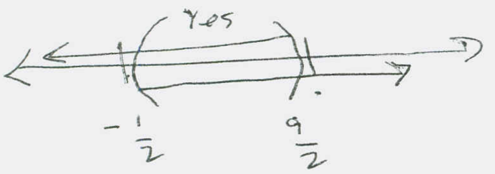
$\left\{-\frac{1}{2}, \frac{9}{2}\right\}$

b. (10 pts) $|2x-4| < 5$

$2x-4<5$ and $2x-4>-5$

$2x < 9$ $\qquad$ $2x > -1$

$\left\{x \mid x < \frac{9}{2} \text{ and } x > -\frac{1}{2}\right\}$



$x \in \left(-\frac{1}{2}, \frac{9}{2}\right)$

c. (10 pts) $|2x-4| \geq 5$

$2x-4 \geq 5$ OR $2x-4 \leq -5$

$2x \geq 9$ $\qquad$ $2x \leq -1$

$\left\{x \mid x \geq \frac{9}{2} \text{ OR } x \leq -\frac{1}{2}\right\}$

~~$x \in \{$~~

$x \in \left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{9}{2}, \infty\right)$

3. (10 pts) Let $f(x) = -3^{2x-7} + 3$. Find the inverse function $f^{-1}(x)$.

$x = -3^{2y-7} + 3$

$-3^{2y-7} + 3 = x$

$-3^{2y-7} = x - 3$

$3^{2y-7} = 3 - x$

$2y - 7 = \log_3(3-x)$

$2y - 7 = \log_3(3-x)$

$2y = \log_3(3-x) + 7$

$y = \frac{1}{2}\left(\log_3(3-x) + 7\right)$

$= f^{-1}(x)$

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4. (10 pts) Find an equation of the line through (-5, -3) and (-1, 5). Point-slope form is preferred.

(x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-3)}{-1-(-5)} = \frac{5+3}{-1+5} = \frac{8}{4} = 2$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y-(-3) = 2(x-(-5))}$$ STOP

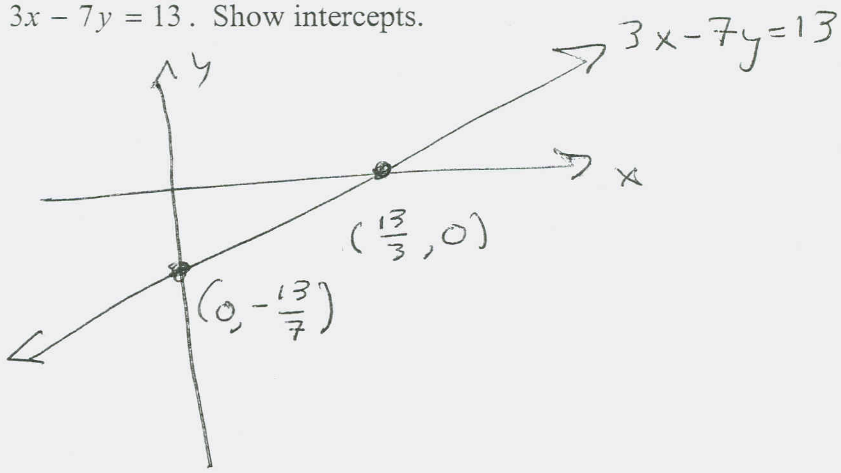
$$y + 3 = 2(x+5)$$

$$y + 3 = 2x + 10$$

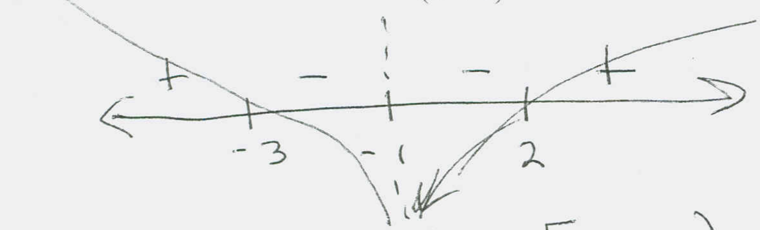
$$y = 2x + 7$$

5. (10 pts) Graph the line $3x - 7y = 13$. Show intercepts.

x	y
0	$-\frac{13}{7}$
$\frac{13}{3}$	0



6. (10 pts) Solve $\dfrac{(x-2)(x+3)^3}{(x+1)^2} \geq 0$.



