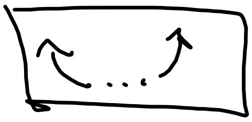
B40 WEEK 9 SOLNS MILLS

1) $g(x) = 4x^4 - 16x^3 + 3x^2 + 26x + 3$

a) 5pts g's end behavior is that of $4x^4$



I'm looking for the diagram. The book teaches

b) 5pts Find the possible # of positive & negative zeros.
There are 2 sign changes in the terms for $g(x)$.
$\Rightarrow$ $\boxed{\text{2 or 0 positive zeros.}}$
$g(-x) = 4x^4 + 16x^3 + 3x^2 - 26x + 3$ has 2 sign changes
$\Rightarrow$ $\boxed{\text{2 or 0 negative zeros.}}$

c) 5pts We list the possible rational zeros of g.

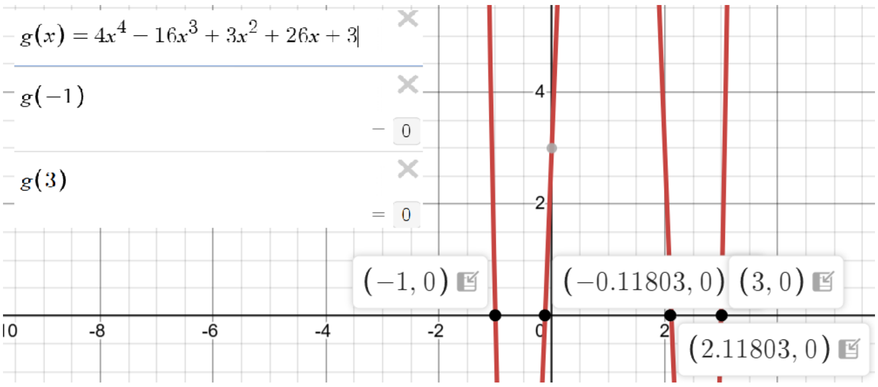
$a_4 = 4$ denom.
$a_0 = 3$ numer.
$\frac{p}{q} = \frac{\text{factors of } 3}{\text{factors of } 4}$

Rational zeros are from this list:

$$\boxed{\pm 1, \pm \tfrac{1}{2}, \pm \tfrac{1}{4}, \pm 3, \pm \tfrac{3}{2}, \pm \tfrac{3}{4}}$$

d) 5pts We find all rational zeros of g and factor as far as we can with them. Here's where a grapher would save you time. Without a grapher, you have up to 6 numbers to plug into f. With Desmos, you can find them before you make any guesses.

METHOD 1 Desmos says $g(-1)=g(3)=0$



$g(x) = 4x^4 - 16x^3 + 3x^2 + 26x + 3$

We divide by $x-(-1) = x+1$, and $x-3$

$$\begin{array}{r|rrrrr} -1 & 4 & -16 & 3 & 26 & 3 \\ & & -4 & 20 & -23 & -3 \\ \hline 3 & 4 & -20 & 23 & 3 & 0 \\ & & 12 & -24 & -3 & \\ \hline & 4 & -8 & -1 & 0 & \end{array}$$

sweet!

sweet!

The other 2 zeros are ugly. Let's see if they're rational by looking at the depressed polynomial

$4x^2 - 8x - 1$

$a = 4, b = -8, c = -1$

$b^2 - 4ac = 8^2 - 4(4)(-1)$

$= 64 + 16 = 80$

80 is not a perfect square

$8^2 < 80 < 9^2$

$64 < 80 < 81$

So the zeros of $4x^2 - 8x - 1$ are irrational and

$\boxed{x = -1, 3 \text{ are the rational zeros}}$

we factor:

$\boxed{g(x) = (x+1)(x-3)(4x^2 - 8x - 1)}$

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(e) (5pts) We find the zeros of $4x^2-8x-1$.

$a=4, b=-8, c=-1$

By part d, $b^2-4ac = 80$

Simplify $\sqrt{80}$

$$\begin{array}{r|l} 2 & 80 \\ 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ & 5 \end{array}$$

$4\sqrt{5} = \sqrt{80}$

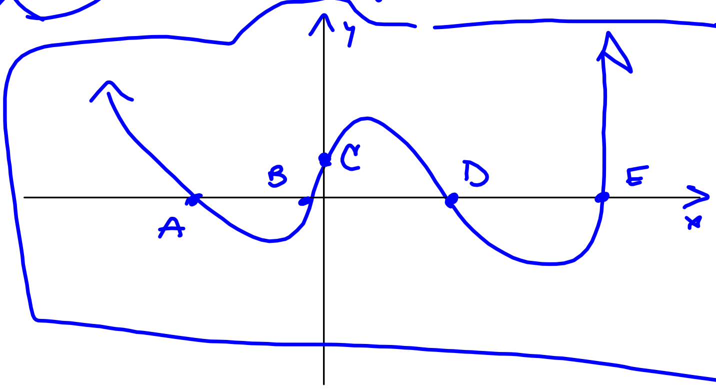
Now, $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a=4, b=-8, c=-1$

