1340 Week 14 Mills

(1) We decompose by partial fractions

(2) (5pts) $\frac{5x+1}{x^2-1} = \frac{5x+1}{(x-1)(x+1)} = \boxed{\frac{A}{x-1} + \frac{B}{x+1}}$ circle this

$\Rightarrow 5x+1 = A(x+1) + B(x-1)$

Setup 2
solve 2
context 1

$x=-1$:
$5(-1)+1 = -4 = B(-1-1) = -2B$

$\Rightarrow B = \frac{-4}{-2} = \boxed{2 = B}$ → circle this...

These 3 things circled suffices as final answer. They don't have to write out $\frac{3}{x-1} + \frac{2}{x+1}$

$x=1$:
$5(1)+1 = 6 = A(1+1) = 2A$

$\Rightarrow A = \frac{6}{2} = \boxed{3 = A}$ → this...

We see this in Calc II This is just inserting comment...

$$\int \frac{5x+1}{x^2-1}\,dx = \int \left(\frac{3}{x-1} + \frac{2}{x+1}\right)dx = 3\ln|x-1| + 2\ln|x+1| + C,$$

not that the above means a whole lot to you, yet.

Setup - 2pts

(5) (5pts) $\frac{x-8}{x^3+8x} = \frac{x-8}{x(x^2+8)} = \boxed{\frac{A}{x} + \frac{Bx+C}{x^2+8} = \frac{x-8}{x^3+8x}}$

$\Rightarrow x-8 = A(x^2+8) + (Bx+C)x$

Solve - 2pts
Context 1pt

$x=0 \Rightarrow -8 = 8A \rightarrow \boxed{A=-1}$

$x=1$: (RANDOM)

$1-8 = -7 = A(1^2+8) + (B(1)+C)(1) = 9A + B + C = -9 + B + C$

$2 = B + C = 2$

$x=2$:

$-6 = 2-8 = A(2^2+8) + (B(2)+C)(2) = 12A + 4B + 2C$

$-6 = 12A + 4B + 2C = -12 + 4B + 2C$

$6 = B + 2C = 6$

$B + C = 2 \rightarrow C = 2-B$

$4B + 2C = 6 \rightarrow 4B + 2(2-B) = 4B + 4 - 2B = 2B + 4 = 6$

$2B = 2$

$\boxed{B = 1}$

$B + C = 2 \rightarrow$

$1 + C = 2 \rightarrow \boxed{C = 1}$

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I shade the bad stuff. They will probably shade the good stuff, because they never listen.

***x*- and *y*-intercepts - Award 1 pt each (2 pts total)**
Corner Points - Award 1 pt each (4 pts total)
Finding Corner Point C (Support) - 2 pts.
Clear Indication of the correct feasible region ("Good Stuff") - 2 pts Are they on the right side(s) of the lines

Q (10 pts) Feasible Region for

$2x - 7y \le 14$
$3x + 2y > 6$
$x \le 7$
$y \le 3$

Shading:

$x \le 7$
$0 \le 7$? $(0,0)$ GOOD

$x = 7$

$2x - 7y \le 14$
$0 \le 14$? Yes
$(0,0)$ GOOD

x	y
0	-2
7	0

$3x + 2y > 6$
$0 > 6$? No
$(0,0)$ BAD

x	y
0	3
2	0

$A = (0,3)$

$y \le 3$
$0 \le 3$?
$(0,0)$ good

Find Corner Point C:

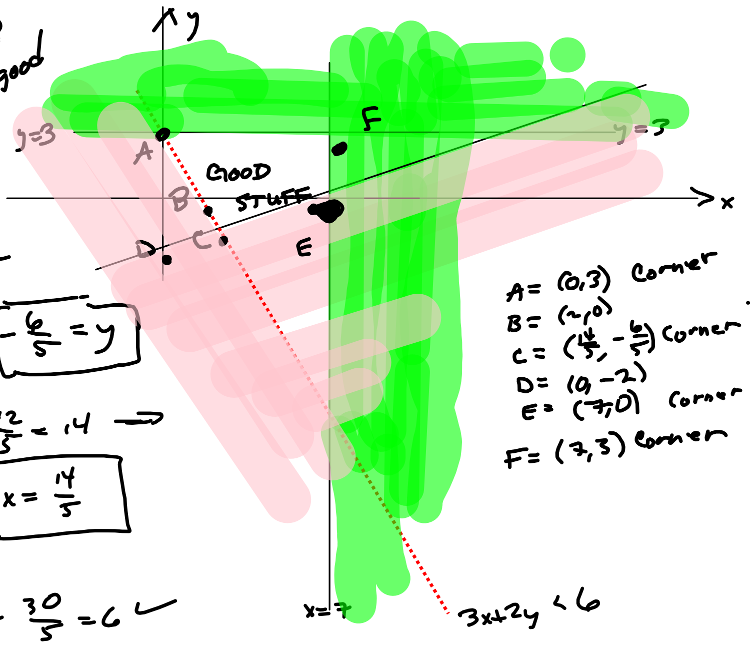
$3(2x - 7y = 14)$
$-2(3x + 2y = 6)$

$6x - 21y = 42$
$-6x - 4y = -12$
$-25y = 30$
$y = -\frac{30}{25} = \boxed{-\frac{6}{5} = y}$

$2x - 7y = 2x - 7\left(-\frac{6}{5}\right) = 2x + \frac{42}{5} = 14 \rightarrow$

$2x = -\frac{42}{5} + \frac{70}{5} = \frac{28}{5} \rightarrow \boxed{x = \frac{14}{5}}$

Check $3\left(\frac{14}{5}\right) + 2\left(-\frac{6}{5}\right) = \frac{42-12}{5} = \frac{30}{5} = 6$ ✓



$A = (0,3)$ Corner
$B = (2,0)$
$C = \left(\frac{14}{5}, -\frac{6}{5}\right)$ Corner
$D = (0,-2)$
$E = (7,0)$ Corner
$F = (7,3)$ Corner

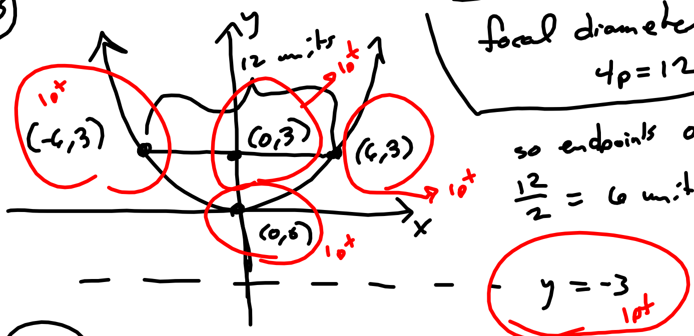
1340 Week 14 Written Mills

3. We find the focal length p, the equation of the directrix, and the focal diameter $4p$, and we label the focus, vertex, directrix, and the latus rectum, including its endpoints.

(a) 5pts $y=\frac{1}{12}x^2 \Rightarrow x^2 = 12y = 4py \Rightarrow p=3$

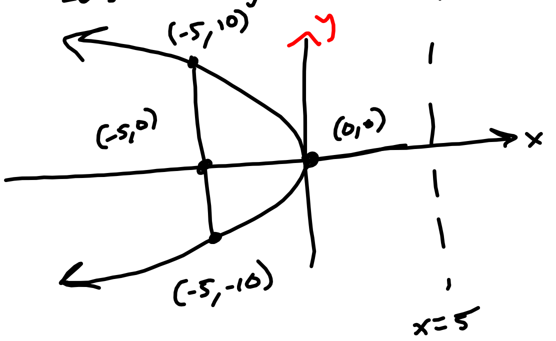
focal diameter: $4p = 12$

Deduct 2 points (1/2 - pt each) for not using ordered pair labels. If you have to count tick-marks, charge them 2 points, iow.



so endpoints of latus rectum are $\frac{12}{2} = 6$ units left & right of the focus.