$x \in (-\infty, -3] \cup [2, \infty)$

7. (10 pts) What is the domain of $\log_5\left(\frac{(x-2)(x+3)^3}{(x+1)^2}\right)$? Hint: You just did most of the work.

$(-\infty, -3) \cup (2, \infty)$

8. Compute the sums:

a. (5 pts) $\sum_{k=1}^{75} 2(-1.07)^{k-1}$

$a = 2,\ r = -1.07,\ n = 75$

$\frac{a(1-r^n)}{1-r} =$

$\frac{2(1-(-1.07)^{75})}{1-(-1.07)}$

$\approx \frac{2(160.8760193)}{2.07} \approx \boxed{155.4357674}$

b. (5 pts) $\sum_{k=1}^{\infty} 5\left(\frac{5}{7}\right)^{k-1}$

$a = 5,\ r = \frac{5}{7}$

$\frac{a}{1-r} = \frac{5}{1-\frac{5}{7}} = \frac{5}{\frac{2}{7}}$

$= (5)\left(\frac{7}{2}\right) = \boxed{\frac{35}{2}}$

9. (10 pts) Use the Binomial Theorem (Pascal's Triangle) to expand $(2x-y)^6$. Best you can earn by brute force is half-credit.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$(2x)^6(-y)^0 + 6(2x)^5(-y) + 15(2x)^4(-y)^2$
$+20(2x)^3(-y)^3 + 15(2x)^2(-y)^4$
$+ 6(2x)(-y)^5 + (-y)^6$
$= 64x^6 + 6(32x^5)(-y) + 15(16x^4)y^2$
$+ 20(8x^3)(-y)^3 + 15(4x^2)(-y)^4$
$+ 12x(-y)^5 + y^6$

$= 64x^6 - 192x^5y + 240x^4y^2 - 160x^3y^3 + 60x^2y^4 - 12xy^5 + y^6$

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10. (10 pts) Let $f(x) = x^2 - 2x + 6$. Simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$. =

$$\frac{(x+h)^2 - 2(x+h) + 6 - [x^2 - 2x + 6]}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 2x - 2h + 6 - x^2 + 2x - 6}{h}$$

$$\cancel{= \frac{2xh - 2xh + h^2}{h}} \qquad = \frac{2xh + h^2 - 2h}{h}$$

$$\cancel{= h(2x} \qquad = \frac{h(2x + h - 2)}{h} = \boxed{2x + h - 2}$$

11. (10 pts) Use synthetic division to find $f(3)$ for $f(x) = x^5 - 3x^4 - 10x^2 + 4x - 10$.

3	1	-3	0	-10	4	-10
		3	0	0	-30	-78
	1	0	0	-10	-26	$\boxed{-88 = f(3)}$

12. (10 pts) Expand the product: $(x - (2 + 3i))(x - (2 - 3i))$

$$= x^2 - (2-3i)x - (2+3i)x + (2+3i)(2-3i)$$

$$= x^2 - 2x + 3ix - 2x - 3ix + 2^2 + 3^2$$

$$= \boxed{x^2 - 4x + 13}$$

13. (10 pts) Write a polynomial (in factored form) with *real* coefficients, of degree 5, that has the given zeros with the given multiplicities:

6

zero	multiplicity
2	1
-3	3
3-7i	1

$$(x-2)(x+3)^3(x-(3-7i))(x-(3+7i))$$

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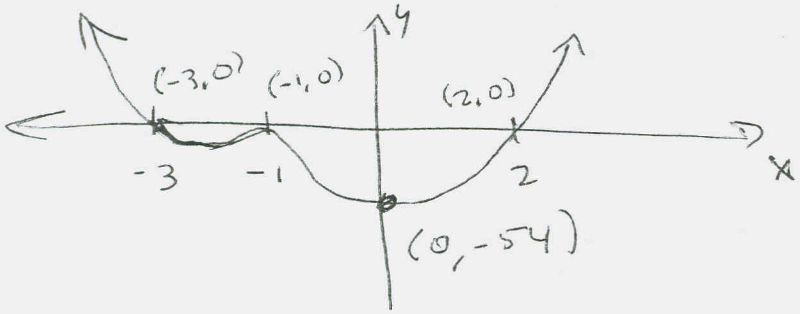
14. (10 pts) Sketch the graph (rough) of $f(x) = (x-2)(x+3)^3(x+1)^2$. Show all intercepts.