$= \dfrac{8 \pm 4\sqrt{5}}{2(4)} = \boxed{\dfrac{2 \pm \sqrt{5}}{2}}$

(f) (5pts) $\boxed{g(x) = (x+1)(x-3)\left(x - \frac{2+\sqrt{5}}{2}\right)\left(x - \frac{2-\sqrt{5}}{2}\right)}$

(g) (5pts) Sketch the graph based on previous work



$A = (-1, 0)$

$B = \left(\frac{2-\sqrt{5}}{2}, 0\right) \approx (-.12, 0)$

$C = (0, 3)$

$D = \left(\frac{2+\sqrt{5}}{2}, 0\right) \approx (2.12, 0)$

$E = (3, 0)$

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(2) $f(x) = 4x^6 - 36x^5 + 146x^4 - 340x^3 + 76x^2 + 776 + 174$

(a) 5pts we provide an end-behavior graphic:

$4x^6$

(b) 5pts we use the given fact that $x = 3-7i$ is a zero of f to split f into the product of a quadratic and a quartic

$x = 3-7i$ is a zero & all coefficients of f are real $\longrightarrow$

$x = 3+7i$ is ALSO a zero.

We split off $(x-(3-7i))(x-(3+7i))$

$= x^2 - (3+7i)x - (3-7i)x + (3-7i)(3+7i)$

$= x^2 - 3x - 7ix - 3x + 7ix + 3^2 + 7^2$

$= x^2 - 6x + 9 + 49 = x^2 - 6x + 9$. We split this factor off by dividing f by it.

$$
\begin{array}{r}
4x^4 - 16x^3 + 3x^2 + 26x + 3 \\
x^2-6x+58 \overline{\big)\, 4x^6 - 40x^5 + 331x^4 - 920x^3 + 21x^2 + 1490x + 174} \\
-\left(4x^6 - 24x^5 + 232x^4\right) \\
\hline
-16x^5 + 99x^4 - 920x^3 + 21x^2 + 1490x + 174 \\
-\left(-16x^5 + 96x^4 - 928x^3\right) \\
\hline
3x^4 + 8x^3 + 21x^2 + 1490x + 174 \\
-\left(3x^4 - 18x^3 + 174x^2\right) \\
\hline
26x^3 - 153x^2 + 1490x + 174 \\
-\left(26x^3 - 156x^2 + 1508x\right) \\
\hline
3x^2 - 18x + 174 \\
-\left(3x^2 - 18x + 174\right) \\
\hline
0!
\end{array}
$$

Sweet!

So $\boxed{f(x) = (x^2-6x+58)(4x^4-16x^3+3x^2+26x+3)}$

Wait a minute! $4x^4-16x^3+3x^2+26x+3 = g(x)$ for #1!

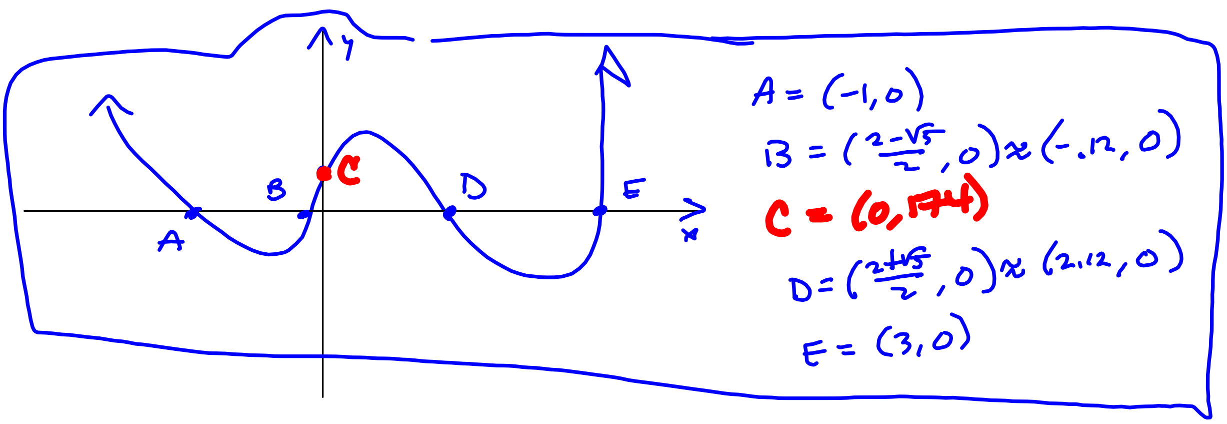
Fact: $x^2-6x+58$ has non-real zeros, and therefore contributes nothing to the x-intercepts of the graph.