(b) 5pts $x = -\frac{1}{20}y^2 \Rightarrow y^2 = 20x = 4px \Rightarrow p = 5$



$4p = 20$

$\frac{20}{2} = 10$

$-\frac{1}{20}y^2$ opens left.

Same scale

For the vertex, they may use $h = \frac{-b}{2a}$ & $k = f\left(-\frac{b}{2a}\right)$, as long as they get the same (h,k) I got, the better way.

(c) (5pts) $y = \frac{1}{8}x^2 - \frac{5}{4}x - \frac{71}{8}$

Find focal length p, focal diameter, and vertex. Show these on the graph

1pt context

$\Rightarrow 8y = x^2 - 10x - 71$

$= x^2 - 10x + 5^2 - 25 - 71$

$= (x-5)^2 - 96 \Rightarrow y = \frac{1}{8}(x-5)^2 - \frac{96}{8}$

$= \frac{1}{8}(x-5)^2 - 12 \Rightarrow$ $\boxed{(h,k) = (5,-12)}$

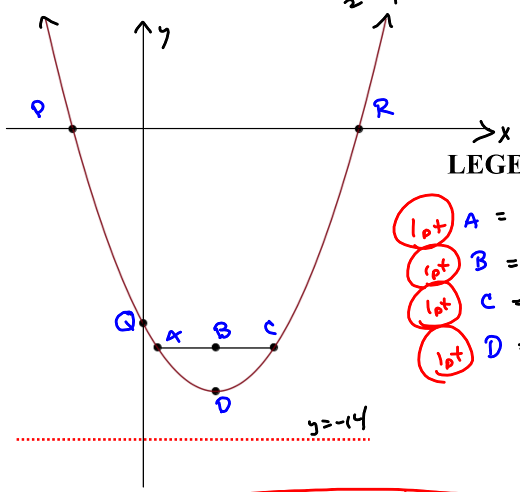
$\Rightarrow 8y = 4py \Rightarrow p = 2$

$p = 2 \Rightarrow$

focal diameter is $4p = 8$

$\frac{8}{2} = 4$

opens up.



LEGEND

1pt A = (1,−10) Endpt. of latus rectum.

1pt B = (5,−10) Focus

1pt C = (9,−10) Endpt. of latus rectum

1pt D = (5,−12) Vertex

(d) (5pts) Find x- & y-intercepts. Show them on the graph

context 1pt

They'll probably use quadratic formula, with incredibly poor style.

$y = \frac{1}{8}x^2 - \frac{5}{4}x - \frac{71}{8}$

y-int: $\left(0, -\frac{71}{8}\right)$

x-int: using previous work:

$= \frac{1}{8}(x-5)^2 - 12 \overset{\text{SET}}{=} 0 \Rightarrow$

$\frac{1}{8}(x-5)^2 = 12 \Rightarrow$

$(x-5)^2 = 96 \Rightarrow$

$x - 5 = \pm\sqrt{96} = 4\sqrt{6}$

$\Rightarrow x = 5 \pm 4\sqrt{6}$

$\frac{1}{8}x^2 - \frac{5}{4}x - \frac{71}{8} = 0 \Rightarrow$

$x^2 - 10x - 71 = 0 \Rightarrow a=1, b=-10, c=-71$

$b^2 - 4ac = 100 - 4(1)(-71) = 100 + 284$

$= 384$

$\Rightarrow x = \frac{10 \pm 8\sqrt{6}}{2(1)}$

$= \frac{5 \pm 4\sqrt{6}}{2}$

2 | 384
2 | 192
2 | 96
2 | 48
2 | 24
2 | 12
2 | 6
3

$\boxed{\begin{aligned} P &= (5-4\sqrt{6}, 0) \text{ x-int} \\ Q &= \left(0, -\tfrac{71}{8}\right) \text{ y-int} \\ R &= (5+4\sqrt{6}, 0) \text{ x-int} \end{aligned}}$

3pts

scratch:

2 | 96
2 | 48
2 | 24
2 | 12
2 | 6
3

1pt: Show P, Q, & R on graph, above.