$f(0) = (-2)(3)^3(1)^2 = -54$



15. (10 pts) The half-life of radioactive Millsium is 35 years (assuming a life span of 3-score-and-10). Build an exponential model $A(t) = A_0 e^{-kt}$. Then use this model to predict the age of a sample that has 5% of its radioactive Millsium remaining.

$A_0 e^{-35k} = \frac{1}{2} A_0$

$e^{-35k} = \frac{1}{2}$

$-35k = \ln(\frac{1}{2})$

$k = \frac{\ln(\frac{1}{2})}{-35} = \frac{\ln 2}{35}$

$k \approx .0198042052$

$A_0 e^{-kt} = .05 A_0$

$e^{-kt} = .05$

$-kt = \ln(.05)$

$t = \frac{\ln(.05)}{-k}$

$= \frac{\ln(.05)}{-\frac{\ln 2}{35}} = -\frac{35 \ln(.05)}{\ln 2} \approx 151.2674833$

$\approx \boxed{151 \text{ yrs}}$

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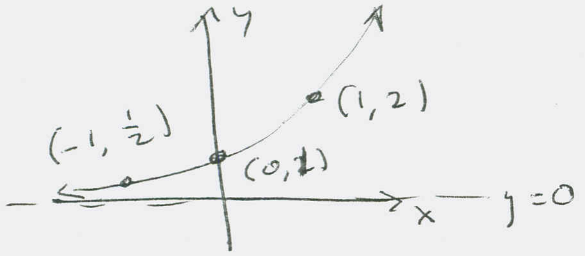
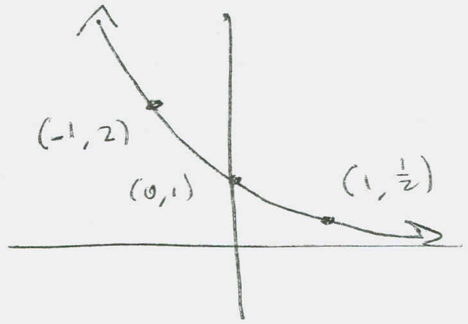
$3 - x = -(x-3)$

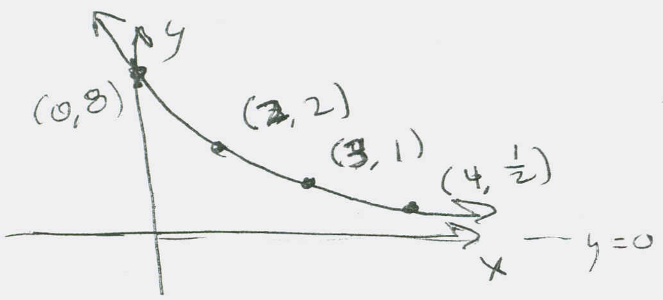
16. (10 pts) Sketch the graph of $g(x) = -3\left(2^{3-x}\right) + 8$ by transforming the function $f(x) = 2^x$.