$x^2-6x+58$ factor definitely makes an impact, but we don't see it at our level of analysis.

$x^2-6x+58$ has no effect on the sign pattern for f, so its sign pattern and overall appearance will be something very much like $g(x)$ ... at THIS level of analysis.

To delve deeper, we'd have to find the highs & lows, because they won't match up. **#2 has a lot more growth potential than #1.**

All you have to do to pass from #1's graph to #2's graph is change the label on the *y*-intercept.



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(3) $R(x) = \frac{3x^2+6x-24}{4x^2+27x+18} = \frac{3(x^2+2x-8)}{4x^2+24x+3x+18} = \frac{3(x+4)(x-2)}{4x(x+6)+3(x+6)}$

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 2$

$= \frac{3(x+4)(x-2)}{(x+6)(4x+3)}$

V.A.: $x=-6$, $x=-\frac{3}{4}$

x-int: $(-4,0)$, $(2,0)$

y-int: $(0, -\frac{24}{18}) = (0, -\frac{4}{3})$

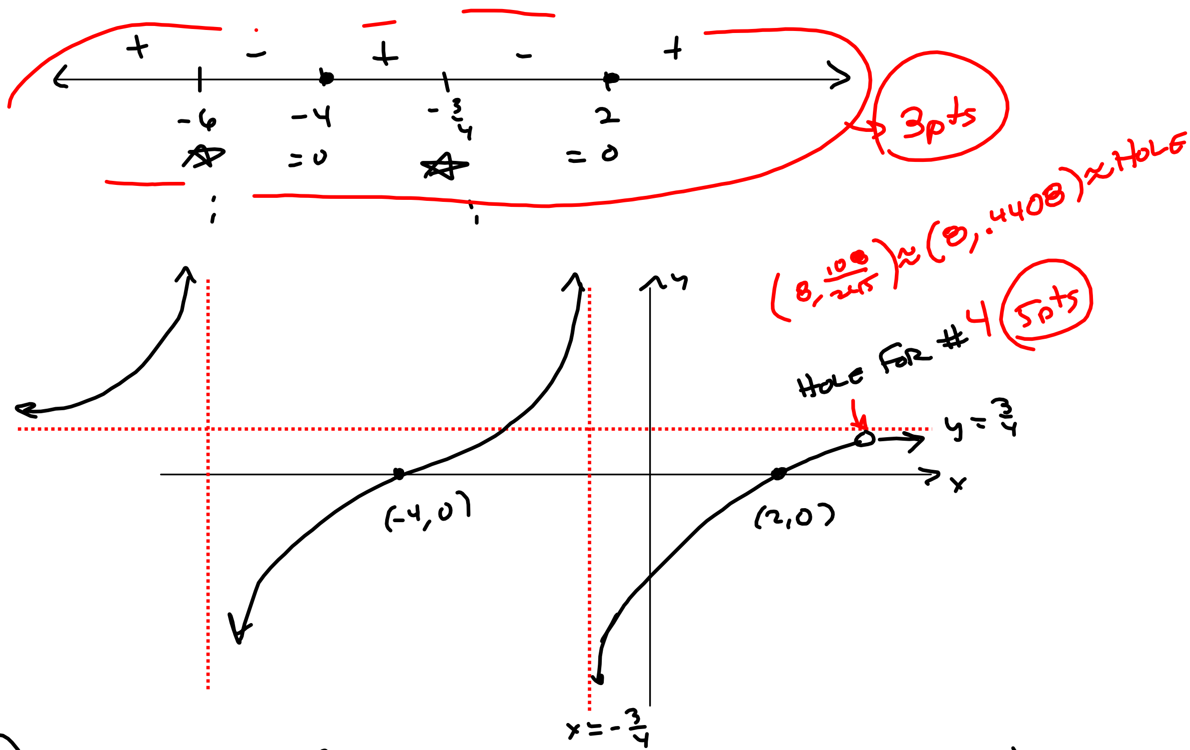
HA: $y=\frac{3}{4}$ → 5 pts

(a) $\boxed{\mathcal{D} = \mathbb{R} \setminus \{-6, -\frac{3}{4}\} =}$

$= (-\infty,-6) \cup (-6,-\frac{3}{4}) \cup (-\frac{3}{4},\infty)$

Either is OK

(b) $\boxed{\text{zeros: } x=-4, 2}$



4) $\hat{R}(x) = \dfrac{3x^3 - 18x^2 - 72x + 192}{4x^3 - 5x^2 - 198x - 144} = \dfrac{3(x^3 - 6x^2 - 24x + 64)}{}$

