(1) We solve the system of linear equations $2x-5y=10$, $7x+2y=28$

3 ways

(2) Graphically:

$2x-5y=10$

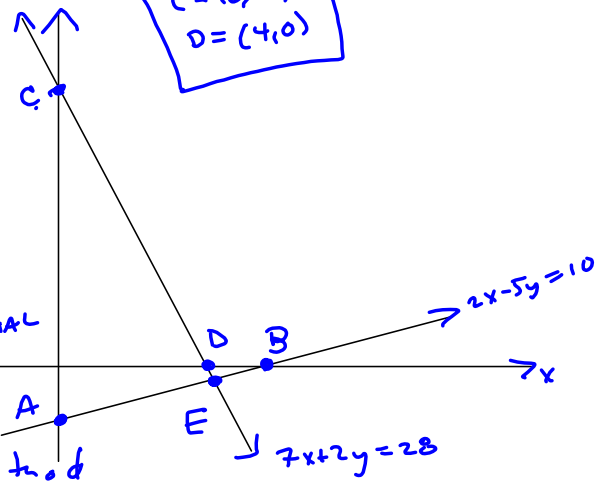
x	y
0	-2
5	0

$A=(0,-2)$
$B=(5,0)$

$7x+2y=28$

x	y
0	14
4	0

$C=(0,14)$
$D=(4,0)$



E = Intersection

Approximately (4.1, -.5) ?

$\approx (4.103, -0.359)$ ≈ ACTUAL

(3) by substitution method

(10 pts) $2x-5y=10 \Rightarrow 5y=2x-10 \Rightarrow y=\frac{2x-10}{5}$

$7x+2y=28$

$\Rightarrow 7x+2\left(\frac{2x-10}{5}\right)$

$= 7x+\frac{4x}{5}-\frac{20}{5}=28$ Times 5 $\frac{14x}{5}-\frac{20}{5}=28$

$\Rightarrow 35x+4x-20=140$

$39x=160$

$x=\frac{160}{39}$

$\Rightarrow y=\frac{2x-10}{5}=\frac{2\left(\frac{160}{39}\right)-10}{5}=\frac{\frac{320}{39}-\frac{390}{39}}{5}$

$=\frac{\frac{-70}{39}}{5}=\frac{-70}{39(5)}=\frac{-70}{195}=\frac{-14}{39}=y$

$(x,y)=\left(\frac{160}{39},\frac{-14}{39}\right)$

$\approx$ (4.102564103, - 0.3589743590)

$\approx (4.103, -0.359)$

(c) (10 pts) Elimination:

$2x-5y=10$
$7x+2y=28$

$-7E1$ $-14x+35y=-70$
$2E2$ $14x+4y=56$
$-7E1+2E2$ $39y=-14$

$y=\frac{-14}{39}$

$\Rightarrow 2x-5y=2x-5\left(-\frac{14}{39}\right)=$

$=2x+\frac{70}{39}=10$

$\Rightarrow 78x+70=390$

$\Rightarrow 78x=320$

$\Rightarrow x=\frac{320}{78}=\frac{160}{39}=x$

7 28
39
252
840
1092

(2) (10pts) Elimination method to solve

$x-2y+z=18$
$2x-3y+3z=38$ E2
$2x-6y+z=37$ E3

E1 $x-2y+z=18$

$-2E1+E2$ $y+z=2$

$-2E1+E3$ $-2y-z=1$

E1 $x-2y+z=18$

E2 $y+z=2$

$2E2+E1$ $z=5$

$y+z=y+5=2$

$\Rightarrow \boxed{y=-3}$

E1 $x-2y+z = x-2(-3)+5 =$

$= x+6+5 = x+11 = 18$

$\Rightarrow \boxed{x=7}$

$\boxed{(x,y,z)=(7,-3,5)}$

$-2E1$ $-2x+4y-2z=-36$

E2 $2x-7y+3z=38$

$y+z=2$

$-2E1$ $-2x+4y-2z=-36$

E3 $2x-6y+z=37$

$-2y-z=1$

$2E2$ $2y+2z=4$

$-2y-z=1$

$\boxed{z=5}$

(a) 3 roots. Dependent, ∞ solns

$$\begin{aligned} x+5y-3z &= -1 \\ 3x+12y-8z &= 3 \\ 2x+7y-5z &= 4 \end{aligned}$$

$$\begin{aligned} 3x+12y-8z &= 3 \\ 2x+7y-5z &= 4 \\ -x-5y+3z &= 1 \end{aligned}$$

E1 $\quad -x-5y+3z = 1$

$-3E1+E2 \quad -3y+z = 6$ } Same

$-2E1+E3 \quad -3y+z = 6$

$-3E1 \quad -3x-15y+9z = +3$

$E2 \quad 3x+12y-8z = 3$

$\quad -3y+z = 6$

$-2E1 \quad -2x-10y+6z = 2$

$E3 \quad 2x+7y-5z = 4$

$\quad -3y+z = 6$

E1 $\quad -x-5y+3z = 1$

$-E2 \quad 3y-z = -6$

$\quad 0 = 0$

$z = \text{any } y$

$3y = z-6$

$y = \frac{z-6}{3}$

$$-x-5y+3z = x-5\left(\frac{z-6}{3}\right)+3z$$

$$= -x-\frac{5z-30}{3}+3z = 1$$

$$\Rightarrow -3x-5z+30+9z = 3$$

$$-3x+4z+30 = 3$$

$$-3x = -4z-27$$

$$-x = \frac{-4z-27}{3}$$

Courtesy KC.

$$(x,y,z) = \left(\frac{4z+27}{3}, \frac{z-6}{3}, z\right)$$

(b) $z=0$:

$(x,y,z) = (-9,-2,0)$

$z=1$:

$(x,y,z) = \left(-\frac{31}{3}, -\frac{5}{3}, 1\right)$

$z=-1$:

$(x,y,z) = \left(-\frac{23}{3}, -\frac{7}{3}, -1\right)$

(4) (10pts) Dependent. No Soln.

$3x+12y-8z=3$
$2x+7y-5z=4$
$-x-5y+3z=3$

$x+5y-3z=-3$
$3x+12y-8z=3$
$2x+7y-5z=4$

E1 $x-5y+3z=1$

$-3E1+E2$ $-3y+z=12$
$-2E1+E3$ $-3y+z=10$

Same, except $12 \neq 10$

$-3E1$ $-3x-15y+9z=9$
E2 $3x+12y-8z=3$
$-3y+z=12$

$-2E1$ $-2x-10y+6z=6$
E3 $2x+7y-5z=4$
$-3y+z=10$

E1 $x-5y+3z=1$
$3y-z=-6$
$-E2$
$0=-2$?!
$-E2+E3$

That's absurd. Either I made a mistake or there never was a solution!

No Solution