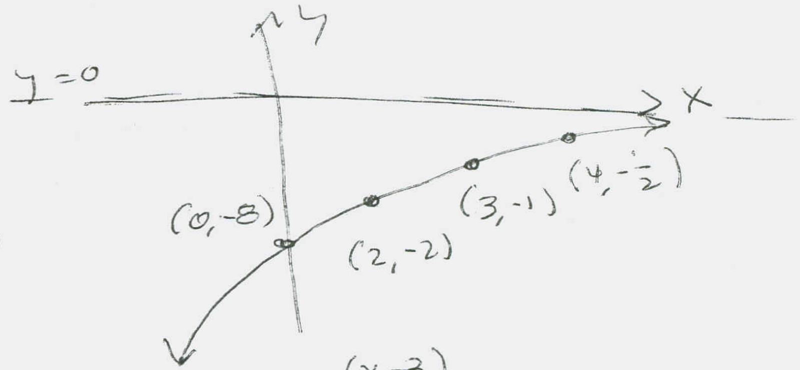
2^x



2^{-x}

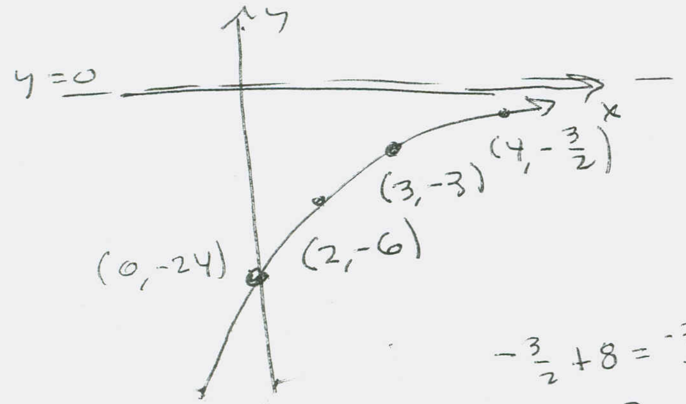


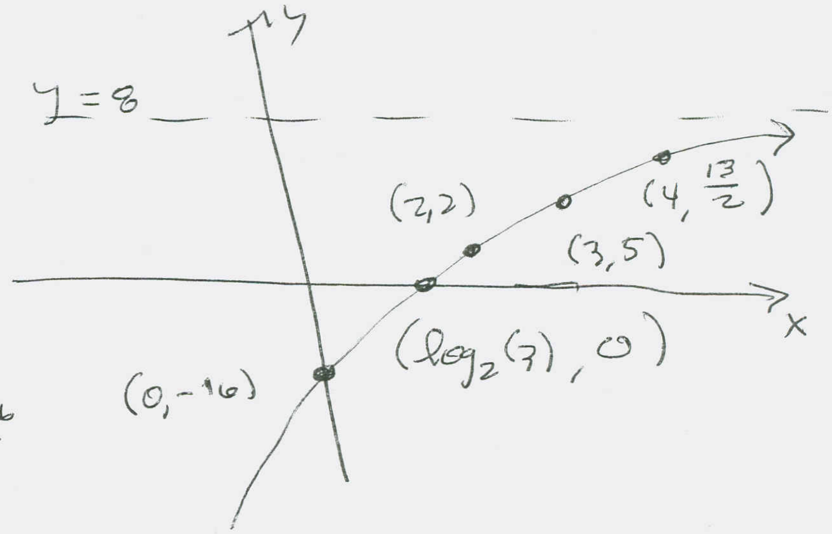
$2^{-(x-3)}$



$-2^{-(x-3)}$

$-3 \cdot 2^{-(x-3)}$





$-\frac{3}{2} + 8 = \frac{-3+16}{2}$

$= \frac{13}{2}$

$-3\left(2^{3-x}\right) + 8 = 0$

$2^{3-x} = \frac{8}{3}$

$3 - x = \log_2\left(\frac{8}{3}\right)$

$-x = \log_2\left(\frac{8}{3}\right) - 3$

$x = 3 - \log_2\left(\frac{8}{3}\right)$

$= 3 - \log_2(8) + \log_2(3)$

$= 3 - 3 + \log_2(3) = \log_2(3)$

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17. (10 pts) **Bonus** Let P = present value (principal), R = Periodic payment, i = interest rate per period, n = total number of periods. If you want to borrow the amount P from a banker, you must make (monthly) payments R. Solve this equation for R to see what the formula is for loan payment amount, when you borrow P dollars.

$$R\left[\frac{(1+i)^n - 1}{i}\right] = P(1+i)^n$$

Simplify your answer as much as possible. The result is the "Loan Amortization" formula, which gives your (monthly) payment as a function of the amount borrowed and the interest rate.

$$R = P(1+i)^n\left[\frac{i}{(1+i)^n - 1}\right]$$

$$= P \cdot \frac{1}{(1+i)^{-n}}\left[\frac{i}{(1+i)^n - 1}\right]$$

$$= \frac{Pi}{(1+i)^{-n}\left((1+i)^n - 1\right)} = \boxed{\frac{Pi}{1-(1+i)^{-n}} = R}$$

Final #16

$-3(2^{3-x}) + 8$

$2^x \longrightarrow 2^{x+3} \longrightarrow 2^{-x+3} \longrightarrow -3(2^{-x+3}) \longrightarrow -3(2^{-x+3}) + 8$