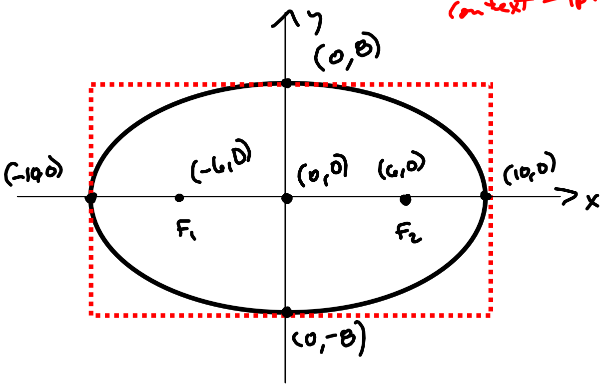
1340 Week 14 Written Mills

(4) We graph the ellipse given by the equation, including the center, foci, and endpoints of the major & minor axes.

(a) (5pts) $\frac{x^2}{100} + \frac{y^2}{64} = 1$

Endpts of major & minor axes – $\frac{1}{2}$pt each
Focus – $\frac{1}{2}$ pt each
Use labels & not tickmarks – 1pt
Context – 1pt

$a^2 = 100$
$\Rightarrow a = 10$
$b^2 = 64$
$\Rightarrow b = 8$
$a^2 - b^2 = 100 - 64 = 36 = 6^2$
$\Rightarrow c = 6 =$ focal length



(b) (5pts) $49x^2 + 25y^2 + 196x - 150y - 804 = 0$

$49x^2 + 196x \quad + 25y^2 - 150y \quad = 804$

$49(x^2 + 4x \quad) + 25(y^2 - 6y \quad) = 804$

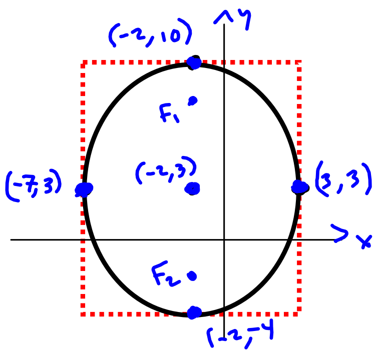
$49(x^2 + 4x + 2^2) + 25(y^2 - 6y + 3^2) = 804 + 49(4) + 25(9)$

$\frac{49(x+2)^2}{(49)(25)} + \frac{25(y-3)^2}{(49)(25)} = \frac{804 + 196 + 225 = 1225}{(49)(25)}$

$\frac{(x+2)^2}{25} + \frac{(y-3)^2}{49} = 1$ 2.5 pts

$\frac{\cancel{1225}}{\cancel{(49)(25)}} = 1$ ✓

$(h,k) = (-2,3)$
$a = 5,\ b = 7$
$b^2 - a^2 = 49 - 25 = 24 = c^2$
$\Rightarrow c = \sqrt{24} = 2\sqrt{6} \approx 4.898979486 \approx$ FOCAL LENGTH



Foci: 2pts
$F_1 = (-2, 3+2\sqrt{6}) \approx (-2, 7.899)$
$F_2 = (-2, 3-2\sqrt{6}) \approx (-2, -1.899)$
Center: $\frac{1}{2}$pt

Deduct a point if they don't use ordered-pair labels.

