1340 WEEK 6 WRITTEN SOLUTIONS H. MILLS

1. Let $s(x) = x^2 - 5x$.

(a) 5pts The net change in s from $x=1$ to $x=5$ is

$$s(5)-s(1) = 5^2-5(5)-(1^2-5(1)) = -1+5 = \boxed{4 = s(5)-s(1)}$$

(b) 5pts The average rate of change from $x=1$ to $x=5$ is

$$\boxed{\frac{s(5)-s(1)}{5-1} = \frac{4}{4} = 1}$$

(c) 5pts We simplify the difference quotient

$$\frac{s(a+h)-s(a)}{h} = \frac{(a+h)^2-5(a+h)-(a^2-5a)}{h}$$

1pt

$$= \frac{a^2+2ah+h^2-5a-5h-a^2+5a}{h}$$

$$= \frac{2ah+h^2-5h}{h} = \frac{h(2a+h-5)}{h} = \boxed{2a+h-5} \quad (h \neq 0)$$

= Difference quotient.

2. 5pts $f(x) = \frac{1}{x-3}$ $\longrightarrow$

$$\frac{f(a+h)-f(a)}{h} = \frac{\frac{1}{a+h-3} - \frac{1}{a-3}}{h} = \frac{\left(\frac{1}{a+h-3}\right)\left(\frac{a-3}{a-3}\right) - \left(\frac{1}{a-3}\right)\left(\frac{a+h-3}{a+h-3}\right)}{h}$$

$$= \frac{\frac{a-3-(a+h-3)}{(a+h-3)(a-3)}}{h} = \frac{1}{h}\left[\frac{a-3-a-h+3}{(a+h-3)(a-3)}\right]$$

$$= \frac{1}{h}\left[\frac{-h}{(a+h-3)(a-3)}\right] = \boxed{\frac{-1}{(a+h-3)(a-3)}} \quad (h \neq 0)$$

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3 5 pts The table gives the population P in a small town from 2002-2020. Figures are for January 1st each year

Year	2002	2004	2006	2008	2010	2012	2014	2016
Pop	3220	3645	4357	4869	5871	6375	6288	5318

Year	2018	2020
Pop	4921	4636

The average rate of change from 2010-2014 is

$$\frac{P(2014)-P(2010)}{2014-2010} = \frac{6288-5871}{4} = \boxed{\frac{417}{4}}$$

4 5 pts The avg. rate of change for $f(x)=2\sqrt{x}$ between $x=a+h$ & $x=a$ is

$$\frac{f(a+h)-f(a)}{h} = \frac{2\sqrt{a+h}-2\sqrt{a}}{h}$$

$$= \left(\frac{2\sqrt{a+h}-2\sqrt{a}}{h}\right)\left(\frac{2\sqrt{a+h}+2\sqrt{a}}{2\sqrt{a+h}-2\sqrt{a}}\right) = \frac{4(a+h)-4a}{h(2\sqrt{a+h}+2\sqrt{a})}$$

$$= \frac{4a+4h-4a}{h(2\sqrt{a+h}+2\sqrt{a})} = \frac{4h}{h(2\sqrt{a+h}+2\sqrt{a})} = \frac{4}{2\sqrt{a+h}+2\sqrt{a}}$$

$$= \frac{4}{2(\sqrt{a+h}+\sqrt{a})} = \boxed{\frac{2}{\sqrt{a+h}+\sqrt{a}}}$$

1340 CONTEXT - .5 MILLS

(5) A pond is filling with H_2O at rate of $10 \frac{\text{gallons}}{\text{min}}$. or words to this effect.

Initially, there are 500 gl of H_2O in the pond.

(a) (5pts) We model V = volume of H_2O in the pond (in gallons) as a function of t = time (in minutes)

$$V = \text{initial} + \left(\begin{matrix}\text{rate of}\\ \text{filling}\end{matrix}\right)(\text{time})$$

$$= \boxed{500 + 10t}$$

(b) (5pts) If the pond holds 2000 gallons, how long will it take to fill it?

