MAT 1340 Week 9 Written Assignment
points Covers Sections 3.4 – 3.7

3.4 – Real Zeros of Polynomials
3.5 – Complex Zeros of Polynomials and the Fundamental Theorem of Algebra
3.6 – Rational Functions
3.7 – Polynomial and Rational Inequalities

This week's assignment contains elements of old Writing Project #3's from previous semesters, but with some subtle differences, for which new videos were made. So there's a mix of old videos and new videos, which I organized into one set for a Week 9 Notes and Videos Page. You can use that, or just click on the videos next to the exercises, below.

For Weekly Written Assignments and WebAssign, we expect students to have access to graphing calculators and computer algebra systems (CAS's). For Written Tests, we only permit scientific calculators. There's a danger of using technology as a crutch.

1. Let $g(x) = 4x^4 - 16x^3 + 3x^2 + 26x + 3$.

   a. (5 pts) Use a simple graphic to describe the end behavior of g. See Video for #1a.

   b. (5 pts) Use Descartes' Rule of Signs to find the number of possible positive zeros g has. How many possible *negative* zeros does g have? See Video for #1b.

   c. (5 pts) What are the possible rational zeros of g? See Video for #1c - #1g.

   d. (5 pts) Find all rational zeros of g. The Factor Theorem says that zeros $x = c$ of g correspond to *factors* $x - c$ of g. Use synthetic division and the rational zeros of g to factor g as far as you can with just the rational zeros. This is where you can save a lot of time with a quick Desmos sketch of the graph, so you can make your first guesses *good* guesses. On a *written* test, I cook the numbers so your rational zeros are all integers, which saves you a lot of time.

   e. (5 pts) The new depressed polynomial is a quadratic polynomial. You can find *its* zeros with the quadratic formula or completing the square. Do so. Now you're ready to...

   f. (5 pts) Write g as the product of linear factors promised to us by the Fundamental Theorem of Algebra. I don't want to see any decimal approximations. If $x = 1 + \sqrt{2}$ is a zero, then $\left(x - \left(1 + \sqrt{2}\right)\right)$ is a factor.

   g. (5 pts) Sketch a quick graph of g that shows all of its intercepts. To keep things in the right place, you may (should) obtain decimal approximations of the irrational zeros you found in part f.

2. Let $f(x) = 4x^6 - 40x^5 + 331x^4 - 920x^3 + 21x^2 + 1490x + 174$.

   a. (5 pts) Use a simple graphic to describe the end behavior of *f*. If you don't know what I mean, they you may benefit from the See Video for #1a.

b. (5 pts) Suppose I told you that $f(x) = 4x^6 - 36x^5 + 146x^4 - 340x^3 + 76x^2 + 776x + 174$ has a complex zero $3 - 7i$. Use this information and long division of polynomials to factor f into the product of a quadratic polynomial and a quartic polynomial. See Video for #2b and #2c

c. (5 pts) Based on your work in part b, you have a quadratic polynomial whose zeros are $3 \pm 7i$. You also have a quartic (4^{th}-degree) polynomial, whose zeros are yet to be determined. This is called the "depressed polynomial."

If all has gone well, the depressed polynomial is the polynomial from #1! So f has the same real zeros as g from #1! Sketch its graph, using only the information from its intercepts. It's identical to #1's graph, with one exception. What's the only difference?

3. Let $R(x) = \dfrac{3x^2 + 6x - 24}{4x^2 + 27x + 18}$. See Video for #3.

   a. (5 pts) What is the domain of *R*?

   b. (5 pts) Find the zeros of R. Also find the *y*-intercept of *R*. These will be labeled points on the graph.

   c. (5 pts) Find any horizontal asymptotes of *R*.

   d. (5 pts) Re-write *R* with its numerator and denominator factored (See parts a and b.). Then provide a sign pattern for *R*. Take care to distinguish between zeros of *R* and vertical asymptotes of *R*, both of which control any sign changes of *R*. Use the parity (sign) of the horizontal asymptote and the *y*-intercept to kick-start your sign pattern.

   e. (5 pts) Render the graph of *R*, showing all intercepts and asymptotes. This is what "Graph *R*" means.

4. Let $\hat{R}(x) = \dfrac{3x^3 - 18x^2 - 72x + 192}{4x^3 - 5x^2 - 198x - 144}$. $\hat{R}$ has the same graph as *R*, with one exception: $\hat{R}$ has a hole. See Video for #4.

   a. (5 pts) Where is the hole? Give your answer as an ordered pair (x, y).

   b. (5 pts) Go back to your graph of R in #3. Add the hole you found in part a, above to its graph. That will suffice in earning credit for the graphs of both *R* and $\hat{R}$. If you wish, you may do a separate graph for $\hat{R}$, showing all intercepts, asymptotes, and the hole.

5. Let $T(x) = \dfrac{3x^3 - 18x^2 - 72x + 192}{4x^2 + 27x + 18}$. T has a pair of vertical asymptotes and a slant (oblique) asymptote. See Video for #5.

a. (5 pts) Use long division to determine the slant asymptote. Call it $s(x)$.

b. (5 pts) Sketch the graph of T, showing all intercepts and asymptotes. Most of the work has already been done, as T has the same denominator as R, and the same numerator as $\hat{R}$.