1340 Week 14 Written Mills

(5) We graph the hyperbola given by the eqn, labeling center, foci, and vertices

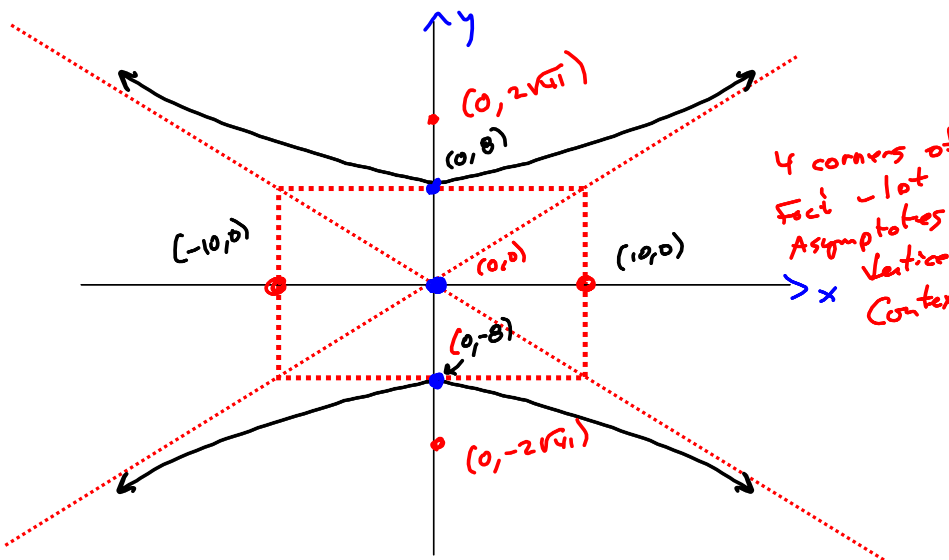
(a) 5pts $\dfrac{y^2}{64} - \dfrac{x^2}{100} = 1$

$a = \sqrt{64} = 8$, $b = \sqrt{100}$.

$c^2 = a^2 + b^2 = 64 + 100 = 164$

$c = \sqrt{164} = 2\sqrt{41}$

$2 \,|\, 164$
$2 \,|\, 82$
41



4 corners of box - 1pt
Foci - 1pt
Asymptotes - 1pt
Vertices - 1pt
Context - 1pt

1340 **Week 14 Written** Mills

b 5pts $25x^2 - 36y^2 - 150x - 144y - 819 = 0 \Rightarrow$

$25x^2 - 150x \quad - 36y^2 - 144y \quad = 819$

$\Rightarrow 25(x^2 - 6x \quad) - 36(y^2 + 4y \quad) = 819$

$\Rightarrow 25(x^2 - 6x + 3^2) - 36(y^2 + 4y + 2^2) = 819 + 25(9) - 36(4)$

$\Rightarrow 25(x-3)^2 - 36(y+2)^2 = 819 + 225 - 144 = 1044 - 144 = 900$

$\Rightarrow \frac{25(x-3)^2}{25(36)} - \frac{36(y+2)^2}{25(36)} = 819 + 225 - 144 = 1044 - 144 = \frac{900}{25(36)}$

$$\Rightarrow \boxed{\frac{(x-3)^2}{36} - \frac{(y+2)^2}{25} = 1}$$ 2 pts

check:

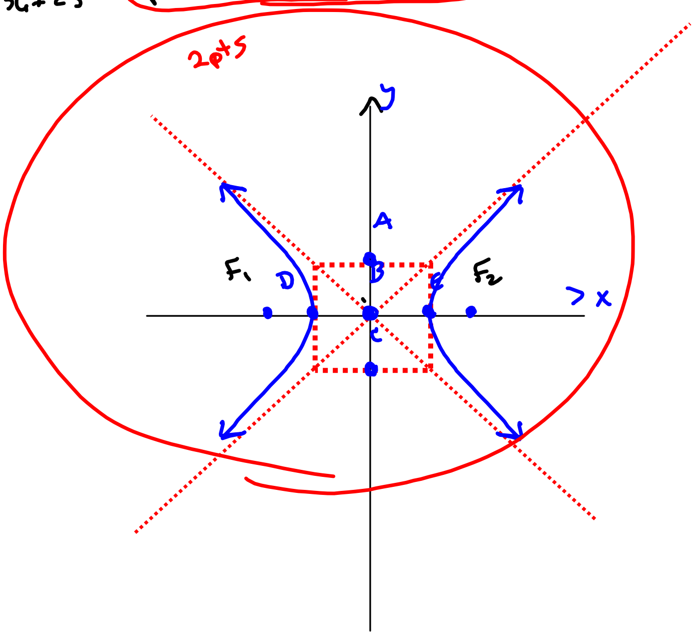
$\begin{array}{r} 25 \\ \times 36 \\ \hline 150 \\ 750 \\ \hline 900 \end{array}$ ✓