We solve $V(t) = 2000 \longrightarrow$

$500 + 10t = 2000 \longrightarrow$

$10t = 1500 \longrightarrow$

$\boxed{t = 150 \text{ minutes}}$

(d) (5pts) Assume you stop when the pond is full. What's the domain of $V(t)$?

$$\boxed{\begin{matrix}\mathcal{D}(V) = [0, 150]. \\ \mathcal{R}(V) = [500, 2000]\end{matrix}}$$

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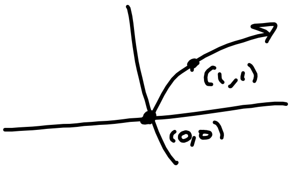
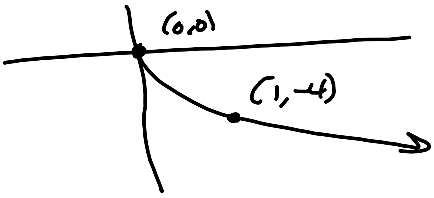
We graph $g(x) = -4\sqrt{-5x-20} + 64$

by transforming the graph of $f(x) = \sqrt{x}$:

(0) $f(x) = \sqrt{x}$



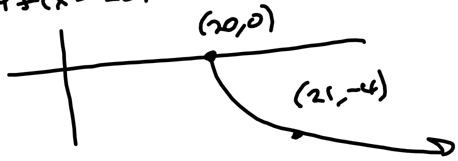
(1) $-4f(x) = -4\sqrt{x}$ $\quad y \longmapsto -4y$



Scratch: $g(x) = -4\sqrt{-5(x+4)} + 64$

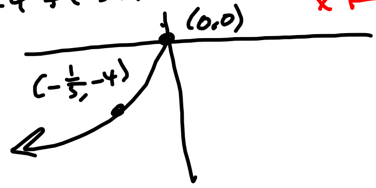
(3) METHOD 1:

$-4f(x-20)$ $\quad x \longmapsto x+20$



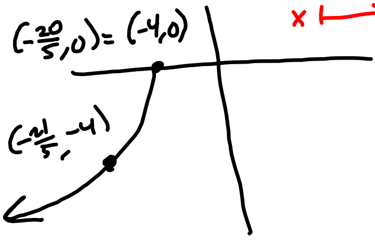
(2) METHOD 2:

$-4f(-5x) = -4\sqrt{-5x}$ $\quad x \longmapsto -\frac{1}{5}x$



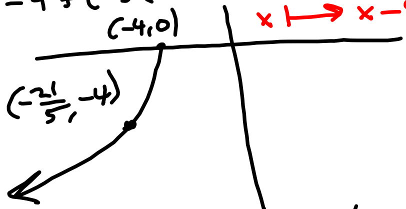
(3) METHOD 1

$-4f(-5x-20)$ $\quad x \longmapsto -\frac{1}{5}x$



(3) METHOD 2

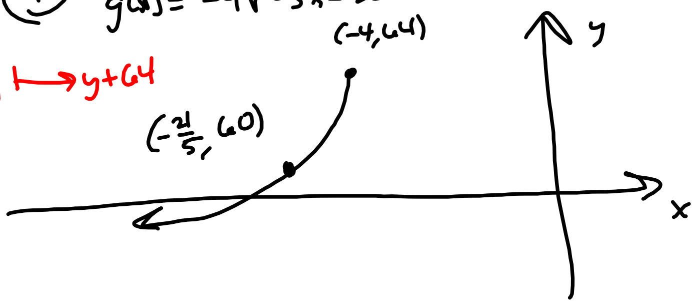
$-4f(-5(x+4)) = -4\sqrt{-5(x+4)}$ $\quad x \longmapsto x-4$



$-\frac{1}{5} - 4 = -\frac{1}{5} - \frac{20}{5} = -\frac{21}{5}$

(4) $g(x) = -4\sqrt{-5x-20} + 64 = -4\sqrt{-5(x+4)} + 64$

$y \longmapsto y+64$



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7 5pts Bonus

$g(x)$ has no y-int.

$g(x) = 0 \implies$

$-4\sqrt{-5x-20} + 64 = 0$

$\implies -4\sqrt{-5x-20} = -64$

$\implies \sqrt{-5x-20} = \frac{-64}{-4} = 16$

$\implies -5x-20 = 16^2 = 256$

$\implies -5x = 236 \to 276$

$\implies x = \frac{236}{-5} \implies \boxed{\left(-\frac{236}{5}, 0\right) \text{ is x-int}}$ $236 \to 276$