We know it has same zeros as R, so: $x = -4$ & $x = 2$:

$$\begin{array}{r|rrrr} -4 & 1 & -6 & -24 & 64 \\ & & -4 & 40 & -64 \\ \hline 2 & 1 & -10 & 16 & 0 \\ & & 2 & -16 & \\ \hline & 1 & -8 & 0 & \end{array}$$

$x - 8 = 0 \longrightarrow x = 8$ is zero. $\longrightarrow$

Support for #4 (Finding the hole @ $x=8$)

See Graph for #3

$(8, R(8))$ is the hole.

$$R(8) = \frac{3(8+4)(8-2)}{(8+6)(4(8)+3)} = \frac{3(\cancel{12})(6)}{\cancel{14}(35)} = \frac{3(36)}{7(35)} = \frac{108}{245} \approx$$

$\approx$ 0.4408163265

(5) $T(x) = \dfrac{3x^3 - 18x^2 - 72x + 192}{4x^2 + 27x + 18} = \dfrac{3(x+4)(x-2)(x-8)}{(x+6)(4x+3)}$

(a) (5pts) Long division for slant asymptote:

$$\begin{array}{r} \frac{3}{4}x - \frac{153}{16} \\ 4x^2 + 27x + 18 \overline{\smash{)}\, 3x^3 - 18x^2 - 72x + 192} \\ -\left(3x^3 + \frac{81}{4}x^2\right) \sim\sim \\ \hline -\frac{153}{4}x^2 + \sim\sim \end{array}$$

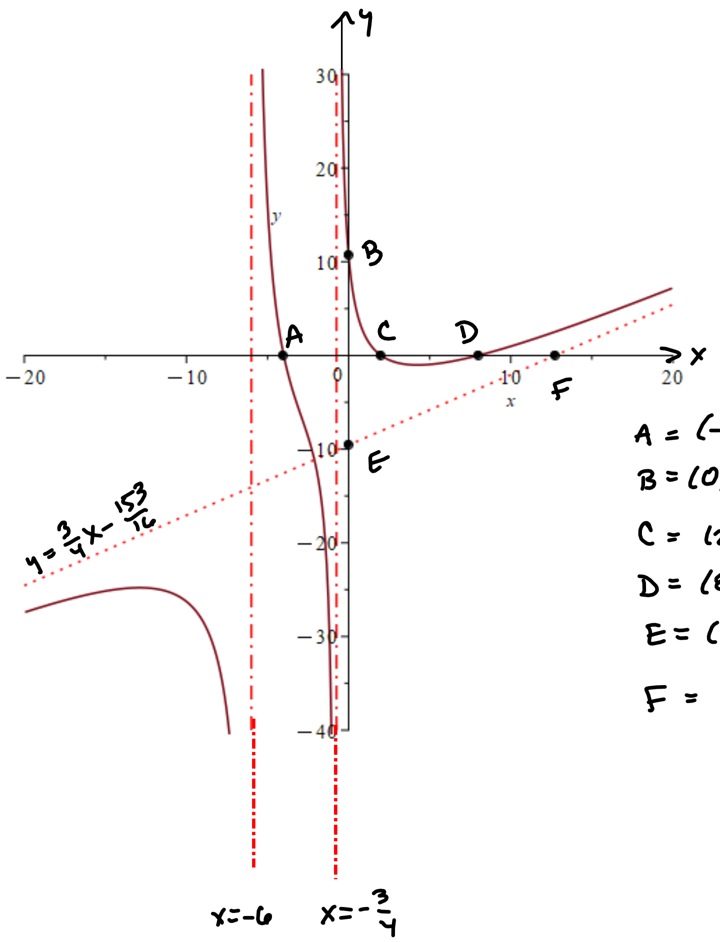
$\dfrac{3x^3}{4x^2} = \dfrac{3}{4}x$

$-18 - \dfrac{81}{4} = -\dfrac{72 - 81}{4} = \dfrac{-153}{4}$

$\dfrac{-\frac{153}{4}x^2}{4x^2} = -\dfrac{153}{16}$

$\boxed{y = \frac{3}{4}x - \frac{153}{16} \text{ is SA.}}$

Generous w/ partial credit



$A = (-4, 0)$

$B = (0, \frac{32}{3}) = (0, 10.\overline{6})$

$C = (2, 0)$

$D = (8, 0)$

$E = (0, -\frac{153}{16}) = (0, -9.5625)$

$F = (\frac{51}{4}, 0) = (12.75, 0)$