$(h,k) = (3,-2)$

$a = 6, b = 5$

$c = \sqrt{a^2+5^2}$ = focal length $= \sqrt{6^2+5^2}$

$= \sqrt{36+25} = \sqrt{61} \approx$ 7.810249676 Either one.



$A = (3,3)$

$B = (3,-2)$ Center

$C = (3,-7)$

$D = (-3,-2)$

$E = (9,-2)$

$F_1 = (3-\sqrt{61},-2)$ vertex

$F_2 = (3+\sqrt{61},-2)$

1340 Week 14 Written Mills

(6) Use Pascal's Triangle or the Binomial Theorem for the following

(a) (5 pts) Use Pascal's Triangle to expand $(3x-2y)^4$

context −1pt

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

− 1pt

$(3x)^4 + 4(3x)^3(-2y) + 6(3x)^2(-2y)^2 + 4(3x)(-2y)^3 + (-2y)^4$ — 1pt

$= 3^4x^4 + 4(3^3)x^3(-2)y + 6(3^2x^2)((-2)^2y^2) + 4(3x)((-2)^3y^3) + ((-2)^4y^4$

$= 81x^4 + 4(-2)(27)x^3y + 6(9)(4)x^2y^2 + 12(-8)xy^3 + 16y^4$

$= 81x^4 - 216x^3y + 216x^2y^2 - 96xy^3 + 16y^4$ → 2pts

$$\begin{array}{r} {}^{5}27 \\ 8 \\ \hline 216 \end{array}$$

context −1pt

(b) (5pts) We find the coefficient of x^9 in the expansion of $(2x-3y)^{16}$

OR $\binom{16}{7} = \binom{16}{9}$

$\binom{n}{k} = \binom{n}{n-k}$

9th term is $\binom{16}{9}(2x)^9(-3y)^7$ = $11440(2)^9x^9(-3)^7y^7$ → 2pts

$= -11{,}440(2)^9(3)^7x^9y^7$ = $-12809871360x^9y^7$

If they get the numerical part right, but don't get the y^7, don't hurt them. It's a technicality you might point out, but don't deduct.

Don't want to do this w/o a calculator.

⟹ coefficient of x^9 is $-12809871360y^7$ → 2pts

(Easy to miss the y^7.)

$\binom{16}{9} = \dfrac{16!}{9!(16-9)!} = \dfrac{16!}{9!7!} = \dfrac{\cancel{16}^{8}\cdot 15^{5}\cdot \cancel{14}\cdot 13\cdot \cancel{12}^{2}\cdot 11\cdot \cancel{10}^{5}}{\cancel{7}\cdot 6\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}}$ = $80(26)(11)(5)$

Can be done by hand.

$= (80)(26)(55) = 208(55) = 11440$

$$\begin{array}{r} 208 \\ \times 55 \\ \hline 1040 \\ 10400 \\ \hline 11440 \end{array}$$

1340 **Week 14 Written** **Mills**

360360

7 5pts Choose 5 from 15 and arrange

$P(15,5) = \frac{15!}{(15-5)!} = \frac{15!}{10!} = 15\cdot14\cdot13\cdot12\cdot11 = \boxed{360360}$

Context - 1pt
P(15,5) - 2pt
360,360 - 2pt

8 5pts Choose 4 from 10 : Context - 1pt

$C(10,5) = \binom{10}{4} = \frac{10!}{4!6!} = \frac{10\cdot\cancel{9}^{3}\cdot\cancel{8}^{1}\cdot7}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = \boxed{210 \text{ ways}}$

2pts either one is OK

1pt

1pt