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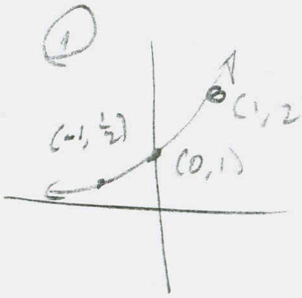
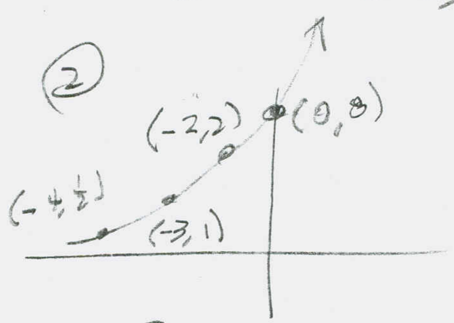


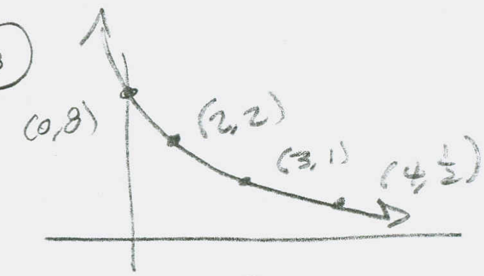
2^x

2



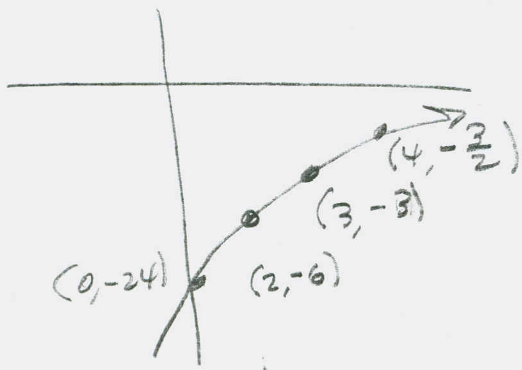
2^{x+3}

3



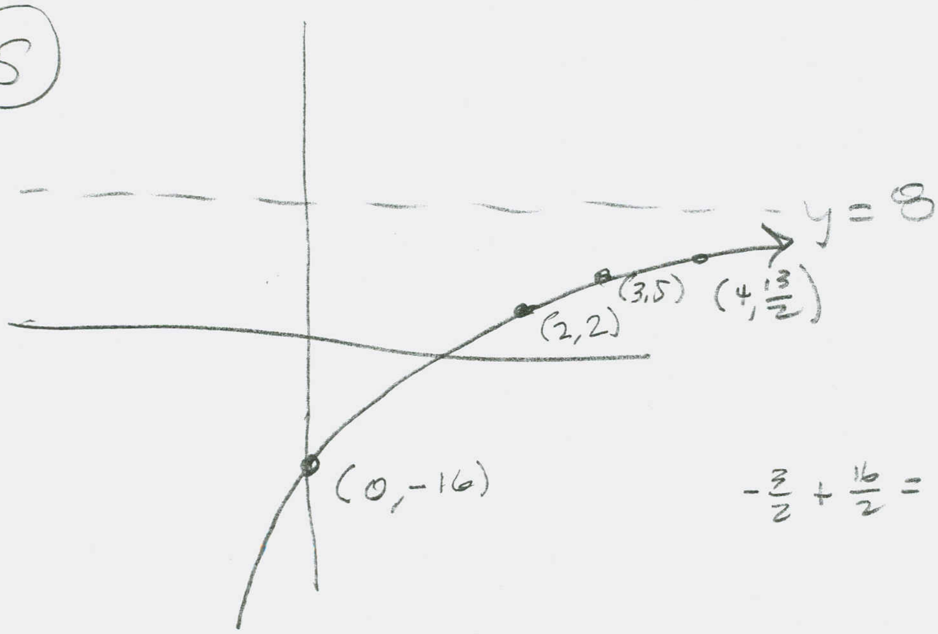
2^{-x+3}

4



$-3(2^{3-x})$

5



$-\frac{3}{2} + \frac{16}{2} = \frac{13}{2}$